Semi-Markov Credit Risk Modeling for a Portfolio of Consumer Loans in the Kenyan Banking Industry

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Abstract

Based on simulations of implied values for credit worthiness over a period of 5 years for 1000 consumers, we establish a case for the semi-markov models as a proxy for internal credit risk models for a portfolio of consumer loans. With ample calibration, we prove the robustness of the semi-markov models in forecasting probabilities of default and loss given default. With a view of credit risk as a reliability problem, we generate credit risk indicators as qualifications of adequacy of a loan portfolio. This informs prospective holding of capital based on forecast delinquencies as opposed to the current retrospective practice that relies on the trigger event of default. We use Monte-Carlo simulation techniques to generate consumer ratings and adopt this to the Merton model to derive the initial probability transition matrix. Initial consumer rating is in accordance with industry practice using a credit score sheet backed by the logit model. The banking credit function could espouse the study results to fulfill regulatory credit risk capital requirements for consumer loans in line with the Central Bank of Kenya Prudential Risk Guidelines or banks in other jurisdictions compliant with the Basel banking framework

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1. Introduction

This study seeks to respond to the need for better credit risk modeling for a portfolio of consumer loans in the Kenyan banking sector. To do this, the study will briefly elucidate the credit risk models currently in use by Kenyan bankers and seek to modify them through adapting the Semi-Markov approach to modeling credit risk. The study seeks to empirically establish a case for the adoption of the Semi-Markov credit risk framework in modeling through modeling credit rating migration patterns and establishing how the modeling of credit risk influences the solvency and capital adequacy of banks in Kenya in light of the Basel solvency requirements

Credit risk[3] management has been noted as the single most important role of a banks' management owing to their nature of business. Credit creation is the main income generating activity of banks, (Kargi, 2011). However, the downside to credit creation is the inherent credit risk that the bank is exposed to. Increasing variety in the types of counterparties and the expansion in the variety of the forms of obligations has necessitated the jump of credit risk management to the forefront of risk management activities carried out by firms in the financial service industry, (Ali Fatemi, 2006).

The financial crisis of 2008-2009 revealed but one thing. An improper estimation of credit risk can lead to dramatic effects on the world's economy, (Munnixl, 2011). A better estimation of credit risk is therefore important, a phenomenon addressed through credit risk modeling, (Bluhm, 2002), (Duffie, 2003), (Giesecke, 2004), (Lando, 2004), (McNeil, 2005). (Munnixl, 2011), distinguish two fundamentally different approaches to modeling credit risk: the structural and the reduced form models. Structural models have a long history, going back to the work of Black and Scholes (1973) and Merton (1974). Reduced form models attempt to capture the dependence of default and recovery rates on macroeconomic risk.

The Kenyan banking sector has experienced a boom in the last few years marked with growth in net assets, branch network, regional expansion, growth in level of loans issued and an increase in the level of depositors, which is not atypical to the pre-financial crisis banking sector in the developed economies such as the USA.

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[3] ***Credit Risk***: : (Chen & Pan, 2012), defines this as the degree of value fluctuations in debt instruments due to changes in the underlying credit quality of borrowers and counterparties. It is the current or prospective risk to earnings and capital arising from an obligor's failure to meet the terms of any contract with the bank or if an obligor otherwise fails to perform as agreed, (CBK, 2013).

(CBK, 2013), notes in its March, 2013 Credit Report Survey that credit risk is the single largest factor affecting the soundness of financial institutions and the financial system as a whole and lending is the principal business activity for most banks. A view re-echoed by (Kargi, 2011). (CBK, 2013), notes that the total percentage of loans to total assets for the period ended 31st March, 2013 was 57%, which prima facie, is good for business, however poses a potential threat to the industry if more loans became non-performing. Thus the need to effectively manage credit risk given that it's inherent to the business of a bank. Credit risk modeling underpins this management.

With the newly issued risk guidelines, (CBK, 2013), the Central Bank of Kenya identifies internal rating models for banks as being key for effective credit risk management. This study's modeling of credit risk will therefore be a proxy of what a plausible portfolio of consumer loans' internal rating model, for credit risk management, could be. According to (Jacques Jansen, 2007), the credit risk problem can be seen as a reliability problem. In light of this, the rating process, carried out by a rating agency, gives a reliability degree of a firm bond. Moreover, the default state can be seen as a down state and an absorbing state. It is within this framework that Semi-Markov credit risk models become handy. (Limnios N, 2000), specifies a critical application of Semi-Markov processes as being in reliability of mechanical systems. With the hypothesis that the next transition only depends on the immediate last one, this problem falls within the Markov processes framework. However, (Limnios N, 2000) points out that, for a mechanical system, transition between two states usually happens after a random duration, not necessarily discrete time consequently, making the Semi-Markov environment a better fit than the Markov one. The study's results are of paramount importance to commercial banks, whose main business is credit creation, the regulator, CBK, as well as other corporate lenders, for instance corporate bond issuers.

2. Literature Review

The objective of this section is to articulate the conceptual foundations of the study. Section 2.1 surveys existing theoretical and empirical literature on the need for effective credit risk management. Section 2.2 discusses the current credit risk models in use within the Kenyan jurisdiction. Section 2.3 explores the need for better credit risk modeling techniques and establishes a case for the Semi-Markov credit risk models.

2.1.The Need for Effective Credit Risk Management

(CBK, 2013), annotates that credit risk is the current or prospective risk to earnings and capital arising from an Obligor's failure to meet the terms of any contract with a bank or if an obligor otherwise fails to perform as agreed. It further emphasizes that a bank's assets are largely comprised of loans making the management of credit risk extremely important. (Njanike, 2009), establishes that poor credit risk management was the chief reason that resulted in the demise of over ten banks in Zimbabwe during the 2003/2004 bank crisis in the southern African nation. The same can be said of the banking crisis in Kenya in the 1980s and in Spain in the 1990s.

While agreeing with (Njanike, 2009), (C.Marrison, 2002) articulates that the main activity of bank management is not mobilization of deposits and issuance of credit, however, risk management is paramount. He outlines that effective credit risk management reduces the risk of customer default. Moreover, they both add that the competitive advantage of a bank is dependent on its capability to handle credit valuably.

Conducting a similar study in Spain, (De Juan, 2008), argues that banking failures were caused by poor credit risk management which was aggravated by the concentration of the loan portfolio in the group in which the bank itself belonged. Fredrick (2012), while using the CAMEL[4] model as a proxy for credit risk established that credit risk management had an impact on the financial performance of commercial banks. He cites that the goal of credit risk management is to maximize a bank's risk adjusted rate of return through maintenance of credit risk exposure within acceptable limits. He articulates the need for credit risk management to be at the center of banks operations and cries foul at the lack thereof

2.2. Current Models and the Case for Semi-Markov Models

(CBK, 2010), points to the application of the CAMEL rating system, an international benchmark, by the Central bank of Kenya in analyzing the soundness of financial institutions. (Fredrick, 2012), recognizes that numerous prior studies have examined the efficacy of the CAMEL ratings and they generally conclude that publicly available data combined with regulatory CAMEL ratings can identify and/or predict problem or failed banks. However, in a case study for the American

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[4] **CAMEL:** refers to an acronym for Capital Adequacy, Asset Quality, Management, Liquidity and Sensitivity to Market Risk. The model identifies and measures the different aspects of a financial institution as stipulated in the acronym, aggregates them to obtain a single value which forms the basis of a rating, (CBK, 2012)

International Assurance-Vietnam, (AIA), it was established that the CAMEL model overlooks the provision as well as allowance for loan loss ratios. Heuristics modeling has also been identified as a key component of most Kenyan banks' credit risk models. However, (Kithinji, 2010), alludes to the fact that subjective decision-making by the management of banks may lead to extending credit to business enterprises they own or with which they are affiliated, to personal friends, to persons with a reputation for non-financial acumen or to meet a personal agenda, such as cultivating special relationship with celebrities or well-connected individuals.

(Valle, 2013), identifies three broad methodologies to model credit risk; structural form models (SFM), reduced form models (RFM) and factor models (FM). SFM are based upon the Black and Scholes theory for option pricing and the Merton model. (Linda, 2004), on the other hand identifies two broad methodologies to modeling credit risk, an options-theoretic structural approach pioneered by Merton (1974) and a reduced form approach utilizing intensity-based models to estimate stochastic hazard rates. However, they both concur that the structural approach models the economic process of default, whereas reduced form models decompose risky debt prices in order to estimate the random intensity process underlying default. Consequently, RFM mainly focuses on the accuracy of the probability of default (PD), such that it is more important than an intuitive economical interpretation.

Under the Merton's structural model, default occurs after ample early warning, (Linda, 2004). Consequently, default occurs after a gradual descent in the assigned behavioral value for consumers or asset values for firms; to the default point. This implies that the PD steadily approaches zero as the time to maturity nears, (Valle, 2013). More realistic credit spreads are obtained from reduced form models (RFM) or intensity-based models, (Linda, 2004). This holds since; whereas structural models view default as the outcome of a gradual process of deterioration in asset values/behavioral value, intensity-based models view default as a sudden, unexpected event, thereby generating PD estimates that are more consistent with empirical observations, (Linda, 2004). This study uses a reduced form model for credit risk.

(Valle, 2013) notes that RFM can be classified as an individual level reduced form model (ILRFM) and portfolio reduced form model (PRFM).He further points out that the former is based on a credit scoring system (two-state or multistate), and the latter assumes an intensity jump process. The study takes the PRFM approach. PRFMs are reported to perform better in capturing the properties of credit risk, (Cheng, C, & Zhang, 2009). Within the PRFMs, Discrete Time Markov Processes (DTMP) and Continuous Time Markov Processes (CTMP) have been used in empirical studies to model credit risk spread as two components; PD and LGD, (Valle, 2013). The suitability of Markov processes in modeling credit risk has been challenged with notable problems being; the underestimation of migration probabilities by DTMPs, dependence of the current state where the current state may depend on various previous states assigned to a firm or consumer and not only in the previous one, the waiting time in a state; among others, (Linda, 2004). The Semi-Markov processes have been postulated as a solution to some of the DTMPs and CTMPs weaknesses, (Duffie, 2003) (G. D'Amico J. J., 2005), (G.D'Amico, 2009), (Monteiro, Smirnov, & Lucas, 2006), (Banachewicz & Lucas, 2007). This study models credit risk within the Semi-Markov framework.

2.3.The Case for Better Credit Risk Modeling Techniques

(Chen & Pan, 2012), indicate that the new Basel Capital Accord explicitly places the onus on banks to adopt sound internal credit risk management practices to assess their capital adequacy requirements. The Central Bank of Kenya (CBK) adopted the Risk Based Supervisory (RBS) approach in 2004 in cognizance of the limitations inherent in the traditional approach which prescribed a common supervisory approach to all institutions irrespective of differences in business activities conducted and risk appetites adopted, CBK, (2013). In managing credit risk, the CBK recommends that banks must receive sufficient information to enable a comprehensive assessment of the true risk profile of the borrower or counterparty. At a minimum, among the factors the bank should consider is the borrower's credit rating/report from a licensed credit reference bureau, CBK, (Risk Management Guidelines, 2013).

However, the ratings are bound to change, a factor that raises the credit risk to the bank, and which the CBK risk management guidelines don't provide for. The CBK guidelines however note that an important tool in monitoring the quality of individual credits, as well as the total portfolio, is the use of an internal risk rating system which will allow more accurate determination of the overall characteristics of the credit portfolio, concentrations, problem credits, and the adequacy of loan loss reserves, CBK, (Risk Management Guidelines, 2013). However, no explicit mention of the working and parameterization or nature of such internal models is mentioned.

In its prudential guidelines, the CBK stipulates that capital requirements for a specific institution may increase or decrease depending upon its risk profile. An institution's minimum capital requirement (MCR) is calculated by dividing its Core and Total Capital by the sum of the value of its Risk-Weighted Assets for Credit Risk, Market Risk and Operational Risk, to arrive at the minimum Tier One and Regulatory capital adequacy ratios respectively, (CBK/PG/03, 2013).

Under (CBK/PG/03, 2013), the Internal Capital Assessment Adequacy Planning (ICAAP) requires that institutions ensure that they at all times plan their capital ahead for a minimum of three years in order to establish and maintain on an ongoing basis an adequate level of capital, which would include an appropriate buffer, as determined by the board, above the regulatory required minimum capital. This requires institutions to have in place an appropriate and proportionate capital management strategy. Hence the need to monitor exposure to different risks, especially credit risk . Of interest for this study is the lack thereof of robust models for forecasting capital requirements especially for credit risk purposes; given the nature of banks business; credit creation, (Kargi, 2011) The CBK requires that an institution's Capital Adequacy Ratio must be at least 12%, of which 8% is Core Capital. In addition to the above minimum capital adequacy ratios of 8% and 12%, institutions are required to hold a capital conservation buffer of 2.5% over and above these minimum ratios to enable the institutions to withstand future periods of stress, (CBK/PG/03, 2013).

(CBK/PG/04, 2013), classifies loans, the major asset of banking institutions, into five categories: normal, watch, substandard, doubtful and loss. Classification is based on the number of days the loan is past its due repayment date. (CBK/PG/04, 2013), portends that the CBK will conduct an on-site examination providing a list of reclassified accounts, some of which will be downgraded from categories earlier classified by the institution. No account from this list will be upgraded by the institution without sufficient justification. Consequently, any classification should be in line with that of the Regulator. Based on the classification, different amounts of provisioning are to be maintained. However, a prudent practice is to provide for more, in order to limit the downside risk of excessive exposure to non-performing loans. Incisive as this might be, Could internal models aligned to the regulators requirements be able to capture exposure levels at different periods? Which would then inform capital adequacy and hence level of provisions made by a bank?

The strict regulation may explain the laxity in research in the area of credit risk modeling within the African jurisdiction. The non-multifariousness of most internal models due to the heavy reliance on regulatory provisions could explain the little or no use of intricate credit risk models. However, even in light of regulation, the need to model credit risk, with its being the paramount risk that influences the capital levels of banks, is palpable. That less has been done is also ostensible.

A Semi-Markov framework will be adopted in modeling credit risk for a portfolio of consumer loans, as a proxy for an internal rating model for banks. For this study, initial rating of consumers is done through an initial score sheet that is backed by a logit model.

3. Empirical Model

This study seeks to empirically establish a case for the adoption of the Semi-Markov credit risk framework in through modeling credit rating migration patterns and establishing how the modeling of credit risk influences the solvency and capital adequacy of banks in Kenya in light of the Basel solvency requirements. (Ross, 2007) Defines a Semi-Markov process by supposing that a process can be in any one of N states $1, 2, \ldots, \mathrm{N}$, and that each time it enters state i it remains there for a random amount of time having mean μ_{i} and then makes a transition into state j with probability p_{ij} Such a process is called a Semi-Markov process. With the view of the credit risk problem as a reliability problem, the process $\mathrm{Z} = \{\mathrm{Z}(\mathrm{t}), \mathrm{t} \geq 0\}$ is assumed to be a Semi-Markov process with kernel $\mathbb{Q}$. It describes the evolution of a consumer from one credit rating to another in time $\mathrm{t} \geq 0$. The main reliability indicators are identified as:

The availability function defined as:

$\mathrm{A_i}\ (\mathrm{t}) = \mathrm{P}\ \left(\mathrm{Z}\ (\mathrm{t}) \epsilon \mathrm{U}\ |\mathrm{Z}(0) = \sum_{\mathrm{j} \epsilon \mathrm{U}} \emptyset_{\mathrm{ij}}(\mathrm{t})\ , \mathrm{i} \in \mathrm{I}\ \mathrm{i}\right), \mathrm{i} \in \mathrm{I}$ .... (i)

$\emptyset_{\mathrm{ij}}s$: The transition probability functions for the Z process.

The reliability function giving the probability that the system is always working in the time interval$[0, \mathrm{t}]$:

$\mathrm{R_i}\ (\mathrm{t}) = \mathrm{P}\ (\mathrm{Z}\ (\mathrm{t}) \epsilon \mathrm{U}\ |\mathrm{Z}(0) = \mathrm{i}), \mathrm{i} \in \mathrm{U}$ .... (ii)

The maintainability function giving the probability that the system is down at time 0 and that the system will leave the set D within the time t,

$\mathrm{M}(\mathrm{t}) = 1 - \mathrm{P}(\mathrm{Z}(\mathrm{u}) \epsilon \mathrm{D}, \forall \mathrm{u} \in (0, \mathrm{t}])$... .... (iii)

(Jacques & Raimondo, 2007), delineate migration as the successive movement of credit ratings, which are estimates of the probability of default. They use the Standards and Poor’s rating model to examine a firm's rating. This model has eight kinds of ratings, (Radu Neagu, 2009). Where the states are in decreasing order depending on the reliability of their debts and the default state D.

(Jacques & Raimondo, 2007), stipulate that in order to apply reliability models in a credit risk environment, based on the S&P classification, then the first seven states should be considered as 'good' states and the D state; the default state, the only 'bad' state and apply a Semi-Markov reliability model to the credit risk problem. State D is an absorbing state. They argue that in this case, only the $R(t)$ function is useful in this environment citing functions $A(t)$and $M(t)$ as meaningless. This argument was adopted in this study. $R_i(t)$ gives the probability that the system was always working up to the time t given that the system was in working state I at time 0

(G.D'Amico, 2009) State that in order to consider dependence of the rating evaluation from the lapse of time in which a firm remains in the same rating a homogeneous Semi-Markov process is introduced. Both (Jacques & Raimondo, 2007)and (D'Amico, Di Biase, Janssen, & Manca, 2009) identify the following reliability indicators as key parts of the model:

$\emptyset_{ij}(t)$ and $\emptyset_{ij}(s,t)$ which represent respectively the probabilities of being in the state j after a time t starting in the state i at time 0 in the homogeneous case and starting at time s in the state i in the non-homogeneous case. The Semi-Markov environment takes into account the different probabilities of state changes during the permanence of the system in the same state.

$R_i(t) = \sum_{j\epsilon U} \emptyset_{ij}(t)$ and $R_i(s,t) = \sum_{j\epsilon U} \emptyset_{ij}(s,t)$ which represent respectively the probabilities that the system never goes into default state in a time t in the homogeneous case and from time s to time t in the non-homogeneous case.

Both (D'Amico, Di Biase, Janssen, & Manca, 2009), (G. D'Amico G. D., 2010) and (Jacques & Raimondo, 2007) again agree on the following possible indicators useful that can be derived from the model.

$\varphi_{ij}(t) = P[Z(t) = j \mid Z(0) = i]$: The probability of a consumer being in the rank value j after a time t starting with the rank value i at time 0 which enables the accounting for the different transition probabilities during the permanence of the firm in the same rating.

$1 - H_i(t) = 1 - \sum_{j=1}^{m} Q_{ij}(t)$. This is the stay on probability function representing the probability that in a time interval t there was no new rating evaluation for the consumer who started with rank i at the starting time.

$\varphi_{iD} = \frac{p_{iD} - Q_{iD}(t)}{1 - H_i(t)}$ Which gives the probability that next transition of a consumer who entered the rank value i at time 0 and stayed on in the same rank till time t, will be in the default state.

$R_i(t) = P[Z(h) \epsilon\, U\ \forall h = 0,1, \dots, t | Z(0) = i] = \sum_{j \epsilon U} \emptyset_{ij}^{r}(t)$: Which is the reliability function. It represents the probability that a consumer will never go into the default state in a time t.

These indicators are adopted for the study.

The application of the formulated Semi-Markov migration model is dependent on data availed from existing ratings. Credit rating data is used to generate the initial transition matrix P. With inadequate rating data available, and the confidential nature of consumer loaners' information, the need to rate using a standard rating model for the different loaners, for homogeneity in rating in terms of variants, was apparent.

This study adopted a logistic regression model to establish the initial rating of a consumer, which was in line with the current practice at majority of Kenyan Banks. If x denotes the number of factors (their number being K) and b the weights attached to them, the score obtained on scoring instance i is:

$$Score_i = b_1 x_{i1} + b_2 x_{i2} \dots + b_K x_{iK} = \boldsymbol{b}'\boldsymbol{x_i} \quad \text{(iv)}$$

Where $\boldsymbol{b}$ and $\boldsymbol{x}$ are column vectors such that;

$$\boldsymbol{x_i} = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ . \\ x_{iK} \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ . \\ b_K \end{bmatrix}$$

$$Prob\ Default_i = F(Score_i)$$

This study then defines F as the logistic distribution function $\Lambda(z)$ defined as

$$\Lambda(z) = \exp(z) / 1 + \exp(z) \,.$$

Applying this to the above result:

$$Prob\ Default_i = \Lambda(Score_i) = \frac{\exp(\boldsymbol{b'x_i})}{1+\exp(\boldsymbol{b'x_i})} = \frac{1}{1+\exp(-\boldsymbol{b'x_i})} \quad \text{(v)}$$

Andrade and Thomas (2005) suggest that using a consumer's behavioral score as a surrogate for credit worthiness of the borrower, one can adopt corporate structural models, the Merton model being most notable, to model for consumer credit risk. Consequently, consumers are assigned an initial behavioral value commensurate with the attained score. Subsequent rating is done using the Merton model for simulated values of the behavioral scores. The assumed period for the simulations is the past 5 years

In the Merton model, (Merton, 1974) the value V of the firm is modeled with a Black and Scholes stochastic differential equation with trend u and instantaneous volatility, (Jacques & Raimondo, 2007).

$$V(t) = V_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W(t)}, \quad \text{(v)}$$

V_0 being the value of the firm at time 0 and $W = (W(t) | t \epsilon [0, T])$ a standard Brownian motion.

If $V_i(t)$ corresponds to the behavioral score of consumer i at time t, as postulated by (Andrade & Thomas, 2004), $V_i(t)$ satisfies:

$$d\, V_i(t) - \mu_i + \sigma_i dW, \quad \text{(vi)}$$

μ_i is the drift of the process, corresponding to a natural drift in credit worthiness caused in part by the account and the customer ageing and so improving. $\sigma_i dW$ is a Brownian motion describing the natural variation in behavioral score. This study sought to rate consumers using the Merton model in light of the classification of loans (CBK, CBK/PG/04, 2013).

For this study;

$I = \{AAA, AA, A, BBB, BB, B, CCC, D\}$. Consequently; $U = \{AAA, AA, A, BBB, BB, B, CCC\}$ and $D = \{D\}$. The CBK provides the following loan classification based on the number of days the loan is past its due repayment date:

$N = \text{Normal}, W = \text{Watch}, SS = \text{Sub} - \text{Standard}, D = \text{Doubtful and } L = \text{Loss}$

To link the state space I with the current loan classification in the Kenyan Banking industry, the following events are identified: $N = \{AAA, AA, A\}$, $W = \{BBB, B\}$, $SS = \{B, CCC\}$, $D = L = \{D\}$

4. Findings and Analysis

Our empirical analysis establish a case for the adoption of the semi-markov modeling of credit risk for a portfolio level consumer loans, as a plausible internal credit rating model for the Kenyan banking industry. Table I presents the generated initial transition matrix based on the simulations on 1000 consumers over 5 years.

Table I: Initial Transition matrix P

P	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.93129	0.06044	0.00504	0.00148	0.00164	0.00009	0.00000	0.00001
AA	0.00464	0.94420	0.04326	0.00519	0.00100	0.00165	0.00002	0.00005
A	0.00051	0.01505	0.94403	0.02950	0.00697	0.00330	0.00004	0.00060
BBB	0.00030	0.00295	0.03704	0.90384	0.04110	0.00976	0.00105	0.00397
BB	0.00023	0.00148	0.00572	0.04727	0.85624	0.05887	0.00908	0.02111
B	0.00000	0.00096	0.00195	0.00351	0.03377	0.89002	0.02404	0.04575
CCC	0.00000	0.00004	0.00474	0.00535	0.01258	0.03479	0.85292	0.08958
D	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

Table II and III represent the transition probabilities obtained by solving the evolution equation for some times, in the homogeneous case. The transition probabilities were generated from the initial transition matrix P, at different times for$(0 \leq \mathrm{t} \leq 12)$. Each $\emptyset_{ij}(t)$ represents the probability of a consumer being in the rank value j after a time t starting with the rank value i at time 0

Table II: Probabilities $\emptyset_{ij}(3)$

$\emptyset_{ij}(3)$	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.80851	0.15974	0.02087	0.00532	0.00441	0.00087	0.00006	0.00019
AA	0.01231	0.84446	0.11640	0.01706	0.00408	0.00491	0.00021	0.00059
A	0.00157	0.04065	0.84637	0.07682	0.02064	0.01039	0.00059	0.00297
BBB	0.00088	0.00938	0.09609	0.74676	0.09736	0.03050	0.00410	0.01496
BB	0.00062	0.00444	0.01931	0.11127	0.63849	0.13706	0.02376	0.06505
B	0.00004	0.00269	0.00630	0.01326	0.07863	0.71263	0.05566	0.13079
CCC	0.00002	0.00049	0.01245	0.01473	0.03135	0.08151	0.62299	0.23646
D	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

Table III: Probabilities $\emptyset_{ij}(7)$

$\emptyset_{ij}(7)$	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.61190	0.29068	0.06603	0.01617	0.00926	0.00404	0.00043	0.00144
AA	0.02260	0.68420	0.22039	0.04332	0.01303	0.01172	0.00110	0.00370
A	0.00371	0.07754	0.69613	0.13727	0.04458	0.02537	0.00297	0.01245
BBB	0.00195	0.02281	0.17073	0.53262	0.14630	0.06744	0.01157	0.04663
BB	0.00122	0.01037	0.04801	0.16517	0.38227	0.19812	0.04113	0.15372
B	0.00022	0.00564	0.01617	0.03447	0.11364	0.47548	0.07867	0.27572

CCC	0.00012	0.00221	0.02311	0.02862	0.05080	0.11805	0.33879	0.43830
D	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

For the homogeneous case, the following transition probabilities were generated from the initial transition matrix P and subsequent transition matrices i.e. $P^n; 1 \leq n \leq 12$, for each $\emptyset_{\mathrm{ij}}(\mathrm{t})$ at different times t; $(1 \leq \mathrm{t} \leq 12)$, as presented in Tables IV and V. Each $\varphi_{\mathrm{ij}}(t)$ the probability of a consumer being in the rank value j after a time t starting with the rank value i at time 1 which enables the accounting for the different transition probabilities during the permanence of the firm in the same rating

Table IV: Probabilities $\boldsymbol{\varphi_{\mathrm{ij}}(3)}$

$\boldsymbol{\varphi_{\mathrm{ij}}(3)}$	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**D**
AAA	0.65325	0.12907	0.01686	0.00430	0.00357	0.00070	0.00005	0.00015
AA	0.01038	0.71160	0.09809	0.01438	0.00343	0.00414	0.00018	0.00050
A	0.00133	0.03427	0.71350	0.06476	0.01740	0.00876	0.00050	0.00250
BBB	0.00065	0.00695	0.07122	0.55347	0.07216	0.02261	0.00304	0.01109
BB	0.00039	0.00281	0.01219	0.07024	0.40304	0.08652	0.01500	0.04106
B	0.00003	0.00190	0.00446	0.00938	0.05564	0.50423	0.03939	0.09254
CCC	0.00001	0.00030	0.00774	0.00915	0.01948	0.05064	0.38706	0.14691
D	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

Table V: Probabilities $\varphi_{ij}(5)$

$\varphi_{ij}(5)$	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.70287	0.23492	0.04195	0.01027	0.00686	0.00224	0.00020	0.00064
AA	0.01818	0.75850	0.17451	0.03004	0.00821	0.00825	0.00058	0.00178
A	0.00265	0.06117	0.76474	0.11176	0.03330	0.01785	0.00162	0.00692
BBB	0.00143	0.01611	0.13934	0.62617	0.12951	0.05027	0.00779	0.02942
BB	0.00094	0.00742	0.03388	0.14710	0.48780	0.17895	0.03421	0.10969
B	0.00012	0.00423	0.01111	0.02416	0.10266	0.57843	0.07200	0.20731
CCC	0.00006	0.00126	0.01841	0.02244	0.04349	0.10671	0.45781	0.34982
D	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

The credit indicators $R_i(t)$, giving the probability that the 'system' was always working up to the time t given that the system was in working state I at time 0 and the stay on probability function, $1 - H_i(t) = 1 - \sum_{j=1}^{m} Q_{ij}(t)$; representing the probability that in a time interval t there was no new rating evaluation for the consumer starting with rank i at the starting time are presented in Table VI and VII respectively.

Table VI: $R_i(t)$,

$R_i(t)$	AAA	AA	A	BBB	BB	B	CCC	D
1	0.99999	0.99995	0.99940	0.99603	0.97889	0.95425	0.91042	0.00000
2	0.99993	0.99976	0.99841	0.99101	0.95712	0.91065	0.83213	0.00000
3	0.99981	0.99941	0.99703	0.98504	0.93495	0.86921	0.76354	0.00000

4	0.99962	0.99890	0.99525	0.97819	0.91262	0.82990	0.70328	0.00000
5	0.99936	0.99822	0.99308	0.97058	0.89031	0.79269	0.65018	0.00000
6	0.99901	0.99735	0.99051	0.96228	0.86815	0.75751	0.60327	0.00000
7	0.99856	0.99630	0.98755	0.95337	0.84628	0.72428	0.56170	0.00000
8	0.99802	0.99505	0.98422	0.94394	0.82477	0.69294	0.52475	0.00000
9	0.99737	0.99361	0.98052	0.93407	0.80371	0.66338	0.49181	0.00000
10	0.99661	0.99195	0.97647	0.92381	0.78315	0.63553	0.46235	0.00000
11	0.99572	0.99009	0.97208	0.91324	0.76313	0.60930	0.43592	0.00000
12	0.99472	0.98803	0.96737	0.90240	0.74367	0.58459	0.41213	0.00000

Table VII: $\mathbf{1 - H_i(t)}$

$\mathbf{1 - H_i(t)}$	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**D**
1	0.93129	0.94420	0.94403	0.90384	0.85624	0.89002	0.85292	1.00000
2	0.80797	0.84266	0.84301	0.74116	0.63125	0.70757	0.62129	1.00000
3	0.65325	0.71160	0.71350	0.55347	0.40304	0.50423	0.38706	1.00000
4	0.49236	0.56921	0.57348	0.37778	0.22423	0.32320	0.20654	1.00000
5	0.34607	0.43174	0.43856	0.23656	0.10938	0.18695	0.09456	1.00000
6	0.22691	0.31086	0.31969	0.13637	0.04708	0.09789	0.03721	1.00000
7	0.13885	0.21269	0.22255	0.07264	0.01800	0.04655	0.01260	1.00000
8	0.07932	0.13843	0.14820	0.03587	0.00615	0.02016	0.00368	1.00000
9	0.04232	0.08580	0.09458	0.01647	0.00189	0.00797	0.00093	1.00000

10	0.02109	0.05069	0.05793	0.00706	0.00053	0.00289	0.00020	1.00000
11	0.00983	0.02858	0.03412	0.00283	0.00013	0.00096	0.00004	1.00000
12	0.00428	0.01539	0.01934	0.00107	0.00003	0.00029	0.00001	1.00000

PRFMs, Discrete Time Markov Processes (DTMP) and Continuous Time Markov Processes (CTMP) have been used in empirical studies to model credit risk spread as two components; PD and LGD, (Valle, 2013). Consequently, the study focused on the PD and LGD components of the Basel formula for computing regulatory credit risk capital. The credit risk capital Basel formula is provided as part of the Appendix. The formula calibrates for suitable standardized values of MF, ρ_i and an α for computing EAD.

From the Semi-Markov model adopted, the computed $\emptyset_{ij}(t)'s \ for \ (0 \leq \text{t} \leq 12)$, are analogous to $PD_i's$ in the formula. However, it is the ability of the Semi-Markov model to predict the probabilities of default over longer durations that makes it appealing for forecasting. This is in sync with the Internal Capital Assessment Adequacy Planning (ICAAP) requirement for institutions to ensure that they at all times plan their capital ahead for a minimum of three years, (CBK/PG/03, 2013). Each $\emptyset_{ij}(t)$ represents the probability of a consumer being in the rank value j after a time t starting with the rank value i at time 0. The study results generate default probabilities for periods greater than three years. Consequently, determining the level of capital reserves to be held due to credit risk is facilitated. Meanwhile, aside from holding capital due to default, the study results facilitate the holding of capital for other loan classifications by providing probabilities of consumer loans being in the other states that would trigger provision. Provisioning is also done prior to occurrence of loss event, further protecting the firm against delinquent events,

The study illustrates the applicability of the model through Customer A, B and C who were randomly selected, ; appraised as per the metrics in the credit evaluation sheet; assigned initial probabilities of default based on their initial scores and assigned initial implied values V_0. Appendix I provides a summary of their details.

Appendix II provides summary of the reserve required for a portfolio of the three customers A, B and C in three years' time, denoted $Reserve_1$. Of interest to a bank apart from the probability of default after a given period of time, is the probability that in a time interval t there is no new rating evaluation for a consumer starting with rank i at the starting time. This is represented as the probability $1 - H_i(t)$, presented in Table 4.12. It is the stay on probability. Consequently, it is possible for a bank to compute the capital reserves for a portfolio of consumer loans after say 3 years, assuming the stay on probabilities over the three years. This provides the expected reserve if the consumers credit worthiness doesn't deteriorate nor improve over a given interval of time. Appendix II provides capital reserves computed after an interval of three years, given the stay-on probabilities for the sample consumers, A, B and C, denoted $Reserve_2$. Moreover, a bank would be concerned with the permanence of a consumer in a state, their subsequent movement to a different state and the effect of this on the amount of capital reserves required. This is represented by each $\varphi_{ij}(t)$, the probability of a consumer being in the rank value j after a time t starting with the rank value i at time 1 which enables the accounting for the different transition probabilities during the permanence of the firm in the same rating. Appendix II provides the capital reserve requirement at time 3 for the portfolio of sample consumers, denoted $Reserve_3$. Finally, a bank's credit risk function is at all times concerned about the soundness of its portfolio of consumer loans given the assumed probabilities of default. To establish the extent of exposure at any time in future, the Semi-Markov credit risk indicator provides the probability that the consumer has no default in a time t starting in the state i at time 0. As evident from the values provided in Table 4.11, there is less than 10% chance that any consumer loan will default in the first year. In fact, the highest probability of default is for a consumer initially rated CCC, with probability 0.08958. The probabilities of having no default deteriorates with time as expected. However, up until time three, the probability of default for a consumer in any rating is still below 40%, an indication that there is less than 40% chance of the portfolio of consumer loans becoming non-performing in the next three years. Further inferences over different durations can be made similarly.

A comparison of the adequacy of reserves provided through the Semi-Markov approach and the current Kenyan banking industry practice was apparent however not feasible. Apart from the lack of data upon which to base such analysis, there was also the need for a common time frame. Majority of Kenyan banks' forecasts for credit risk is over a period of 1 year. Meanwhile, classification of loans into the separate classes i.e. Normal, Watch, Sub-Standard, Doubtful and Loss, is a retrospective process that follows after a consumer fails to make good their loan repayments. To the contrary, the Semi-Markov model is a prospective model. Though the results of the Semi-Markov credit risk model may be reliable, the fact that the data values were simulated may not be representative enough of the Kenyan banking industry. Nevertheless, the fact that this model is better in forecasting credit risk indicators for a portfolio of consumer loans is evident, which attains the objective of the study: Establishing a case for the adoption of the Semi-Markov credit risk framework in modeling of credit risk for a portfolio of consumer loans through modeling credit rating migration patterns and how this influences the solvency and capital adequacy of banks in Kenya in light of the Basel solvency requirements.

5. Conclusions

With considerable progress having been made in the area of modeling consumer credit risk, the use of RFMs to model credit spreads has been acclaimed as more realistic to other models. RFMs view default as a sudden, unexpected event, thereby generating PD estimates that are more consistent with empirical observations, (Linda, 2004). Consequently, they are preferable. The Basel Accord recommends that banks have an internal rating model for their credit risk exposures. Meanwhile, the CBK Risk Guidelines note that an important tool in monitoring the quality of individual credits, as well as that of the portfolio, is the use of an internal risk rating system which will allow more accurate determination of the overall characteristics of the credit portfolio, concentrations, problem credits, and the adequacy of loan loss reserves, (CBK, 2013).

The study concludes that indeed there is a need to model credit risk for effective credit risk management by banks. The inadequacy of the current risk management practice among Kenyan banks is apparent. Non-multifarious and highly subjective credit risk models have consistently been used and their inability to adequately capture credit risk and forecast the probability of default over longer durations has been established. It is concluded that this is a distressing trend since it implies inadequacy of capital reserves held by banks for credit risk. Under (CBK/PG/03, 2013), the Internal Capital Assessment Adequacy Planning (ICAAP) requires that banks ensure that they at all times plan their capital ahead for a minimum of three years in order to establish and maintain on an ongoing basis an adequate level of capital, which would include an appropriate buffer, as determined by the board, above the regulatory required minimum capital.

The study further concludes that there is need for robust internal credit risk models. To respond to the need, the study adopted a PRFM, the Semi-Markov model, given the ability of PRFMs to model credit risk spread as two components; PD and LGD, (Valle, 2013). The study sought to pitch for the case of the Semi-Markov credit risk models in light of the aforementioned regulatory requirements and the need for more robust credit risk models. Initial credit scoring of randomly selected consumers was done in line with the current practice in the Kenyan banking sector. To each initial credit score, an implied value which acts as the proxy for credit worthiness of the specific consumer; was then assigned. Subsequent rating was done through the Merton model through which the initial transition matrix was generated assuming past historical values for credit worthiness for a portfolio of consumer loans. The initial transition matrix was then espoused to the Semi-Markov environment. The study concludes; from the analysis, results and discussion; the Semi-Markov models not only respond to the existent need for better credit risk modeling but go as far as forecasting for periods beyond the required regulatory minimum of three years.

Whether the capital reserves computed from the Semi-Markov framework are more sufficient than the existent capital reserves for portfolios of consumer loans computed through standard industry practice could not be verified in the study. This was due to the reluctance by banks to provide such information. Nonetheless, from the study results and discussion, the Semi-Markov framework facilitates better prediction of default probability, the extent of exposure and hence facilitates adequate capital provision prior to occurrence of loss event i.e. default. Lack of data to facilitate the modeling process, was the only challenge to the generation of results and the proceeding analysis. The use of *Monte-Carlo* simulated data however facilitated passable deductions.

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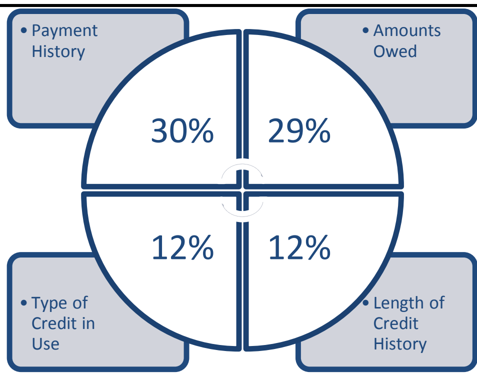
7. Appendices

7.1. Appendix I: Consumer A. B and C



Inferences

Overall % =83%

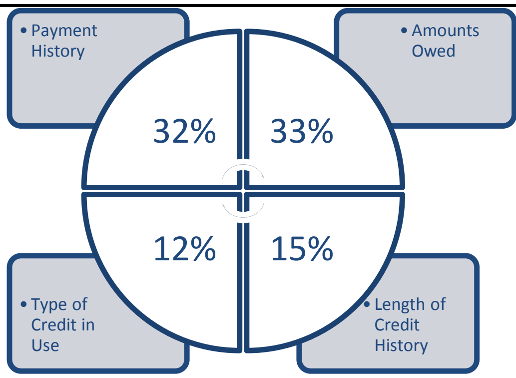
Implied Vo=Overall%*200=166

Initial Scoring=(3)

Initial Rating=BBB

Initial Probability of Default=4.74%

Consumer A



Inferences

Overall % =91%

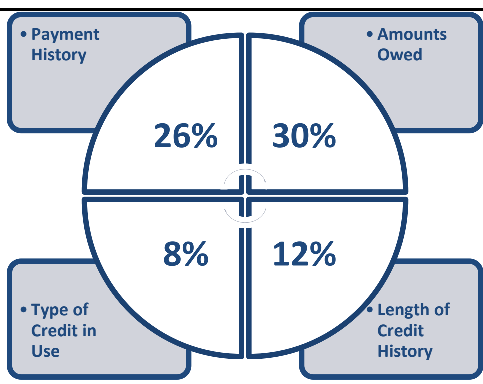
Implied Vo= Overall%* 200=184

Initial Scoring=(7)

Initial Rating=AA

Initial Probability of Default=0.09%

Consumer B



Inferences

Overall % =76%

Implied Vo= Overall%* 200=152

Initial Scoring=(2)

Initial Rating=BB

Initial Probability of Default=11.92%

Consumer C

7.2. *Appendix II: Reserves*

Reserve	Amount (KES)
$\boldsymbol{Reserve_1}$.	283,143
$\boldsymbol{Reserve_2}$.	144,532
$\boldsymbol{Reserve_3}$.	186,378

7.3.Appendix III: Basel Credit Risk Capital Formula

The Basel II regulatory capital formula for credit risk is as stipulated below:

$$Credit\ Risk_{reg\_Cap} = \sum_{i=1}^{N} LGD_i * EAD_i * \left[\Phi\left(\frac{\Phi^{-1}(PD_i) - \sqrt{\rho_i}\Phi^{-1}(0.001)}{\sqrt{1-\rho_i}}\right) - PD_i\right] * MF(M_i, PD_i)$$

Where:

$Credit\ Risk_{reg\_Cap} = the\ Internal\ Risk\ Based\ Credit\ risk\ regulatory\ capital$

LGD_i
$= Loss\ given\ default\ for\ consumer\ i;\ proportion\ of\ exposure\ lost\ if\ default\ occurs$

$EAD_i = Exposure\ at\ default\ for\ consumer\ i$

$PD_i = Probability\ of\ default\ of\ consumer\ over\ a\ period\ of\ 1\ year$

$\rho_i = Correlation\ of\ 'assets'. For\ the\ study, this\ is\ the\ correlation\ of\ the$

$consumer\ behavioral\ values\ MF\ =$
$Maturity\ Factor. It\ captures\ the\ incremental\ credit\ risk\ capital\ due$

$to\ credit\ \boldsymbol{migration}$

7.4. Appendix IV: Score Sheet

Payment History [35%]

Other Accounts

Account Type	Assign 1 or 0	Initial Amount Owing (KES)	Repayment Amount Monthly (y_i) (KES)
Credit Account			
Retail Account			
Installment Loan			
Finance Company Account			
Mortgage Account			
Total	$\boldsymbol{x/6}$		$\boldsymbol{q = \{y/Monthly\ Income\}}$

Payment History

Other Accounts

Value of q	Initial Owing (KES)	Amount
> 0.3	*75%*	
$= 0.3$	*50%*	
< 0.3	*25%*	
Total		

$\text{Total} = \left\{\frac{x}{6} * \langle 1 + q \rangle\right\}$	

Payment History [35%]

Public Record and Collection Items

Event	Assign 0 or 1	$\boldsymbol{z/5}$

Date of event	Assign t%
$0-360$	*100%*
$361-720$	*75%*
$721-1080$	*50%*
>1080	*25%*

$$\boldsymbol{Total} = \frac{\boldsymbol{z}}{\boldsymbol{5}} * \langle(\boldsymbol{1}+\boldsymbol{t_1}) + \cdots (\boldsymbol{1}+\boldsymbol{t_z})\rangle$$

Delinquencies

How late (Days):	☐ *0-30*	☐ *31-60*	☐ *61-90*	☐ *>90*
Assign (d %)	*25%*	*50%*	*75%*	*100%*

How much was owed: o

Initial Loan Amount: P

Delinquency Date:	$0-360$	$361-720$	$721-1080$	>1080
Assign (t %)	*100%*	*75%*	*50%*	*25%*

Number of Delinquency Cases in the last 1 year: Assign 0 or 1 (A total of n cases)

$$Total = \frac{1}{\boldsymbol{n}}\left\{\langle(\boldsymbol{1}+\boldsymbol{d_1}) * {}^{\boldsymbol{o_1}}/_{\boldsymbol{p_1}} * (\boldsymbol{1}+\boldsymbol{t_1})\rangle + \cdots + \langle(\boldsymbol{1}+\boldsymbol{d_n}) * {}^{\boldsymbol{o_n}}/_{\boldsymbol{p_n}} * (1+t_n)\rangle\right\}$$

$$\boldsymbol{Payment\ History}\ \text{Overall}\ \boldsymbol{Total} = \left[\left\{\frac{\boldsymbol{x}}{\boldsymbol{6}} * \langle\boldsymbol{1}+\boldsymbol{q}\rangle\right\} + \frac{\boldsymbol{1}}{\boldsymbol{n}}\left\{\langle(\boldsymbol{1}+\boldsymbol{d_1}) * {}^{\boldsymbol{o_1}}/_{\boldsymbol{p_1}} * (\boldsymbol{1}+\boldsymbol{t_1})\rangle + \cdots + \langle(\boldsymbol{1}+\boldsymbol{d_n}) * {}^{\boldsymbol{o_n}}/_{\boldsymbol{p_n}} * (\boldsymbol{1}+\boldsymbol{t_n})\rangle\right\} - \frac{\boldsymbol{z}}{\boldsymbol{5}} * \langle(\boldsymbol{1}+\boldsymbol{t_1}) + \cdots (\boldsymbol{1}+\boldsymbol{t_z})\rangle\right] * \boldsymbol{35}\%$$

Amounts Owed

Account Type	Nº of Payments Made f_i	Initial Amount Owing (P_i) (KES)	Proportion Outstanding During New loan Term (K_i)

Credit Account
Retail Account
Installment Loan
Finance Co Account
Mortgage Account
Total

$$Where\ K = \frac{Initial\ Amount\ Owing - \mathrm{N^o}\ of\ Payments\ made * Repayment\ Amount}{Initial\ Amount\ Owing}$$

$$K_i = \frac{P_i - f_i * Repayment\ Amount}{P_i}$$

$$Amounts\ \boldsymbol{Owed\ Total} = \left[\frac{\mathbf{1}}{\boldsymbol{Total\ Accounts\ Outstanding}} * \langle \mathbf{1} - \sum_{\forall\, \boldsymbol{i}} \boldsymbol{K_i} \rangle\right] * \mathbf{35}\%$$

Length of Credit History [15%] and Types of Credit in Use [15%] sections will be scored from these two prior sections.