Strathmore UNIVERSITY

INSTITUTE OF MATHEMATICAL SCIENCES (IMS)

MASTER OF SCIENCE IN BIOMATHEMATICS

END OF SEMESTER EXAMINATION

BMA 8104: BIOMATHEMATICS I

Date: March 8, 2016 Duration: $2\frac{1}{2}$ Hours

Answer question ONE and any other TWO questions.

Question I (20 marks)

1. (a) Describe the process of mathematical modeling of a real world system. (5 Marks)

   (b) Why is mathematical modeling important in relation to disease dynamics . (2 Marks)

   (c) Consider the following system of differential equations:

$$\begin{aligned} \frac{dX}{dt} &= x(1-x-2y) = f(x,y) \\ \frac{dY}{dt} &= y(1-2x-y) = g(x,y) \end{aligned}$$

   Determine the Jacobian of the system for steady-state at (0,0) and determine the stability of the system at this point. (5 Marks)

   (d) Describe R_0 and discuss its importance in relation to disease modeling. (2 Marks)

   (e) Discuss the compartmental model which best fits the dynamics of Pneumonia transmission with treatment interventions. (6 Marks)

Question 2 (20 marks)

2. Suppose there is a lake with some fish that are attractive to fishermen. We wish to model the fish-fishermen interaction given the following assumptions:

   - Fish grow exponentially in the absence of fishing
   - The presence of fishermen depresses fish growth at a rate jointly proportional to the fish and fishermen populations.
   - Fishermen are attracted to the lake at a rate directly proportional to the amount of fish in the lake
   - Fishermen leave the lake at a rate directly proportional to the number of fishermen already there.

   i. Describe all the variables and parameters carefully and state any assumptions that you make about the parameters. (6 Marks)

   ii. Draw the interaction diagram and formulate a mathematical model for this situation. (8 Marks)

   iii. Suppose sharks are introduced in this lake and that these sharks feed on the fish, formulate new model for the interactions of the three populations. (6 Marks)

Question 3 (20 marks)

3. The dynamics of the transmission of polio is assumed to be governed by the following system of nonlinear ordinary differential equations:

$$\begin{aligned} \frac{dS}{dt} &= \Lambda - \beta SI - \gamma SE - (\mu_S + \nu)S \\ \frac{dE}{dt} &= \beta SI + \gamma SE - (\mu_E + \delta)E \\ \frac{dI}{dt} &= \delta E - (\mu_I + \alpha)I \\ \frac{dV}{dt} &= \nu S - \mu_V V \end{aligned} \tag{1}$$

   i. Describe the variables and parameters in the model (8 Marks)

   ii. Draw a diagram that would represent the disease dynamics as represented by the equations (6 Marks)

   iii. State possible investigations that would be done using the model (2 marks)

   iv. How can this model be improved, show using a conceptual diagram. (4 marks)

Question 4 (20 marks)

4. In order to model the transmission of Measles, the total population (N) is divided into four classes: Susceptible (S), Exposed (E), infected (I) and Recovered (R). The transitions among these four classes is depicted in Figure 1 in which

- Individuals are equally likely to be infected by the infectious individuals in a case of contact except those who are immune.
- Recovered individuals are permanently immune.
- Infectious individuals are detected early and isolated for immediate treatment and education
- The Newborns are assumed to be susceptible
- The population is homogeneously mixing.

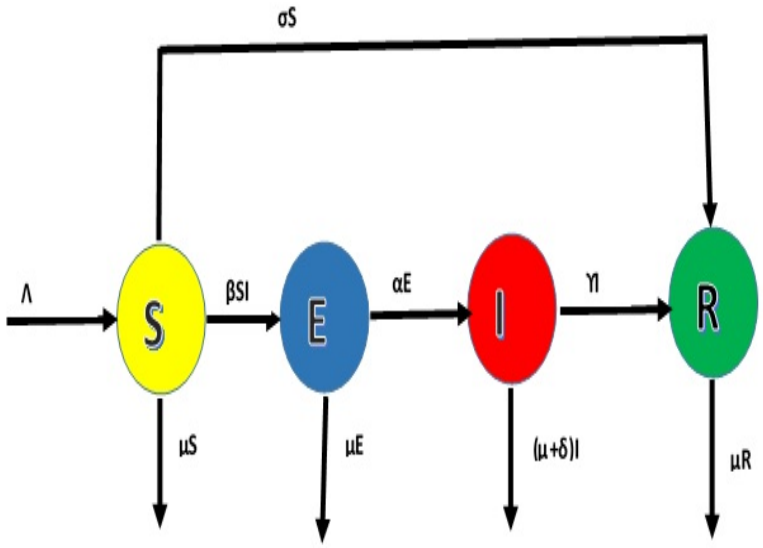

Figure 1: Dynamics of measles in population level

i. Describe the variables and parameters stating any assumptions that you make about the parameters. (6 marks)

ii. Write down the equations for measles dynamics as depicted in the diagram. (4 marks)

iii. Discuss how the epidemic can be eliminated given the basic reproduction number R_0 is:

$$R_0 = \frac{\Lambda\beta\alpha}{\mu(\mu+\alpha+\sigma)(\mu+\gamma)}$$

(2 marks)

iv. Discuss how the model can be improved, draw the conceptual diagram and write new equations for the improved model. (8 marks)