Strathmore Business School

Bachelor of Supply Chain Management

End of Semester Examinations April 2024

MAT 2204: Quantitative Analysis, Modelling & Decision Making

Date: -/04/2024 **Time: 2 Hours**

Instructions:

1. ***Answer Question One*** *and* ***any other two*** *questions.*
2. *Clearly indicate all the steps in your work.*

Question One (30 Marks)

(a) Define the following terms as used in quantitative analysis:

(i) Multiple regression analysis. [2 Marks]

(ii) Markov chains. [2 Marks]

(iii) Hypothesis testing. [2 Marks]

(iv) A type I error. [2 Marks]

(b) (i) Describe 5 steps for conducting a hypothesis test. [5 Marks]

(ii) An emergency service company claims that they reach their clients in need of their service within 12 minutes. A random sample of 40 such emergency reaction times was obtained as 12.02 minutes with a standard deviation of 1.04 minutes. At $\alpha = 0.05$, can we conclude that the service goal of the emergency service company is being achieved? [4 Marks]

(c) A firm produces two products. These products are processed on three different machines. The time required to manufacture one unit of each of the two products and the daily capacity of the three machines are given in the table below.

	Time (minutes)		Machine Capacity (Minutes Per Day)
	Product 1	Product 2	
Machine One	2	3	440
Machine Two	4	3	470
Machine Three	2	5	430
Profit Per Unit, $	4	3	

(i) Formulate a Linear programming problem to determine how much of products 1 and 2 should be produced to maximize the total profit. [4 Marks]

(ii) Determine the maximum profit using the graphical method. [4 Marks]

(d) Part time students admitted to a certain Bachelor's Degree program in a certain University are considered to be 1^{st} year students until they complete 15 credits successfully. Then they are classified as 2^{nd} year students and may begin to take more advanced courses and work on their thesis required for graduation. Past records indicate that at the end of each year, 10% of the 1^{st} year students (F) drop out of the program (D) and 30% become 2^{nd} year students (S). Also 10% of the 2^{nd} year students drop out of the program and 40% graduate (G) each year. Students that graduate or drop never return to the program.

(i) Find the transition matrix p. [3 Marks]

(ii) What is the probability that a 1^{st} year student graduates within 4 years? Drops out within 4 years? [2 Marks]

Question Two (20 Marks)

The data below provide the sick days reported in a month for various age groups of staff of a certain company

Staff	1	2	3	4	5	6	7	8	9	10
Age *(X)*	30	32	35	40	48	50	52	55	57	61
Sick Days *(Y)*	2	1	0	2	5	4	5	6	8	7

From the given data:

(i) Compute the *Karl Pearson's* coefficient of correlation and interpret the results. [7 Marks]

(ii) Obtain the coefficient of determination and explain the results. [3 Marks]

(iii) Obtain the regression equation linking age to sickness. [8 Marks]

(iv) Estimate the age of a staff whose sick days are 16 in a month. [2 Marks]

Question Three (20 Marks)

(a) An insurance firm classifies drivers as low-risk if they are accident-free for 1 year. Past records indicate that 98% of the drivers in the low-risk category (L) one year will remain in that category the next year, and 78% of the drivers who are not in the low-risk (L') one year will be in the low-risk category the next year.

(i) Draw a transition diagram to represent the given information. [2 Marks]

(ii) Find the transition matrix p. [2 Marks]

(iii) If 90% of the drivers are in the low-risk this year, what is the probability that a driver chosen at random will be in the low-risk category next year? Year after next? [6 Marks]

(b) A manufacturing firm produces two products P_1 and P_2. Each of these products must be processed through two different machines. One machine has 24 hours of available capacity, and the second machine has 16 hours of available capacity. Each unit of product P_1 requires 2 hours of time on both machines. Each unit of product P_2 requires 3 hours on the first machine and 1 hour on the second machine. The incremental profit is \$ 7 per unit of product P_1 and \$ 6 per unit of product P_2, and the firm can sell as many units of each product as it can manufacture.

(i) Formulate a linear program to represent the given piece of information. [4 Marks]

(ii) Using graphical method, determine how many units of product P_1 and P_2 should be produced within the limits of available machine capacities. [6 Marks]

Question Four (20 Marks)

(a) Describe the four components of a linear programming problem. [8 Marks]

(b) Explain the four categories of a linear program. [4 Marks]

(c) A transition matrix for a certain Markov chain is given by

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}.$$

Find the stationary matrix, S. [4 Marks]

(d) A telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 58 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$? [4 Marks]

Question Five (20 Marks)

(a) Data used to predict job performance (Y) of Chevy mechanics based on mechanical aptitude test scores (X_1) and test scores from personality test that measures conscientiousness (X_2), are given in the table below.

Y	X_1	X_2
1	40	25
2	45	20
1	38	30
3	50	30
2	47	28
3	55	30
3	53	34
4	55	36
4	58	32
3	40	34
5	55	38
3	48	28
3	45	30
2	55	36
4	60	34

(i) Fit the least squares multiple regression equation of job performance (Y) on mechanical aptitude test scores (X_1) and personality test scores that measures conscientiousness (X_2). [12 Marks]

(ii) Predict the job performance scores for aptitude test score, $X_1 = 72$ and personality test scores $X_2 = 53$. [3 Marks]

(b) A manufacturer claims that its rechargeable batteries are good for an average of more than 1,000 charges. A random sample of 100 batteries has a mean life of 1002 charges and a standard deviation of 14. Is there enough evidence to support this claim at $\alpha = 0.01$? [5 Marks]