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HEDGING LONGEVITY RISK USING LONGEVITY SWAPS

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

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ABSTRACT

Longevity-linked securities provide the desirable hedging instruments to insurers and annuity providers, and on the other hand, diversification benefits to their counterparties such as the reinsurers and banks. In Kenya however, the longevity market is not in existence and as such this paper seeks to model a longevity swap in the Kenyan context and determine its effectiveness in hedging longevity risk. The longevity swap is modeled using the distortion approach; Wang transform. With the projected cash flows for the floating and the fixed leg, the swap value is calculated, which gives the amount the insurer or annuity provider will have to pay to get into the contract.

In determining the hedge effectiveness, sensitivity analysis was conducted on the parameters including the interest rates, cohort ages, term of the swap against the swap premium and the swap value. The results were that the premium increases with the term and entry age of the reference cohort. In terms of the swap value which is the difference between the present values of the floating against the fixed leg, as the population gets older the swap gets cheaper and therefore becomes more effective to hedge for lower ages.

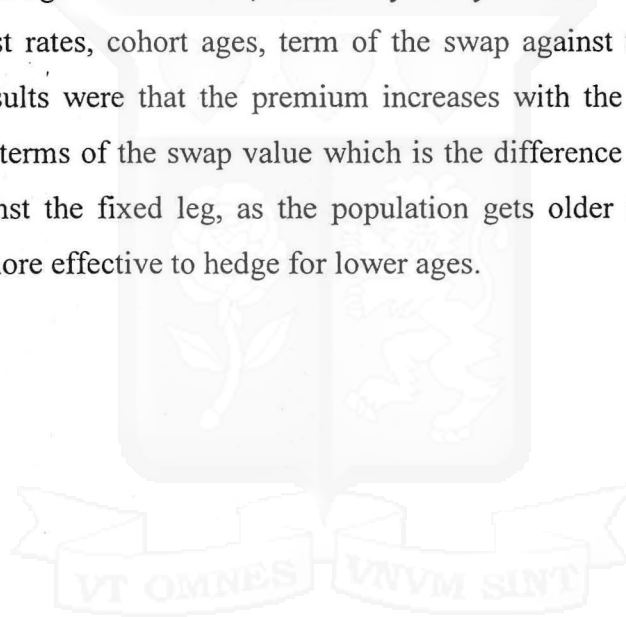


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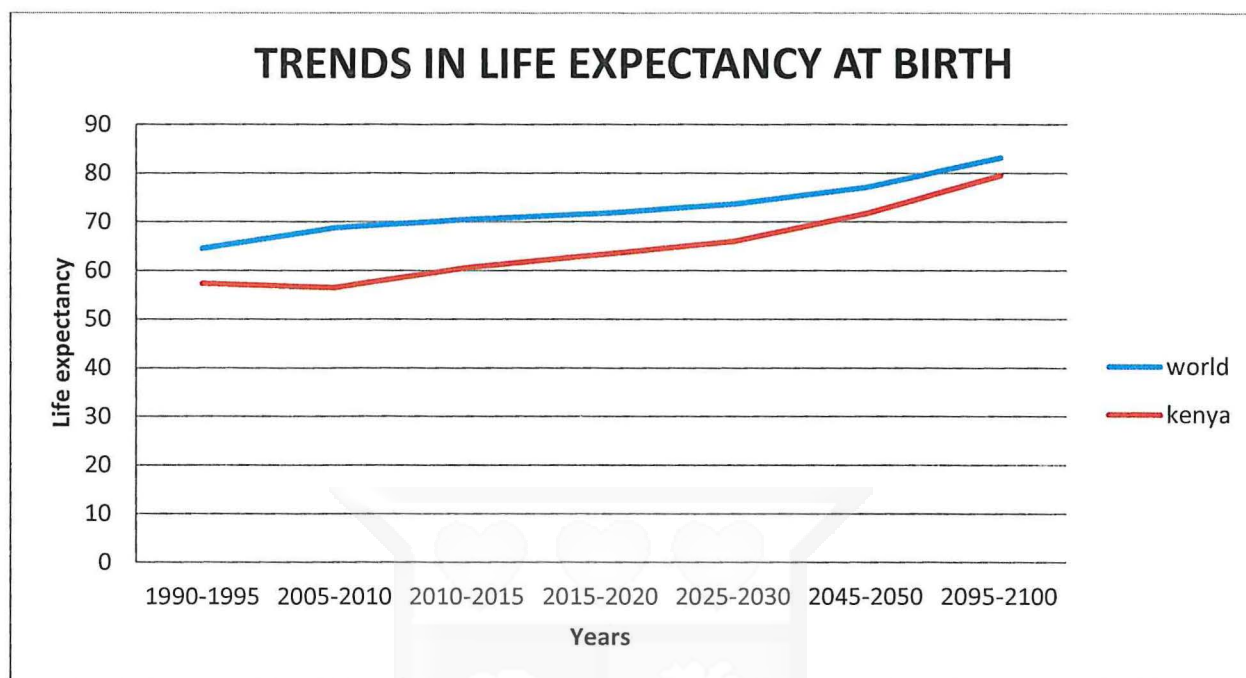
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Figure 1: Comparison of trends in life expectancy at birth between Kenya and the world



Source: : United Nations, Department of Economic and Social Affairs, Population Division (2015)

Figure 1 above shows the trends in life expectancy of both the world and Kenya over the years into the expected future. The life expectancy of both Kenya and the World are increasing and expected to increase even more to the year 2100 although that of the World is higher than Kenya's. This goes to show that with increasing life expectancy longevity risk is increasing as well and should be managed.

1.2 Problem Statement

There is increased uncertainty about longevity risk and the adequate amount of capital to hold in case of adverse experience (Njenga, 2011). Increasing longevity risk have meant benefits need to be paid much longer than expected, increasing the value of the sponsor's obligation to members (Loeys et al., 2007). In practical terms, greater capital has to be constituted to balance the long term risk, and this itself is a challenge (Barrieu et al., 2012).

In recent years, developing countries, including Kenya have experienced the decline in mortality rates and increase in life expectancy. Such trends in mortality reductions and increase in life expectancy present longevity risk to insurers and annuity providers.

As a result, it has become paramount for insurance companies and pension funds to find a suitable and effective way to hedge, or to transfer part of the longevity risk to reinsurers and the financial markets.

1.3 Research Objectives

1. To model a longevity swap
2. To determine the effectiveness of using swaps in hedging longevity risk

1.4 Research Questions

1. How is a longevity swap modeled?
2. How effective is the use of swaps in hedging longevity risk?

1.5 Significance of the study

This project would be important to practitioners as well as academicians and researchers by contributing to the already existing knowledge on longevity risk and longevity swaps as a way to hedge this particular risk. For the practitioners it will enable them to come up with reasonably priced products while incorporating the longevity risk in it.

The most affected by longevity risk are life insurers and pension providers. The results of this project will enable them find suitable ways to manage the longevity risk effectively.

Chapter 2: LITERATURE REVIEW

2.1 Introduction

In real-world markets, empirical evidence shows that hedging reduces the cost of capital market imperfections including the costs of financial distress and external financing as well as achieving a reduction in cash flow variability (Stulz, 1993). For the insurance industry, reduction of the cost of capital is the primary source of value creation from hedging and more so longevity hedging (Wills and Sherris, 2010).

2.2 Theoretical framework

2.2.1 Longevity swaps

Longevity swaps as stated earlier are agreements between two parties to exchange fixed payments for floating payments that vary with the mortality experience of an underlying reference population. Longevity swaps are mostly structured as reinsurance transactions. The floating payments of the swap match the mortality experience of the insured population. The swap is indemnity based and hedges the actual experience of the lives, covering both systematic and idiosyncratic risk, to eliminate any basis risk (Meyricke & Sherris, 2014).

According to Dowd et al., (2006) the most basic case for a longevity swap (survivor swap) would involve the exchange of a single preset payment for a single random mortality-dependent payment. Suppose that at time 0, two firms enter into an agreement to swap a preset amount $S(t)$ for a random amount $K(t)$ at some future time t . As with a conventional forward rate agreement (FRA), $K(t)$ can be interpreted as a coupon associated with an implicit notional principal, and to keep mutual credit risks down, it makes sense for the agreement to specify that the two parties exchange only the net difference between the two payment amounts. Such that;

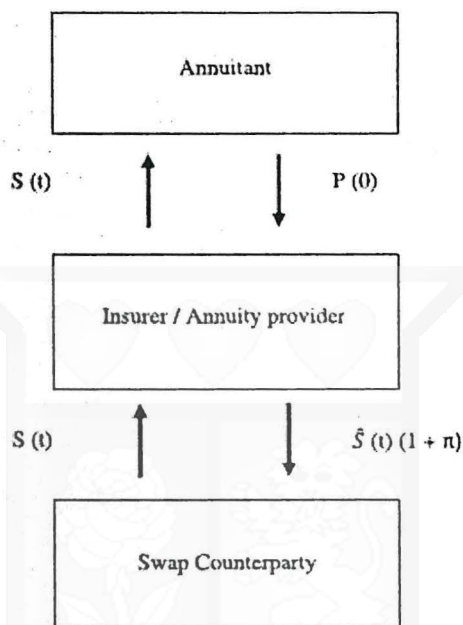
$$A \text{ pays firm B an amount} = K(t) - S(t) \text{ if } K(t) > S(t) \quad (1)$$

$$B \text{ pays A an amount} = S(t) - K(t) \text{ if } S(t) > K(t) \quad (2)$$

Where $S(t)$ is related to the number of people from a specified reference population (e.g., the whole population or the number of annuity holders at time 0) who have survived to time t .

Firm **A** benefits if $S(t)$ turns out to be high relative to $K(t)$ and loses if $S(t)$ turns out to be low: firm **A** has a long exposure to $S(t)$, while **B** has a short exposure to $S(t)$. Wills and Sherris, (2010) summarize the cash flows of a longevity swap as;

Figure 2: summary of cash flows of a longevity swap



Source: Sherris and Wills (2010)

Advantages of longevity swap according to Roy, (2012) are that they are independent of market risk but can still take advantage of market opportunities, they can be de-risked even in the absence of sufficient funding and credit risk is mitigated through collateralization.

Daitch (2013) further stated that a longevity swap allows one to manage the longevity risk much more efficiently, with no up-front premium and no immediate impact on the balance sheet. He moreover adds that a longevity swap can protect the income statement from unexpected costs arising from:

- Mortality improvements at a higher rate than priced
- Errors in the base table
- Basis error if characteristics of annuity block differs from basis used to create the firm's mortality table
- Volatility associated with a heterogeneous bloc

2.2.2 Pricing a longevity swap

In pricing a mortality linked or a longevity hedging instrument engineered by corporations the question that arises according to MacMinn, Brockett and Blake (2006) is how these instruments can be priced. This is so because longevity securities unlike the conventional fixed-income securities cannot be valued using the standard spot yield curve and zero- arbitrage (or net present value) methods due to market incompleteness. To solve this, a premium will have to be paid for some market participants to bear the longevity risk.

The prospective market for longevity derivatives contains more short than long investors because the amount of longevity risk supplied is larger than the amount of longevity risk demanded (Loeys et al., 2007). Investors who are therefore willing to take on this risk demand compensation for it.

There are different approaches to pricing longevity risk. A common feature of the proposed pricing methods is that the market price of longevity risk is determined largely by the expected volatility of the underlying survival rates (Wills and Sherris, 2010).

I. Distortion Approach to Pricing

In this distortion approach the Wang Transform is applied. It is a market based equilibrium pricing method that unifies the finance and insurance pricing theories(Wang, 2002).

This approach distorts the distribution of the survivor index to create suitable risk-adjusted expected values (or certainty equivalents) which can be discounted at the risk-free rate (Cipra, 2010). Blake et al., (2006) add that the extent of the risk adjustment should reflect the market prices of risk for other assets in the market place which permit trading of other incomplete market risks.

The current best estimates of the mortality rates are converted used to their risk-neutral counterparts by the Wang transform. The Wang transform, (Wang, 2002), essentially is an application of the Capital Asset Pricing Model to not normally distributed variables. The approach determines the standardized form of a series of probabilities (z-scores) as if they were normally distributed. These z-scores are uniformly shifted by an amount λ , which stands for the

market price of risk. The shifted z-scores are then transformed back, again using the standard normal distribution.

According to S. S. Wang (2002) one fortunate property of the Wang transform is that normal and lognormal distributions are preserved. That is;

- > If F has a Normal(μ, σ^2) distribution, F^* is also a normal distribution with $\mu^* = \mu - \lambda\sigma$ and $\sigma^* = \sigma$.
- > If F has a lognormal(μ, σ^2) distribution such that $\ln(X) \sim \text{Normal}(\mu, \sigma^2)$, F^* is another lognormal distribution with $\mu^* = \mu - \lambda\sigma$ and $\sigma^* = \sigma$.

II. Risk-Neutral Pricing

According to Blake et al., (2006) this approach is based on a long-established financial economic theory that states that even in an incomplete market, if the overall market is arbitrage free, then there exists at least one such risk-neutral measure Q that can be used to calculate fair prices. The risk-neutral approach to pricing can also be applied to the longevity bonds engineered using swaps and forwards.

Therefore assuming an arbitrage-free environment there exists a risk-neutral measure Q allowing risk-free discounting using the same discount factor $d(t,0)$;

$$V(LB) = \sum_{t=1}^T d(0, t) \cdot E_Q(S(t) | \Omega_0) \quad (3)$$

Where $E_Q(S(t) | \Omega_0)$ is the expected value of $S(t)$ under the risk-neutral measure Q conditional on the information Ω_0 available at time 0 (Cipra, 2010).

This risk-neutral approach to pricing has been favored by many authors. They include Miltersen and Persson (2005) who introduce the concept of forward force of mortality rate without any limitations on the dependence between the stochastic behavior of the forward force of mortality rate and the stochastic behavior of the forward rate use this approach under a pricing measure Q . Cairns, Blake, & Dowd (2006) also apply a range of risk neutral frameworks for pricing and hedging mortality risk that allow for both interest and mortality factors to be stochastic.

The advantage of this approach is that it provides natural benchmark valuations, and one can also argue heuristically that they will become easier to justify in any given context as markets become less incomplete over time. However the use of arbitrage-free methods is always problematic if markets are incomplete, as is certainly the case with mortality derivatives markets. Moreover many of the assumptions underpinning these frameworks such as liquid and frictionless markets do not hold in practice (Cairns et al., 2006).

III. Sharpe ratio approach.

The Sharpe ratio method is based on parallels with the capital market. This method assigns a Sharpe ratio and determines the forward longevity premium by assuming that the total of $(1 + \pi)PV [H(t)]$ equals the present value of the floating leg. (LoeysAC et al., 2007) explains this method as the procedure JP Morgan intended to use in the pricing of its q-forwards.

Milevsky, Promislow, & Young (2007) also propose an instantaneous Sharpe ratio to determine the mortality risk premium. They show that in a stylized setting when the mortality distribution is unknown the Law of Large Numbers does not suffice to show that the risk per policy goes to zero. Therefore given the systematic or equivalently non-diversifiable risk, they use the Sharpe Ratio to develop a premium pricing method given aggregate mortality risk.

Using the comparison to the Sharpe ratio in the financial market which is the ratio of the expected excess return and the return volatility;

$$SR^{Market} = \frac{E[R] - R_f}{\sigma(R)} \quad (5)$$

Sharpe ratio in the insurance context can be defined as the excess payoff above the expected payment, divided by the standard deviation of the risky payment,

$$SR^{Insurance} = \frac{N(1 + L) - E[W_N]}{\sigma[W_N]} \quad (6)$$

The longevity risk loading L will be set so that the Sharpe Ratio is consistent with other asset classes in the economy.

In this study the model used to price longevity swap will be the Wang transform. This is because the transform has desirable properties compared to the other two approaches. The Wang transform is fairly easy to numerically compute since many softwares have Φ and Φ^{-1} as inbuilt functions (Wang, 2002). According to Lin & Cox, (2008) it has a clear economic interpretation since it can recover the capital asset pricing model (CAPM) for underlying assets and the Black-Scholes formula for options. In addition, the transformed distribution in the Wang transform reflects risk aversion of insurers and investors to risks that cannot be hedged (Lin and Cox, 2008).

However, the Sharpe approach is not appropriate since peculiarities of equity markets may not be present in the longevity market (Bauer, Börger, & Rußs, 2010). For the risk neutral approach its assumptions do not hold in practice such as liquid and frictionless market.

2.3 Empirical evidence

2.3.1 Modeling a longevity swap

Dowd et al., (2006) first discussed survivor swaps (SS) an agreement to exchange cash flows in the future based on the outcome of at least one survivor index as instruments for managing, hedging, and trading mortality-dependent risks. In their study they also investigated vanilla SSs a reminiscent of vanilla interest rate swaps, and suggest how their premiums and values might be determined in an incomplete market setting using the distortion approach.

Barrieu et al., (2012) in understanding, modeling and managing longevity stated that longevity swaps mainly take two forms, depending on whether they are index-based or customized. They gave differing longevity swaps arranged by JP Morgan in 2008.

First a customised swap transaction; In July 2008, JP Morgan executed a customized 40 year longevity swap with a UK life insurer for a notional amount of GBP 500 million. The life insurer agreed to pay fixed payments, and to receive floating payments, replicating the actual benefit payments made on a closed portfolio of retirement policies. The swap is before all a hedging instrument of cash flows for the life insurer, with no basis risk. At the same time, JP Morgan entered into smaller swaps with several investors that had agreed to take the longevity risk at the end.

The investors have access to the appropriate information to enable them to assess the risk of the underlying portfolios. The back-to-back swap structure of this transaction meant that JP Morgan had no residual longevity exposure. The longevity risk is transferred from the insurer to the investors, in return for a risk premium.

Second is a standardized transaction; In January 2008, JP Morgan executed a 10 year standardized longevity swap with the pension insurer Lucida for a notional sum of GBP 100 million; with an underlying risk determined by the LifeMetrics index for England and Wales. The structure of the swap enabled a value-hedge for Lucida, who agreed to keep the basis risk.

Van Rooijen, (2013) also looks at hedging using longevity swaps. Van Rooijen forecasted mortality rates with the well-known Lee-Carter model and then priced longevity index swaps using Monte Carlo simulations and the equivalent utility pricing principle. The results showed that the hedging costs involved were lower than the decrease in liabilities leading to the conclusion that longevity index swaps provide a profitable opportunity for hedging longevity risk

Recently in Ghana Omari-Sasu et al., (2016), explored a hypothetical hedging strategy based on longevity swaps for the Social Security and National Insurance Trust (SSNIT) pension scheme. They used the Cairns-Blake-Dowd model to forecast future mortality rates of pensioners from age 71 to 90. With the forecasted mortality rates, longevity swap contract was designed whereby realized mortality rates were swapped with the forecasted expected mortality rates. The payout structure under the swap ensured that the SSNIT's liability was completely hedged against longevity risk.

2.3.2 Effectiveness of longevity hedging using longevity swaps

Significant underestimates of past longevity improvements and still high uncertainty about future mortality have elevated longevity to a high profile risk for pension funds, insurers, and other companies (Loeys et al., 2007). As a result of this, Loeys et al., went on to look into the existing market and its potential to provide an effective longevity hedge. They conclude that pension funds and life insurance companies are the ones that have the potential to participate in a longevity market but the existing markets provide no effective hedge for longevity and mortality risk.

On the other hand, Coughlan et al., (2011) looked at basis risk as an important consideration when hedging longevity risk with instruments based on longevity indices, since the longevity experience of the hedged exposure may differ from that of the index. They come up with a framework for developing an informed understanding of the basis risk, appropriately calibrating the hedging instrument and evaluating hedge effectiveness. From their results, based on their case studies, they conclude that high levels of hedge effectiveness should be achievable with appropriately-calibrated, static, index-based longevity hedges.

Ngai and Sherris, (2011) also investigates the effectiveness of static hedging strategies for longevity risk management using longevity bonds and derivatives (q-forwards) for the retail products: life annuity, deferred life annuity, indexed life annuity and variable annuity with guaranteed lifetime benefits. Results show that static hedging using q-forwards or longevity bonds reduce longevity risk substantially for life annuities, but significantly less for deferred annuities. For inflation indexed annuities static hedging of longevity is less effective because of inflation risk. Variable annuities provide limited longevity protection compared to life annuities and indexed annuities, as a result longevity risk hedging adds little value for these products.

Barrieu et al., (2012) in support of using capital markets argued that despite the limited activity on longevity derivatives, using the capital markets to transfer part of the longevity-risk is complementary to traditional reinsurance solutions, and would thus seem to be a natural move. They state that up to 2012, almost all longevity capacity had been provided by the insurance and reinsurance markets. Whilst this capacity facilitated demand, exposure to longevity risk was and is still high. It is therefore clear that insufficient capacity exists in traditional markets to absorb any substantial portion of the risk, and only capital markets are a potential capacity provider

Knowledge gap

Longevity swaps is one strategy used in hedging against longevity risk. Around the world, Blake et al., (2006) report that several insurance companies already have entered into longevity/mortality swaps on an over-the-counter basis. The parties consist of life insurance companies and investment banks. These transactions have been made mostly in the United Kingdom, United States of America and Netherlands and none in the developing countries as yet.

In Kenya, longevity risk is a growing concern as life expectancy as seen in Figure 1 is increasing overtime. With this, insurance companies as well as pension and annuity providers seek an efficient way to effectively mitigate or transfer this risk. In line with this Loeys et al., (2007) stated that the existing markets provide no effective hedge for longevity and mortality risk. On the other hand Coughlan et al., (2011) argued that high levels of hedge effectiveness should be achievable with appropriately-calibrated, static, index-based longevity hedges. This study therefore seeks to bridge this gap.



Chapter 3: RESEARCH METHODOLOGY

This chapter clearly states the research methods used in this study.

3.1 Research design

This study is a descriptive study as it gives information on the use of longevity swaps to hedge longevity risk and to further to determine its effectiveness in longevity hedging.

It is quantitative since the data collection methods to be used will generate numerical data. These will include survival rates and market price of risk.

3.2 Population

The target population is the insured Kenyan adults from age 60 and above. This is because the retirement age in Kenya is 60. The mortality rates are given per age of the individual. The sampling method used is stratified sampling since the population in this study contains people of heterogeneous characteristics the population is divided into strata according to their ages and those born in the same time period (cohorts). The benefit of stratified random sampling is that it provides better comparison and hence representation across strata.

3.3 Data collection

The data collection is secondary data. Market interest rates and bond yields collected from the Nairobi Securities Exchange. The mortality rates were collected from the Kenya mortality tables for the period 2007 to 2010. Market annuity price is from the insurer's annuity quotation.

The frequency of the data is annual.

3.4 Data analysis

The dependent variable of this study is the price of the longevity swap while the independent variables are market interest rates, market price of risk, maturity term, survival rates given by the best estimate mortality and the cohort.

3.4.1 The model

The model used is the Wang transform. It converts the currently expected floating payments into their risk neutral equivalents using a specific market price of risk from the insurance market, and then determines the premium. The main property of Wang's method is that it transforms the underlying distribution in such a way that prices are discounted expected values (Lin & Cox, 2008)

Dowd et al., (2006) proposes this as a distortion valuation approach to pricing longevity swaps more specifically in pricing the premium.

With Wang transform;

If $\Phi(x)$ is the standard normal cumulative distribution function, the Wang distortion operator g_λ is;

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) + \lambda]$$

Where u is a probability between 0 and 1 and λ is the market price of risk reflecting the level of systematic risk (S. S. Wang, 2002). This implies that u can be transformed such that;

$$u^* = g_\lambda(u)$$

The distribution of the best estimates for the survival rates (p) is filled in as u :

$$F^*(p) = \Phi[\Phi^{-1}(F(p)) + \lambda]$$

The Wang transform adds the risk of longevity to the survival rates, which makes these rates risk neutral as no additional premium is required for the longevity risk when these rates are used. The risk neutral rates are the basis of the present value determination of the floating payments (PV [S(t)]): they are multiplied with the annuity amount, and discounted with the risk-free interest rate structure.

The present value of the fixed payments (PV[H(t)]) is determined by discounting the best estimate mortality rates at the initiation of the swap against the prevailing market interest rate

curve. Since the preset payment is $(1+\pi)H(t)$ and the floating payment is $S(t)$, the value to the preset payer is

$$\text{swap value} = PV[S(t)] - (1 + \pi)PV[H(t)]$$

At the initiation of the swap, the premium is set such that both sides of the swap are equal (the initial value of the contract is zero).

$$\pi = \frac{PVS(t)}{PVH(t)} - 1$$

A sensitivity analysis will be conducted on the resulting data. This is done by evaluating the change in swap value resulting from a change in the parameters of the pricing model. The influence of changes in different parameters on the price of the swap is examined where the other parameters are held constant.

The parameters examined are;

1. Interest rates which will be used to discount the survival probabilities. Movements in the interest rates in the model indicate the level of compensation per unit risk.
2. Market price of risk: This is determined by the difference between the market annuity price and the purchase price of an annuity based on the expected survival rates.
3. Cohort – this refers to grouping of the sample population born in the same time period. In determining the premium, the cohort effect is used to determine whether the price increases or decreases with age
4. Maturity of the longevity swap will be used to determine its influence on the price of the swap given whether it is a long or short period.

Chapter 4: RESEARCH FINDINGS AND DISCUSSION

4.1 Assumptions of the model

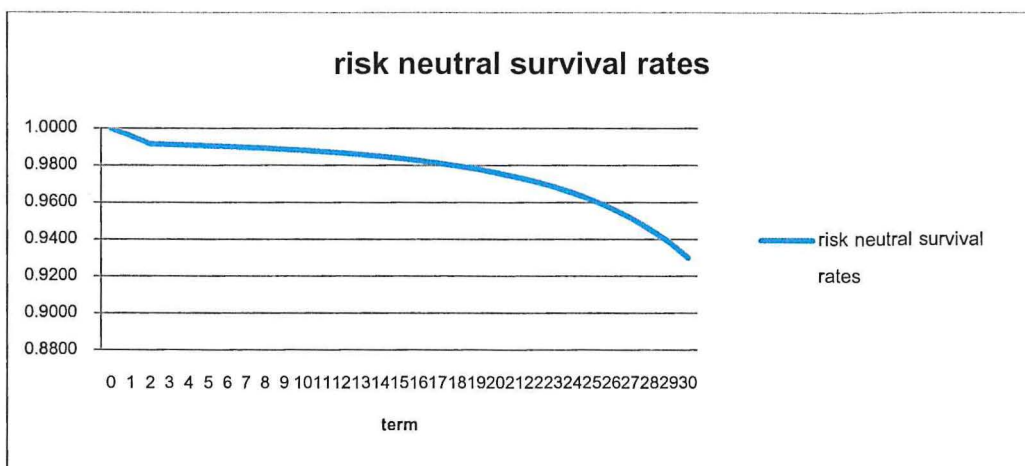
The model used is the Wang transform (Wang, 2002). The swap cohort was male aged 60 with a term of 30 years. The market risk was calculated as the difference between the annuity market price taken from the annuity quotation of ICEA Lion for a male aged 60 with 6% contribution rates and the annuity expected price. In discounting the fixed leg the yields from a 23 year bond are used and assumed to remain constant for the last years. For the interest rates the risk margin of 3% from the bond yields was applied.

4.2 Fitting the model

Survival rates

Mortality rates for annuity male q_x (graduated rates) were obtained from the Kenya mortality tables (KE 07-010 mortality tables). The probability of survival (p_x) was then calculated as $1 - q_x$. In using the Wang transform the underlying distribution of the best estimate survival probability p is transformed making the survival rates risk neutral as longevity risk is added to it. Wang transform assumes a normal distribution of the risk neutral survival rates. The risk neutral rates obtained from Figure 3 decreases over the years with increase in age. At time 0 the probability of survival is 1, and it is higher at younger ages than older ages. Towards the end of the term of the swap, i.e between years 26 – 30, it decreases at faster rate.

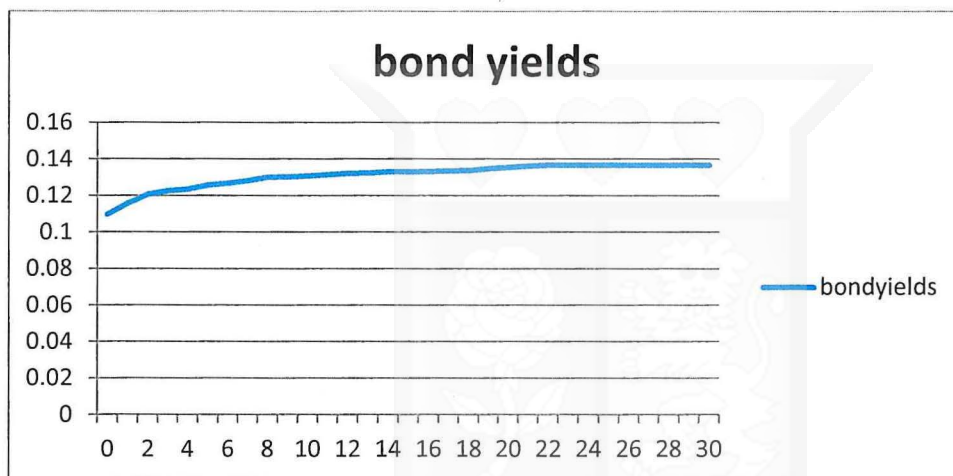
Figure 3: risk neutral survival rates



Interest rates

The interest rates are used in discounting the floating payments. They are obtained from the bond yields less the risk margin assumed at 3%. The bond yields are risk free rates for the 23 year bond on the NSE (Nairobi Securities Exchange) issued by the government. For the 23 years the yield continue to rise but for the remaining 7 years the bond yields were assumed to remain constant over time as shown in the Figure 4.

Figure 4: Bond yields



Fixed leg and the floating leg

The fixed leg is the calculated by discounting the best estimate mortality rates and annuity amount using the risk free rates. Given the survival rates the floating payments are the obtained as;

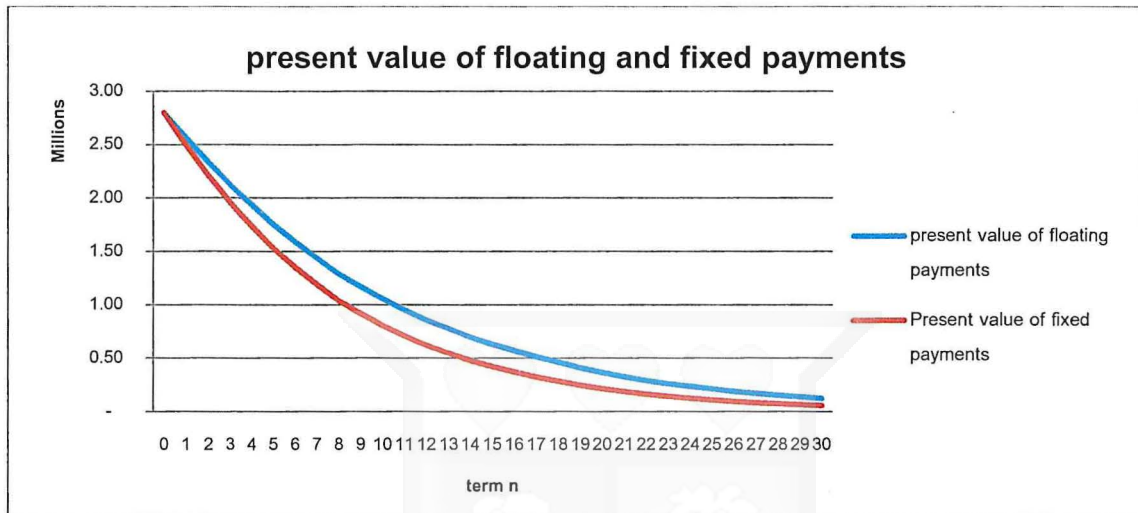
$$\begin{aligned} PVS(t) &= (\text{risk neutral survival rates} * \text{annuity amount}^2) * \text{discounting factor} \\ &= (F^*(p) * a_{60})V^n \end{aligned}$$

Both the floating leg and fixed leg are reducing over time with the floating leg being higher than the fixed as indicated in Figure 5. However, at initiation of the contract i.e time 0 the values of both fixed and floating cash flows are equal. The counterparty pays the insurer whenever the floating rate exceeds the fixed rate the insurer expected to pay out to the annuitants.

² The annuity amount was obtained from the ICEA Lion annuity quotation for a 60 year old male (single life) with 0 guaranteed period and 0% annual escalation

In this case the floating leg exceeds the fixed amount from year 2 to the end of the period, but the difference increases as time increases but starts to close up towards the end of the term.

Figure 5: present value of fixed and floating payments



Swap value

The swap value is calculated as the difference between the present value of the floating payments and the present value of fixed payments. At initiation of the contract the value of the swap is 0 as the floating and fixed payments are equal.

Figure 6: The graph of the swap value against the term of the swap

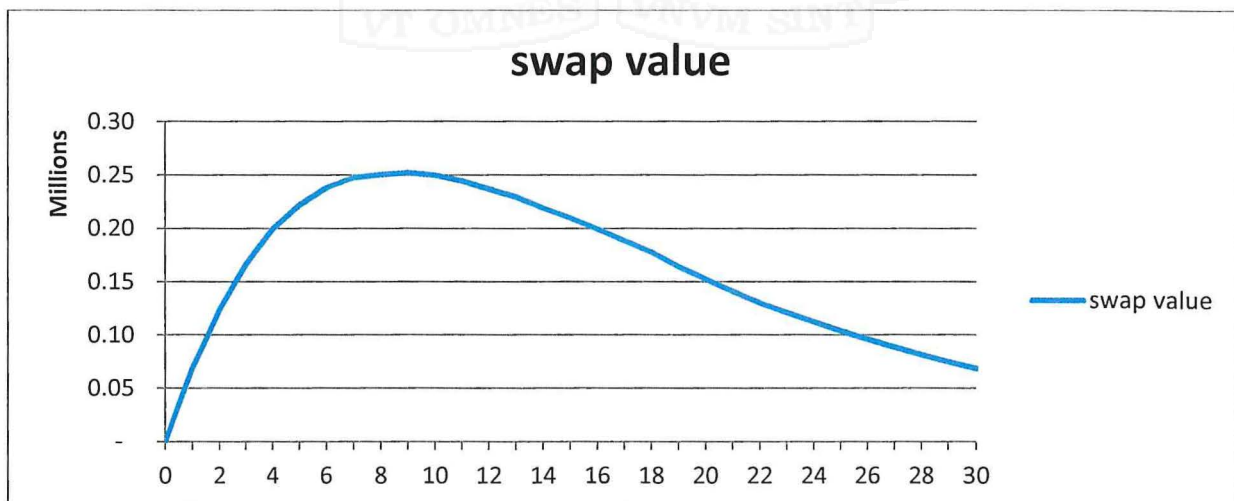
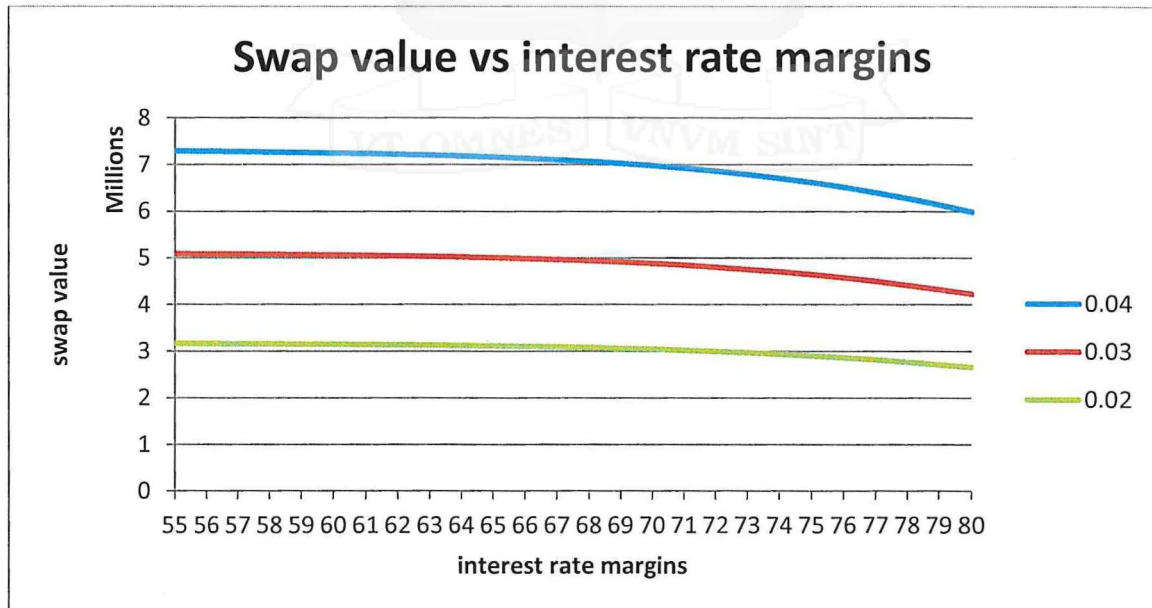


Table 2: Sensitivity of swap value to changes in the risk margin and the cohort ages

	5,060,285.03	0.04	0.03	0.02
entry age				
	55	7294831	5088994	3168443
	56	7288238	5084629	3165904
	57	7280754	5079677	3163026
	58	7272241	5074048	3159753
	59	7262532	5067628	3156022
	60	7251422	5060285	3151751
	61	7238666	5051856	3146848
	62	7223973	5042150	3141200
	63	7207003	5030942	3134675
	64	7187355	5017968	3127120
	65	7164563	5002921	3118355
	66	7138096	4985448	3108171
	67	7107347	4965149	3096333
	68	7071644	4941576	3082577
	69	7030257	4914242	3066613
	70	6982411	4882627	3048131

Figure 8: The graph of swap value against changes in interest rate margins



Chapter 5: CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

In describing the mechanics of a longevity swap the distortion approach(Wang, 2002) was considered as the pricing model. The Wang transform method is based on a normal transformation of the survival probabilities and discounts the two legs of the swap back to the moment of initiation of the contract, enabling the determination of the swap value.

A sensitivity analysis was performed. For the premium resulting from the Wang transform method, the effects of changes in interest rates, the market price of risk, the maturity and the cohort the swap is based on were considered. The relationship between the interest rates and the swap value is an inverse relationship as increase in interest rates results in decrease in swap value. An increase in maturity causes increasing swap value, as the uncertainty about survival rates further in the future is larger. In consideration to the cohort when the swap is based on younger cohorts, the swap value decreases along with the mortality rates. Westland (2009) in her findings also concluded that a longer maturity of the contract comes with a higher price of the longevity swap. This makes sense because at the initiation of the swap, the accurateness of mortality rate estimates will be better for the nearby maturities.

From the findings, it is possible to conclude that a longevity swap is effective in hedging longevity risk. In a longevity swap the receiver is compensated for decreases in mortality rates beyond the expected decreases while he has to pay money when the actual mortality rates turn out to be higher than the expected.

5.2 Limitations

The Wang transformation requires a value for the market price of risk parameter (λ) which is determined by the difference between the market annuity price and the purchase price of an annuity based on the expected survival rates. The main drawback from this is that; market prices do not only contain the best estimate of future mortality and the market price of longevity risk, but also the market price of interest rate risk and a margin for costs. Another is that different insurers will have different prices for annuities (Westland,2009). One other limitation of the distortion approach is that it assumes a normal distribution for the risk neutral survival rates.

5.3 Recommendations

For Kenya the longevity market is not in existence compared to other countries such as the UK. It is therefore necessary to develop the market if hedging using longevity swaps is to be an effective option. This is because in its existence the market will allow for more efficient pricing and distribution of risk and permit a growth in size of market that involve longevity risk. A longevity market will also increase liquidity for the parties involved (Loeys et al., 2007).

When the market in Kenya exists and more information on the pricing is available, it will also enable an examination of the hedge effectiveness when the environment changes during the term of the swap.



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