Strathmore UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
END OF SEMESTER EXAMINATION
EMT 2211 ENGINEERING MATHEMATICS II

Date: 15th December, 2022 Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

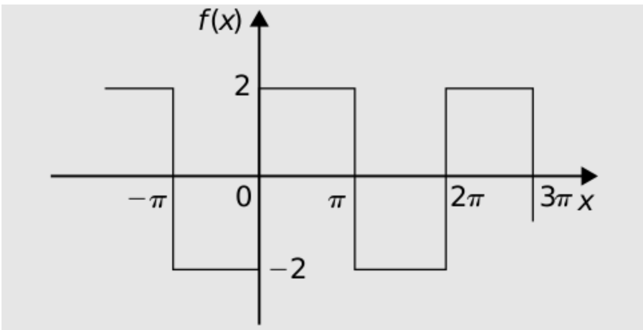
(a) Find each of the following Laplace transforms, showing, where appropriate, the technique used.

(i) $\mathscr{L}\{1 + 2t - \frac{1}{3}t^4\}$ [2 Marks]

(ii) $\mathscr{L}\{e^{3t} + 3\sin 2t\}$ [2 Marks]

(b) The equation $\frac{dv}{dt} = -(av + bt)$, where a and b are constants, represents an equation of motion when a particle moves in a resisting medium. Solve the equation for v given that $v = u$ when $t = 0$. [3 Marks]

(c) Obtain the Fourier series for the square wave shown below. [4 Marks]



(d) Use Euler's method to obtain a numerical solution of the differential equation $\frac{dy}{dx} = x + 2y$, given the initial conditions that at $x = 0$, $y = 0$, for the range $x = 0(0.1)0.4$. Approximate $y(0.4)$ [4 Marks]

(e) A transformation in three dimensional space is defined by the following 3×3 matrix, where x is a scalar constant.

$$\mathbf{C} = \begin{pmatrix} 2 & -2 & 4 \\ 5 & x-2 & 2 \\ -1 & 3 & x \end{pmatrix}$$

Show that $\mathbf{C}$ is non-singular for all values of x. [3 Marks]

(f) Solve the system of simultaneous equations by manipulating their augmented matrix into reduced row echelon form. [4 Marks]

$$\begin{aligned} x + 3y + 5z &= 6 \\ 6x - 8y + 4z &= -3 \\ 3x + 11y + 13z &= 17 \end{aligned}$$

(g) The oscillations of a heavily damped pendulum satisfy the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$$

where $x$cm is the displacement of the bob at time t seconds. The initial displacement is equal to +4cm and the initial velocity $\left(i.e.\ \frac{dx}{dt}\right)$ is 8cm/s. Solve the equation for x. [5 Marks]

(h) Find the inverse Laplace transform, showing, where appropriate, the technique used.
. [3 Marks]

$$\mathscr{L}^{-1}\left\{\frac{s}{s^2 - 6s + 10}\right\}$$

QUESTION TWO (15 MARKS)

(a) In determining a Fourier series to represent $f(x) = x$ in the range $-\pi$ to π, Fourier coefficients are given by:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cos nx\, dx$$
$$\text{and} \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx\, dx$$

where n is a positive integer. Show by using integration by parts that $a_n = 0$ [5 Marks]

(b) (i) Determine the Fourier series for the function $f(\theta) = \theta^2$ in the range $\pi < \theta < \pi$. The function has a period of 2π. [7 Marks]

(ii) For the Fourier series in part (i), let $\theta = \pi$ and show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. [3 Marks]

QUESTION THREE (15 MARKS)

(a) Show that the solution of the differential equation: $x^2 - 3y^2 + 2xy\frac{dy}{dx} = 0$ is $y = x\sqrt{(8x+1)}$, given that $y = 3$ when $x = 1$. [5 Marks]

(b) In a galvanometer the deflection θ satisfies the differential equation

$$\frac{d^2\theta}{dt^2} + 4\frac{d\theta}{dt} + 4\theta = 8$$

Solve the equation for θ given that when $t = 0$, $\theta = \frac{d\theta}{dt} = 2$. [5 Marks]

(c) Use the the fourth order Runge-Kutta method to solve the differential equation:

$$\frac{dy}{dx} = y - x, \quad y(0) = 2$$

Find $y(0.1)$ given that $h = \triangle_x = 0.1$ [5 Marks]

QUESTION FOUR (15 MARKS)

(a) The velocity of a car, accelerating at uniform acceleration a between two points, is given by $v = u + at$, where u is its velocity when passing the first point and t is the time taken to pass between the two points. If $v = 21m/s$ when $t = 3.5s$ and $v = 33m/s$ when $t = 6.1s$, use matrix inverse to find the values of u and a, each correct to 4 significant figures. [5 Marks]

(b) The 3×3 matric **A** is given below.

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -3 \\ 2 & 4 & 3 \\ -4 & 2 & -1 \end{pmatrix}$$

The matrix **A** is non singular.

(i) Evaluate $\mathbf{A}^2 - \mathbf{A}$. [2 Marks]

(ii) Show clearly that $\mathbf{A}^{-1} = \frac{1}{20}[\mathbf{A} - I]$. [2 Marks]

(c) A d.c. circuit comprises three closed loops. Applying Kirchhoff's laws to the closed loops gives the following equations for current flow in milliamperes:

$$\begin{aligned} 2l_1 + 3l_2 - 4l_3 &= 26 \\ l_1 - 5l_2 - 3l_3 &= -87 \\ -7l_1 + 2l_2 + 6l_3 &= 12 \end{aligned}$$

Use determinants to solve for l_1, l_2 and l_3. [6 Marks]

QUESTION FIVE (15 MARKS)

(a) Find each of the following Laplace transforms or inverse Laplace transforms, showing, where appropriate, the technique used.

(i) $\mathscr{L}\left\{2e^{3t}(4\cos 2t - 5\sin 2t)\right\}$ [2 Marks]

(ii) $\mathscr{L}^{-1}\left\{\frac{5s+1}{s^2-s-12}\right\}$ [3 Marks]

(b) Prove that [3 Marks]

$$\mathscr{L}\{\cosh at\} = \frac{s}{s^2 - a^2}$$

(c) Solve the following pair of simultaneous differential equations [7 Marks]

$$3\frac{dx}{dt} - 5\frac{dy}{dt} + 2x = 6$$
$$2\frac{dy}{dt} - \frac{dx}{dt} - y = -1$$

given that at $t = 0$, $x = 8$ and $y = 3$

END OF PAPER

Formula sheet

Function, $f(t)$	Laplace transforms $\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$
1	$\frac{1}{s}$
k	$\frac{k}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$t^n (n = 1, 2, 3, \cdots)$	$\frac{n!}{s^{n+1}}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$

Function, $e^{at} f(t)$	Laplace transforms $\mathscr{L}\{e^{at} f(t)\}$
$e^{at} t^n$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$
$e^{at} \cosh \omega t$	$\frac{s-a}{(s-a)^2-\omega^2}$

Euler's First Order RK	$y_{i+1} = y_i + k_1 h, \quad k_1 = f(x_i, y_i)$
Classical Fourth Order RK	$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k4)h$ where $k_1 = f(x_i, y_i); \quad k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$ $k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right); k_4 = f\left(x_i + h, y_i + k_3 h\right)$

Fourier series for periodic functions	
Over period 2π	$f(x) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ where $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx; \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$ and $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx; \quad (n = 1, 2, 3, \cdots)$
Over period L	$f(x) = a_0 + \Sigma_{n=1}^{\infty}[a_n \cos\left(\frac{2\pi nx}{L}\right) + b_n \sin\left(\frac{2\pi nx}{L}\right)]$ where $a_0 = \frac{1}{L} \int_{-L/2}^{L/2} f(x)\, dx; \quad a_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \cos\left(\frac{2\pi nx}{L}\right)\, dx$ and $b_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \sin\left(\frac{2\pi nx}{L}\right)\, dx; \quad (n = 1, 2, 3, \cdots)$