2024

Modeling the dependence between equity returns and foreign exchange rates in Kenyan financial markets - a copula approach.

Lanya, Jacob O.
Strathmore Institute of Mathematical Sciences
Strathmore University

Modeling the Dependence Between Equity Returns and Foreign Exchange Rates in Kenyan Financial Markets:A Copula Approach

By

Jacob O. Lanya, 136352

Submitted in Partial fulfillment of the requirements for the degree of Master of Science degree in Mathematical Finance and Risk Analytics at Strathmore University

Institute of Mathematical Sciences
Strathmore University
Nairobi, Kenya

February, 2024

DECLARATION AND APPROVAL

DECLARATION

I declare that this work has not been previously submitted for the award of a degree by this or any other university. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Student's Name: **Jacob O. Lanya**

Signature: .... Date: 29/02/2024

APPROVAL

The thesis of Jacob O. Lanya was reviewed and approved for examination by:

Signature:......................... Date: 29/02/2024

Dr. Stanley Odhiambo Sewe

Lecturer, Institute of Mathematical Sciences

Strathmore University

ACKNOWLEDGEMENTS

I would like to take this opportunity to express my deepest appreciation to all those who made it possible for me to complete this thesis.

I am highly indebted to my supervisor Dr. Stanley Sewe for his guidance and support as well as for the useful remarks, comments, and engagements throughout the study.

My thanks also go to my fellow classmates and all other people not mentioned for the support and encouragement.

Lastly, and most importantly I would like to thank the Almighty God for being the main source of success throughout the entire course.

ABSTRACT

The dependence structure between financial assets is critical in risk management and portfolio diversification. Using the copula approach, this study sought to investigate the dependence structure between the Kenyan stock market and the foreign exchange market over the period 2011 to 2022. We first estimated marginal distributions by the ARMA-GARCH model with normal, student t, and skewed t distributions. To model dependence between the return series, we used elliptical copulas (Gaussian, student t) and Archimedean copulas (Gumbel, Clayton, Frank). We also compared the best-fit copula with the time-varying student t copula and time-varying Gaussian copula. The best-fit copula was later used to measure portfolio tail risk. Parameter estimation was based on inference for margins technique. The best-fitting marginal model and copula were selected using AIC and BIC. Our findings show that the ARMA(1,1)-GARCH(1,1)-t distribution model was the best fit for margins. Student t copula was the best fitting copula. We found the existence of symmetric dependence and tail dependence between the variables. Portfolio risk is measured using VaR. The analysis suggested that the choice between the equally weighted portfolio and the Global Minimum Variance portfolio depends on the investor's risk tolerance and investment objectives. While a more conservative approach might favor the Global Minimum Variance portfolio, investors seeking to balance risk and return might find the equally weighted portfolio or a mix of assets according to the optimal weights more suitable.

Keywords: Dependence, Copula, Foreign Exchange, Equity returns, Portfolio risk, Marginal distributions, Value at Risk

TABLE OF CONTENTS

ABBREVIATIONS

VaR	Value at Risk
CVaR	Conditional Value at Risk
IFM	Inference Function of Margins
CML	Canonical Maximum Likelihood
MLE	Maximum Likelihood Estimator
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
MA	Moving Average
AR	Autoregressive
NSE	Nairobi Securities Exchange
Forex	Forex Exchange

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

1 INTRODUCTION

1.1 Background of the study

The relationship between the returns on financial assets may significantly impact decisions on risk management and portfolio diversification. Along with the projected return and risk of each asset, the dependence structure between the assets will impact the portfolio returns distribution and, consequently, the value that investors derive from a certain investment choice (Chen et al., 2004). Modeling the dependence between financial assets has been a subject of debate among academics and finance industry practitioners for quite some time. Understanding the consequences of these relationships is crucial for risk management and portfolio allocation. In the recent past, the world economy has experienced various economic and financial crises that adversely affected the financial markets (Maneejuk and Yamaka, 2019). It is therefore crucial to investigate and establish whether and how markets are linked. This will be useful for investors, policymakers, and financial institutions to plan financial strategies in the likelihood of a potential crisis.

The relationship between stock and foreign exchange markets is subject to diverse interpretations, with compelling arguments supporting either positive or negative correlation. The negative correlation could occur due to the return-chasing effect (Hau and Rey, 2006; Wang et al., 2013). In this case, whenever the equity market becomes bullish, international investors would be attracted which leads to currency appreciation in that country. By contrast, the positive correlation may stem from the effects of portfolio rebalancing and exchange rate exposure (Hau and Rey, 2006; Wang et al., 2013). In this scenario, a bullish market encourages domestic investors to shift their capital to other countries where stock markets present more attractive investment opportunities. This leads to the depreciation of currency in that country. These arguments imply that determining a dependence structure is a matter of empirical investigation rather than theoretical deduction. Consequently, flexible models are essential for accurately representing the dependence structure

Traditionally, modeling of multivariate distributions has been predominantly centered on the multivariate normal distribution. In this case, the measure of dependency is associated with Pearson's correlation coefficient, commonly employed for variables that are jointly normally distributed. However, empirical evidence showing that asset returns are skewed, leptokurtic, and have fat-tailed distributions has been recently presented by Cuñado and de Gracia (2003) and Aloui et al. (2013). Longin and Solnik (2001) showed evidence of multivariate non-normality in negative tails. Poon and Granger (2003) support asymptotically independent models for dependence framework of tails of stock return and further added that extreme dependence occurs as a consequence of heteroskedasticity in the stock return process. There exists evidence of tail area dependence and that during extreme downward movements, markets may exhibit more dependence as compared to upward movements. Furthermore, Andersen et al. (2006) indicated that correlation and volatility are time-varying. Embrechts et al. (2002) found that it is misleading to use linear correlation in a non-elliptical world and enlisted some of the limitations posed by linear correlation in the study of dependence of financial time series.

The limitations of linear correlation have led to research on flexible models of dependence between financial assets. Copula models, introduced by Sklar (1959), have been used to model the dependence structure of multivariate financial time series and found to perform better. Several papers on copula theory and applications have been published, covering various aspects and applications of copulas in different fields. Some notable ones include Joe (1997) and Nelsen (2006), which are the foundational textbooks providing comprehensive introductions to copula theory with a focus on statistical foundations. Cherubini et al. (2004) approached copulas with a focus on their application within mathematical finance while McNeil et al. (2015) discussed copula methods in the context of risk management. Patton (2009) summarized copula applications in financial time series. Kolev et al. (2006) offered a review of copula methods and recent developments while Patton (2012) offered a brief overview of copula-based techniques applicable to both univariate and multivariate time series analysis. There is extensive literature in which copulas have been used to model the linkages between international

markets and dependence between exchange rates and recently, studies have started to consider modeling dependence across markets of different asset types (Kamal and Haque, 2016; Li et al., 2019; Ning, 2010; Sewe et al., 2014; Hu, 2006).
The 2007-2008 global financial crisis demonstrated how risk can spread across global markets (Chiang et al., 2007). Numerous market behaviors demonstrated that as negative news surfaces in one market, it quickly spreads to other markets (Li et al., 2019). As a result, domestic markets are susceptible to the quick spread of global financial crises. The Kenyan economy is more dependent on imports as presented by Wamalwa and Were (2021), so it is crucial to consider how well connected the Kenyan markets are to the world market. The likelihood of the contagion effect occurring is higher when there is a strong correlation between the Kenyan stock market and the foreign exchange market. This study shall investigate the link between the Kenyan stock and foreign exchange markets using parametric copulas. Our focus is the dependence structure between the returns of these markets from a risk management and diversification perspective which is crucial for investors.

1.2 Kenya Financial Markets

1.2.1 Kenya Stock Market

Nairobi Securities Exchange previously known as Nairobi Stock Exchange, was established in 1954. NSE has experienced massive growth since its establishment. Currently, NSE has 65 listed companies. In 1964, the NSE 20 share index was introduced as a benchmark index that represents the 20 best-performing companies in the exchange. NASI was later introduced in 2008 as a complementary index that comprises all companies listed in the NSE. By promoting savings and investment and assisting local and foreign businesses in accessing cost-effective financing, the NSE is playing an important role in the expansion of Kenya's economy. The Nairobi Stock Exchange is regulated by the Capital Markets Authority of Kenya. As of 2022, the equity market was negatively impacted. The decline was attributed to the rising of domestic and international interest rates leading to capital outflows and a shift towards fixed-income assets (Exchange, 2023).

1.2.2 Kenya Foreign Exchange

In 1993, all exchange control laws were repealed and Kenya moved to an exchange rate system that is fully market-determined hence attracting short-term capital inflows. The Central Bank of Kenya releases indicative rates meant to help those exchanging currencies gauge the value of the Kenya shilling. The exchange rate generally is determined by the demand and supply. Foreign exchange helps in the determination of a country's economic health. Fluctuations in foreign exchange affect international investment and have a direct impact on international trade and buying power. The Kenyan shilling depreciated by 3.5% against the US Dollar in 2021 in addition to the 7.7% depreciation in 2020 (Cytonn, 2021). In 2022, Kenya experienced increased inflation, averaging 7.65% for the year. Additionally, the Kenyan shilling depreciated further against the US Dollar ending the year at Kshs. 123.37 compared to Kshs. 113.14 in 2021. Therefore it is crucial for investors to understand if there is the existence of any dependence for the purpose of diversification.

1.3 Problem Statement

The dependence structure between financial markets provides crucial information to investors and policymakers by explaining how one market responds to fluctuations in other markets. Although linear correlation has been widely used to model the dependencies, Embrechts et al. (2002) established that the use of linear correlation is inappropriate and misleading in situations where returns have non-elliptical distributions. Copula models have been found to overcome the shortcomings presented by GARCH, correlation, and conditional correlation.

While copulas have been utilized extensively in empirical studies concerning modeling the dependence between equity and foreign exchange, most have focused on developed and emerging markets (Li et al., 2019; Ning, 2010; Wang et al., 2013) with very little research done on Sub-Saharan Africa markets like Kenya. Furthermore, most empirical research findings rely completely on statistical evaluations of copula models rather than on risk management with the authors not distinctly explaining the significance of dependence between the assets in risk management. Sewe et al. (2014) considered Kenya and applied semi-parametric copula-based multivariate dynamical models (SCOMDY) to investigate the dependence between these two variables. This research seeks to consider the use of parametric copula models to study the relationship between the Kenyan stock and foreign exchange markets and apply the nature of dependence between these variables to estimate portfolio risk.

1.4 Research Objective

The main objective of this study is to investigate the dependence between equity returns and foreign exchange rates in the Kenyan Financial markets using parametric copulas and application of these findings to portfolio risk estimation.

Specific Research Objectives

1. To model the univariate returns from equity and forex markets
2. To model the dependence between equity returns and foreign exchange returns
3. Estimate the portfolio risk of the assets using the best fitting copula

1.5 Research Question

The questions this dissertation will try to answer are:

1. Are there significant tail dependencies between equity returns and foreign exchange rates?
2. If the tail dependencies exist, is there a significant asymmetric or symmetric dependence?
3. Which of the copula models captures the dependence with the best fit?

1.6 Significance of the study

Given the broad research undertaken on dependence structure among financial markets in developed economies, there is a need to extend the same literacy content to African frontier markets such as Kenya. Determining the dependence structure between financial assets is significant for various reasons. Stock markets are a reflection of the growth status of economies. Understanding the dependence between these variables will therefore enable policymakers to understand how one market responds to fluctuations in another market and in turn formulate growth-oriented policies. From an international investor's point of view, it is important to understand dependence for portfolio diversification purposes. Dimitrova (2005) points out that with a one percent decline in the exchange rate, equity markets react with less than one percent. Hence, risk reduction becomes ineffective by diversification with the existence of a positive correlation between markets and diversification can only be possible if the markets are negatively correlated. Academically, studying the interaction between asset markets is appealing since it is at the center of asset pricing as we know CAPM is based on the relationship between risk and expected return (Sharpe, 1964; Markowitz, 1952).

CHAPTER TWO

2 LITERATURE REVIEW

2.1 Dependence between Foreign Exchange and Equity Markets

Extensive research, empirical and theoretical, has been conducted on the co-movements and relationship between equity and currency markets. Stock-oriented model (Branson, 1983) and flow-oriented model (Dornbusch and Fischer, 1980) are the most commonly used models in theoretical studies regarding the interdependence between equity and exchange rates. Stock stock-oriented model postulates that the supply and demand of assets determine the exchange rate. An increase in stock prices encourages demand for local assets from foreign investors thereby appreciating the local currency. Therefore, the stock-oriented model suggests a negative relationship exists between equity prices and exchange rates. On the other hand, the flow-oriented model postulates that a positive relationship exists between exchange rates and stock prices. Dornbusch and Fischer (1980) stated that a country's current account balance is significant when determining the exchange rate. The flow-oriented model assumes that trade balances can be affected by fluctuations in the exchange rate.

Empirically, the study of dependence between stock prices and foreign exchange rates has been extensively done employing diverse datasets and methodologies. However, the findings across studies are varied. Bashir et al. (2016) observed a positive correlation between the stock market and forex in Latin America using Granger causality and a detrended cross-correlation approach. Diamandis and Drakos (2011) conducted cointegration and multivariate Granger-Causality tests and found positive relations in Latin America. In a study focusing on six Asian countries, Lee et al. (2011) utilized the Smooth Transition Conditional Correlation-GARCH (STCC-GARCH) model. Tsai (2012) applied a quantile regression model and found a negative relationship specifically when exchange rates reached extreme levels. Dahir et al. (2018) employing wavelet analysis, revisited the link between stock prices and forex in BRICS nations. They identified a positive relationship in Brazil, Russia, and India, no correlation in China, and bidirectional causality in South Africa. Kim (2003) investigated the long-run equilibrium between stock prices and the dollar exchange rate in the US using a multivariate cointegration and error correlation model. Other researchers have employed multivariate GARCH (Jain and Biswal, 2016; Tastan, 2006) and Vector Autoregressive Mode (Delgado et al., 2018) to explore the dynamic relationship between stock prices and foreign exchange rates.

The use of conventional Pearson correlation is inadequate for measuring dependence across financial markets because it treats positive and negative returns, as well as large and small realizations, equally. This approach can lead to a substantial underestimation of risk from joint extreme events(Poon et al., 2004; Tastan, 2006). In response to these concerns, some researchers have turned to multivariate-GARCH models (Ang and Chen, 2002) or regime-switching models (Ang and Chen, 2002) to more accurately capture the joint dynamics of returns.

2.2 Copula and Dependence structure

The joint distribution function $H(x_1....x_n)$ completely describes the dependence between real valued variables $x_1......x_n$. Multivariate distribution problems with specified univariate marginals have evoked substantial attention (Kolev et al., 2006). The concept of copulas introduced by Sklar (1959) was brought by decomposing *H* into a function that describes the dependence structure and one that describes only the marginal behavior. Therefore, a copula is a function that joins a multivariate distribution to its one-dimensional marginal distribution function. Sklar's theorem is instrumental in elucidating how copulas are essential in defining the relationship between a multivariate distribution function and its univariate marginal distributions.

Copulas have gained popularity in dependence modeling. The main reason for its success is that copula functions allow for simple and flexible modeling of joint multivariate distributions. Secondly, Copulas allows for the modeling of marginal distributions independent of the dependence structure thus giving us more flexibility in model estimation and specification. The invariance of copulas and copula-based methods of dependency to any increasing change of the underlying individual time series is one of its key properties. Since numerous time series are frequently found to be nonlinear and asymmetrically dependent on one another in financial series, the copula technique allows for flexible modeling of these relationships without being constrained by the marginal distribution of the individual time series (Chen et al., 2004).

Copula families, representing a diverse set of joint distribution functions, allow estimation of both symmetric and asymmetric distributions, along with tail dependency. These features find applications in modeling financial returns characterized by skewed, leptokurtic, and fat-tailed distribution. The Gaussian and student t copulas ate the two basic copulas and are referred to as elliptical copulas. Elliptical copulas are copulas of elliptical distribution (Embrechts et al., 2001). Gaussian copulas are copulas of bivariate normal distribution. The Gaussian copula does not capture the tail dependence. On the other hand, student t copulas are copulas of the bivariate t distribution. The degree of freedom parameter in the student t copula makes it possible to capture both upper and lower tail dependence. Both student-t and Gaussian copulas are classified within the symmetric copula family. In regards to the Student-t copula, known for its sensitivity to both upper and lower tails, there exists a symmetric dependence structure, suggesting equality in both tail dependence measures. The major drawback of these elliptical copulas is that they lack closed-form expressions and are constrained to exhibit radial symmetry (Embrechts et al., 2001). In many financial applications, there's a prevalent observation of stronger dependency among extreme losses compared to extreme gains. This indicates that Gaussian and Student's t copulas, which are symmetric, cannot adequately capture such asymmetrical characteristics.

Numerous parametric families of copulas fall under the category of Archimedean copulas, offering a wide array of distinct dependence structures including both symmetrical and asymmetrical dependency. The Archimedean copula can be generated from various functions. For instance, the Gumbel copula was proposed by Gumbel (1960). This copula family is asymmetric and exhibits a greater dependence on the upper tail. Clayton (1978) proposed a family of bivariate distributions tailored for survival data. The Clayton copula exhibits sensitivity to the lower tail and falls under the asymmetric family. The Frank copula was introduced by Frank (1979). The Frank family lacks tail dependence similar to the Gaussian copula and falls under the symmetric copula category. Another Archimedean copula family was proposed by Joe (1997) and is sensitive to the upper tail. While the variety of Archimedean copulas allows for both lower and upper tails estimation in multivariate distributions, they have limitations. For instance, certain families only permit modeling of non-negative dependence between assets. Additionally, a single copula function is utilized to model the entire dependence structure for a multidimensional dataset.

2.3 Application of copulas in Finance

The main motivation for employing copulas in finance is the existence of empirical evidence that financial asset returns have a non-Gaussian distribution. The evidence of non-linearity has broad implications for financial decision-making in derivative pricing, risk and portfolio management, credit risk, study of financial markets contagion among others. This section discusses some of the literature on dependence patterns between equity and foreign exchange markets.

Ning (2010) examined the dependence structure between equity markets and forex markets in G5 countries before and after the introduction of the euro. She used three copula models; the normal copula which is symmetric with zero tail dependence, the student t-copula which has symmetric non-zero tail dependence and the SJC copula which allows for asymmetric tail dependence. Ning (2010) finds significant positive tail dependence between two variables in both periods in each country and symmetric tail dependence. Using the SJC copula, Michelis and Ning (2010), studied the co-movements between TSX index (Canadian stock market) rates of return and the US/Canada exchange rate. They pointed out that the SJC copula function allows for both asymmetric and symmetric tail dependence. Michelis and Ning (2010) found the existence of dynamic and static asymmetric tail dependence between the two sets of returns. The asymmetry is such that there is significantly greater dependence on the lower tail as compared to the upper tail.

Sewe et al. (2014) explored the use of copulas to examine the dependence between equity and foreign exchange in Kenya. They used a semi-parametric copula-based multivariate dynamic model to investigate the dependence. Student's t-copula was found to be the best parametric model to capture the dependence structure in the data. Sewe et al. (2014) found significant symmetric dependence and tail dependence between the Kenyan stock market and foreign exchange. Meanwhile, Kamal and Haque (2016) using five copula functions found the existence of significant asymmetric dependence with upper tail dependence between the foreign exchange market and the stock market in South Asia.

Wang et al. (2013) developed a dependence-switching copula to examine dependence and tail dependence between stocks and foreign exchange and suggested that analyzing cross-market correlation within a time-invariant copula structure may not be appropriate. Employing the dependence-switching copula model developed by Wang et al. (2013), Kumar et al. (2019) investigated the dependence structure between BRICS stocks and foreign exchange markets. They established that dependencies and tail dependence were symmetric in four market conditions except for Russia during the negative regime. They also found that during negative regimes, dependencies were asymmetric but tail dependence was symmetric for all countries. Furthermore Li et al. (2019) also used unconditional and conditional copula models to capture the dependence between the Chinese stock market and the RMB foreign exchange rates and found that conditional copulas outperformed unconditional copulas.

2.4 Time Varying Dependence

Patton (2006b) introduced the study of time-varying conditional copulas by extending Sklar's theorem to conditional distribution. Subsequently, time-varying copulas have been applied in numerous studies and new models have been developed to appropriately model time-varying dependence. Patton (2006b) proposed a copula model for which the copula parameter for time-varying dependence is a parametric function of transformations of the lagged data and an auto-regressive term.

Creal et al. (2013) established a unifying and consistent structure named Generalized Auto-regressive Score (GAS) for introducing time-varying parameters in a time series process. The GAS model uses a scaled score vector as an updating technique for the observation-driven part. The GAS specification demonstrates a correlation process that varies more persistently over time compared to the Patton model. However, the time-varying parameter reacts more strongly with the increased sensitivity to the opposite sign of exchange rate returns if the current correlation estimate is positive. The GAS model has also been shown to exhibit greater sensitivity to observations located in the lower and upper tails.

Jondeau and Rockinger (2006) is another literature that deals with dynamic copulas in which a similar approach to that of Patton is employed. In this literature, the dependence parameter is rendered to be conditional and time-varying. Hafner and Manner (2012) considered a time-varying copula model in which the dependence parameter observes an autoregressive process. The model includes a Gaussian copula with a stochastic correlation process analogous to stochastic volatility models. On the other hand, Chollete et al. (2009), BenSaïda (2018), da Silva Filho et al. (2012), Gurgul and Machno (2016), Garcia and Tsafack (2011) considered regime-switching models for the conditional copula which the functional form of copula are allowed to change over time.

2.5 Estimation of Copula Parameters

When using copula functions to model the joint density of two random variables, we need to consider how copula parameters will be estimated efficiently and correctly. A number of estimation techniques exist in the literature. The first method is the exact maximum likelihood technique where all copula parameters and the marginal parameters are estimated at the same time. The second technique is the inference functions of margins, whereby parameters of the marginals are first estimated and with these parameters, the copula parameters are estimated. The canonical maximum likelihood method, with unknown marginal densities, applies the empirical probability integral transform to get the uniform marginals required for copula parameters estimation. The other techniques are non-parametric ways of copula estimation. The first technique is such that the empirical copula is estimated from the data directly, leaving the whole specification non-parametric. The second method was introduced by Genest and Rivest (1993) and is most appropriate for Archimedean copulas. First, the non-parametric estimate for Kendall's tau is obtained and an estimate of the copula parameter is obtained using the relationship between Kendall's tau and the copula parameter.

2.5.1 Exact Maximum Likelihood Method

Maximum likelihood is an appropriate estimator for parametric marginal distributions (Fan and Patton, 2014). Before obtaining the maximum likelihood estimator, the log-likelihood function is obtained. Considering a d-dimensional distribution and given that the copula function and the marginals are continuous, then the join *pdf* is expressed as

$$f(x_1, ..., x_d) = c(f_1(x_1, \alpha_1).......f_d(x_d, \alpha_d); \theta) \prod_{i=1}^{d} f_i(x_i, \alpha_i) \tag{2.1}$$

which is the product of copula density and the marginal densities with θ being the copula parameter and α the vector parameter of the marginal distribution. The copula density is defined as $c = \frac{\partial^d C}{\partial F_1 ....\partial F_d}$. Therefore the log-likelihood function is given by

$$\begin{aligned} \mathcal{L}(\theta, \alpha) &= \sum_{i=1}^{n} \log \left[c\{F_1(x_{1i}, \alpha_1)........F_d(x_{di}, \alpha_1), \theta\} \prod_{j=1}^{d} f_j(x_{ji}, \alpha_j) \right] \\ &= \sum_{i=1}^{n} \left[\log c\{F_1(x_{1i}, \alpha_1), ...F_d(x_{di}, \alpha_1), \theta\} + \sum_{j=1}^{d} \log f_j(x_{ji}, \alpha_j) \right] \end{aligned} \tag{2.2}$$

Hence by maximization, we obtain the *full maximum likelihood estimator*

$$(\hat{\theta}\hat{\alpha}) = \arg\max \mathcal{L}(\theta\alpha) \tag{2.3}$$

The ML estimator can be shown to be consistent and asymptotically normal under certain regulatory conditions (Cherubini et al., 2004). However, in some situations, it may create computational burden (Fan and Patton, 2014).

2.5.2 Two-Stage Estimator-IFM

Alternatively, a two-stage estimator which is also known as ***inference functions of margins*** could be implemented (Joe, 2014, 1997; Cherubini et al., 2004) though generally referred to as multistage maximum likelihood estimation. This estimator employs a good form of the copula decomposition of a joint distribution. The first stage involves estimating the parameters from the marginal distributions

$$\hat{\alpha_i} = \arg\max_{\alpha} \mathcal{L}(\alpha_i), i = 1, 2, ... \tag{2.4}$$

The second stage is estimating the copula parameter by maximizing the copula density

$$\hat{\theta} = \arg\max_{\theta} \mathcal{L}(\theta) \tag{2.5}$$

Joe and Xu (1996) indicated that the IFM estimator is asymptotically normal and consistent. The multi-stage estimation yields parameters that are less efficient than the one-step maximum likelihood estimator, although Patton (2006a) and Joe (2005) through simulation studies show that the one-stage estimator does not perform well. According to Okhrin et al. (2013), multistage estimation advantage lies in its reduction of the computational complexities.

2.5.3 Canonical Maximum Likelihood Method

Alternatively, when the marginal model is unknown(non-parametric), the marginal distributions are modeled with empirical *cdf* and copula estimation on the ranks of data (Chen and Fan, 2006). In the CML method, empirical probability integral is used to first transform the sample data of interest the sample data of interest and the maximum likelihood estimator is used to estimate the copula parameters. Concerning the parametric case, the difficulty here is that the copula likelihood depends on the marginal parameters α and infinite-dimensional parameter F_i(Patton, 2013). Again, just like the two stages, a loss of efficiency problem occurs. However, canonical maximum likelihood is powerful to the marginals misspecification which may cause biased estimates of copula parameters.

2.5.4 Other Estimation methods

Although the most prevalent estimation method used in literature is Maximum likelihood estimation, other methods have been considered. Bayesian estimation is considered in Arbel et al. (2019) and Smith et al. (2012). Rémillard (2017) considered methods of moments-type estimator where parameters of a given copula family have known mapping to a dependence measure. Oh and Patton (2017) considered simulated methods of moments and Generalized methods of moments where the number of measures of dependence may be more than the number of unknown parameters.

2.6 Model selection and Goodness of Fit

Once the parameters of the copula have been obtained, it is a critical issue to compare the competing models. So far there is no consensus on which copula to use in specific applications or how a particular copula's accuracy can be tested. The goodness of fit tests are used to evaluate the overall fit of the competing copula models and the fitness of the margins. A number of goodness-of-fit tests have been proposed for testing the specification of copula. Examples of this tests include Chen et al. (2004),Fermanian (2005), Genest et al. (2006) and Genest et al. (2009). Chen et al. (2004) and Berg and Bakken (2006) used tests that are based on the probability integral transform. Fermanian (2005), and Genest et al. (2009) assumed tests based on empirical copula process and propose tests based on Kendall's process that implies the use of non-parametric free statistics such as Anderson-Darling and Kolmogorov-Smirnov. Genest et al. (2009) put their emphasis on blanket tests and proposed application of a double parametric bootstrap technique.

Using Anderson-Darling tests and Kolmogorov-Smirnov, Kole et al. (2007) compared the accuracy of different types of copulas. The tests are used to directly compare the fit of the copula on observed dependence. The advantage of using these two traditional tests is that they can be used for any copula of any dimension. Wang (2010) and Savu and Trede (2008) proposed parametric families tests of Archimedean copulas. Savu and Trede (2008) test is based on the classical χ^2 statistic, even though its asymptotic distribution is not χ^2. On the other hand, Wang (2010) proposed two tests that are based on the Fisher transform of the correlation coefficient of a bivariate *U, V*. Scaillet (2007) proposed a non-parametric goodness of fit test for copula that is kernel-based with fixed smoothing parameters.

Besides the various tests that have been proposed, there exist model selection criteria that allow us to rank copulas according to their fit. Akaike information criterion is the most widely used criterion.

2.7 Copulas and Portfolio Management and Risk Analysis

The concept of dependence between assets is a crucial aspect of risk management with the implication on the market risk measure based on the Value-at-Risk concept that requires modeling of dependence between asset returns. For a long time, dependence between assets in a portfolio was assessed by linear correlation coefficient. However, research has revealed significant skewness and excess kurtosis induced by extreme market events hence disagreeing with the normality hypothesis. Copula provides for flexible modeling of multivariate distribution tail behavior. It is of interest to risk management since multivariate distributions' tail behavior explains the concurrent anomaly of asset returns. McNeil et al. (2015) pointed out that copula facilitates a *bottom up approach to multivariate model building*. This is especially important to risk management marginal behavior of individual risk factors are considered than their dependence framework.

2.7.1 Value at Risk and Conditional Value at Risk

VaR has become a common measure for assessing and measuring risk in finance. VaR is commonly defined as a quantile of portfolio distribution. In some instances, the negative of this quantile is considered so that higher levels of risk coincide with higher values of VaR (Rosenberg and Schuermann, 2006). Formally, let X be a random variable with continuous distribution function F to model the return distribution of a risky portfolio over a given time horizon. For a specified probability $1 - \alpha = q$, VaR is defined as the q^{th} quantile of the F distribution.

$$VaR^{\alpha} = F^{-1}(1 - \alpha) \tag{2.6}$$

Artzner et al. (1999) proposed properties for measures of risk and referred to them as ”coherent”. These include homogeneity, monotonicity, subadditivity, and translation invariance. Importantly, VaR does not satisfy the subadditive property unless the underlying risk comes from a normal distribution (Embrechts et al., 2002). Conditional VaR, also referred to as expected shortfall (ES) is a coherent measure of risk that estimates the mean that exceeds VaR.$CVaR^{\alpha}$ and is therefore defined as,

$$CVaR^{\alpha} = E(R/R \geq VaR^{\alpha}) \tag{2.7}$$

The joint multivariate distribution complexity makes it hard to estimate the Value at Risk of a portfolio. Moreover, when the number of assets is increased, computational problems arise. Copula theory has been used to overcome this problem. Copula functions allow for the flexible construction of multivariate distributions with various dependence structures and margins. Copula could be applied to get more realistic multivariate densities as well as combine risks when the marginal distributions are estimated individually.

More recently, copula has been used explicitly to measure portfolio VaR. Karmakar (2017) concluded that BB1 copula is the most appropriate for capturing the dependence structure between several exchange rates in the Indian foreign exchange market and applied it to estimate portfolio risk using VaR and CVaR and global minimum risk portfolio is selected based on efficient frontiers. Wang et al. (2010) also found improved forecasting accuracy for portfolio VaR and Conditional VaR using Clayton and student-t copulas when compared to the Gaussian copula. Huang et al. (2009) demonstrated that copula models like Student-t outperform various benchmarks, including GARCH models and historical simulation when estimating and predicting portfolio Value at Risk. Kole et al. (2007) employed copula to estimate the U.S. stock market and bond indices. They demonstrated that the Student-t copula performs better than Gumbel and Gaussian copulas in capturing downside risk. Gumbel (Gaussian) copulas were found to overestimate (underestimate) the probabilities of extreme losses. Aloui and Aïssa (2016) investigated dependence structures between stock, energy, and currency markets using vine copulas and concluded that vine models result in better VaR forecasts than historical simulation and Gaussian techniques. Kumar et al. (2019) demonstrated the benefits of dependence modeling using the Clayton C-vine model, both in terms of VaR forecasting and portfolio performance.

2.8 Conclusion and Gap Identified in the Reviewed Literature

The literature discussed above highlights the use of copula in modeling dependence between returns on risky assets. Sewe et al. (2014) considers the Kenya market but implements the study using semi-parametric models. Furthermore, most of the existing literature findings rely totally on the use of the Goodness-of-fit test to statistically evaluate copula models rather than to apply the findings for risk management. The authors have not been able to plainly show how dependence between the assets is significant in risk management. Therefore this research considers parametric modeling of dependence in the Kenya context to investigate the existence of dependence between equity and foreign exchange rate markets. The study employs both static and dynamic copula in modeling dependence between these two markets. Furthermore, the best-fit copula is used to estimate the portfolio risk of the two risky assets.

CHAPTER THREE

3 RESEARCH METHODOLOGY

3.1 Data

The study considers the Kenyan stock and foreign exchange markets for analysis. We seek to examine the dependence between the returns of these two markets. The study considers the NSE 20 index as a proxy for the Kenyan stock market. The foreign exchange is represented by the Kenyan shilling against the dollar. The samples will be from the year 2011 to 2022.

3.2 Modeling of Margins

For financial time series, return at time t is given as

$$x_t = \log\left(\frac{P_t}{P_t - 1}\right) \tag{3.1}$$

where P_t is the price at time t. The mean equation of x_t is; (Engle, 1982)

$$x_t = E(X_t|\psi_{t-1}) + \varepsilon_t \tag{3.2}$$

where $E(X_t|\psi_{t-1})$ is the conditional mean of x_t given ψ_{t-1} which is the information at t-1. To describe the conditional mean and the time series dependence, we combine AR(*p)* and MA(*q*) to get an ARMA(*p,q*) model,

$$x_t = \mu + \sum_{i=1}^{p} \phi_i x_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \tag{3.3}$$

where ε_t is the white noise $(0, \sigma^2)$, ϕ_i and θ_j are parameters and μ is a constant. Despite many advantages of ARMA, their assumption of constant variance restricts their applicability. In literature, it is well documented that returns show conditional heteroscedasticity and fat-tails.

ARCH model introduced by Engle (1982) explains volatility clustering and heavy-tailed returns. The GARCH model, a generalized form of the ARCH model, as proposed by Bollerslev (1986), incorporates the ARMA model to describe the error variance. In

Equation 3.3, the ε_t terms are called the innovations of the time series process. They are described by Engle (1982) as an autoregressive conditional heteroscedastic process such that

$$\varepsilon_t = \sigma_t \eta_t \tag{3.4}$$

where $\sigma_t^2 = E(\varepsilon_t^2|\psi_{t-1})$ is the conditional variance of the error that changes over time and η_t is a sequence of iid random variables with mean 0 and variance 1. The variance equation of GARCH(*p,q*) model (Bollerslev, 1986) is given by

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \sigma_{t-j}^2 \tag{3.5}$$

$$\varepsilon_t = \sigma_t \eta_t$$

$$\eta_t \sim i.i.d$$

For modeling the marginal distribution, the ARMA-GARCH model is employed, where GARCH addresses the conditional variance and ARMA tackles the conditional mean. This model effectively captures conditional dependence and conditional heteroscedasticity. The disturbance term η_t will be selected from either Normal, student's t, and skewed student's t distribution, each with their respective density functions given by:

$$f_N(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}(x-\mu)^2/\sigma^2} \tag{3.6}$$

$$f_t(x;\nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi\nu}\,\Gamma(\frac{\nu}{2})} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2} \tag{3.7}$$

$$f_{skt}(x;\nu,\xi) = \frac{2\xi}{1+\xi^2} \left[f_t(\xi x;\nu) I(x<0) + f_t(\frac{x}{\xi};\nu) I(x \geq 0) \right] \tag{3.8}$$

where ν is the degree of freedom

3.3 Copula Function

The copula function allows for the creation of a multivariate distribution that maintains consistent marginal distributions for each variable, thereby offering flexibility in modeling the dependence between random variables. Nelsen (2006) provided an excellent account of the general theory of copula function and Patton (2006b) explained a time-varying specification to represent the dependence structure dynamics. Since our data is bivariate, we define the copula in bivariate form. The copula function can formally be described as follows:

Definition 3.1 *:A 2-dimensional copula is a function C:* $[0,1]^2 \rightarrow [0,1]$ *with the following properties:*

1. C *is grounded i.e* $C(u,0) = C(0,u) = 0$ *for any* $u \in [0,1]$
2. C *has margins i.e* $C(u,1) = C(1,u) = u$ *for any* $u \in [0,1]$
3. C *is* 2*-increasing i.e for all* $0 \leq u_1 \leq u_2 \leq 1$ *and* $0 \leq v_1 \leq v_2 \leq 1$
   $C([u_1, v_1] \times [u_2, v_2] = C(u_2, v_2) - C(u_1, v_2) - C(u_2, v_1) + C(u_1, v_1) \geq 0$

Theorem 3.1 (Sklar's Theorem) *Let H be a 2-dimensional distribution function with margins* F_1, F_2*. Then there exists a 2-copula C such that for all* $(x_1, x_2) \in [-\infty, \infty]^2$

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \tag{3.9}$$

Conversely, if C is a 2-copula and F_1, F_2 *are distribution functions, then function H is a 2-dimensional distribution function with margins* F_1, F_2*. Furthermore, if* F_1, F_2 *are continuous, then C is unique. Otherwise, C is uniquely determined on* $RanF_1 \times RanF_2$

Therefore, from Sklar's theorem, Copulas can be used to represent the dependence concepts as well as the separation of the continuous multivariate distribution functions, the univariate margins, and the multivariate dependence structure.

Corollary 1 *Given H is a bivariate joint distribution function with continuous marginals* F_1, F_2*, then the corresponding Copula C for any* $u \in [0,1]^2$ *can be constructed as,*

$$C(u_1, u_2) = H(F_1^{-1}(u_1), F_2^{-1}(u_2)) \tag{3.10}$$

Note that if continuous random variables u_1, u_2 with distribution functions as above, then *C* is the joint distribution function for the random variables $u_i = F_i(u_i)$, *i*=1,....*n* which are uniformly distributed (Kolev et al., 2006; Nelsen, 2005).

H(x,y)=C(F(x),G(y))

3.3.1 Conditional Copulas

Definition 3.2 (Conditional Copulas, Patton (2006b)) *Let X, Y, and W be continuous random variables. The conditional copula of (X, Y)/W where $X|W \sim F$ and $Y|W \sim G$, is the conditional join distribution function of $U = F(X|w) \sim U(0,1)$ and $V = G(Y|w) \sim U(0,1)$ given W.*

Hence, the conditional bivariate copula represents the joint distribution of the two random variables conditionally *Uniform*(0,1). Generally, *2 dimensional conditional copula* can be obtained from any distribution function such that the conditional joint distribution of the first 2 variables is a copula for every value of conditioning variables.

Theorem 3.2 (Sklar's Theorem for Continuous Conditional Distributions Patton (2006)) *Let H be the joint conditional distribution function of $(X,Y)|W$. Let F be the conditional distribution of $X|W$, G be the conditional distribution of $Y|W$, and $\mathcal{W}$ be the support of W. Assuming F and G are continuous in x and y. Then a unique conditional copula* C *exist such that*

$$H(x,y|w) = C(F(x|w), G(y|w)|w), \forall(x,y) \in [-\infty,\infty]^2, w \in \Omega \tag{3.11}$$

Conversely, if we let F and G be conditional distribution functions of $X|W$ and $Y|W$ respectively and C be a conditional copula, then function H in equation 8 is a conditional bivariate distribution function with marginals F and G.

That the conditional variable W must be the same for both marginal distribution and the copula is the complication that arises from extending Sklar's theorem to conditional distribution. For H to be a joint conditional distribution function, the same conditional variable should be used for F, G, and C.

Conditional copula function

Following Sklar's theorem for continuous conditional distribution, Let H be the joint conditional distribution of (X, Y)/W, F and G be conditional distributions of X/W and

Y/W respectively, and $\mathcal{W}$ be the support of W. Assuming F and G are continuous, a unique conditional copula *C* exist such that for all $x, y \in \mathbb{R}$ and $w \in \mathcal{W}$,

$$H(x, y|w) = C(F(x|w), G(y|w)|w) \tag{3.12}$$

By differentiating Equation 3.12 we obtain conditional joint density function:

$$h(x, y|w) = f(x|w) \cdot g(y|w) \cdot c(u, v|w) \tag{3.13}$$

where $c(u,v|w) = \partial^2 C(u,v|w)/\partial u \partial v$ is the conditional copula density.

3.4 Measures of Dependence

A key characteristic of a good measure of dependence for guiding the type of copula is that it should be scale invariant as described by Nelsen (2006) and should not be affected by strictly increasing transformations of the data. Linear correlation is not scale invariant and is affected by the marginal distribution of data. Pearson's linear correlation coefficient is said to be not a precise and absolute description of the dependence between variables (Embrechts et al., 2002). Given the familiarity of linear correlation as a measure, it is useful to report but we will augment it with the other measures of dependence.

3.4.1 Linear Correlation

This is a measure of the degree of linear relationship between two variables.

Definition 3.3 *Consider two random variables X and Y. The linear correlation coefficient is defined as*

$$\rho(X, Y) = \frac{Cov(X, Y)}{\sqrt{VarXVarY}} \tag{3.14}$$

and $-1 \leq \rho(X, Y) \leq 1$ *where* Cov(X,Y) *is the covariance between X and Y and* Var X *and* Var Y *are the variances of X and Y respectively.*

Linear correlation is a reasonable dependence measure in elliptical distributions which include Normal and Student-t distributions.

3.4.2 Concordance Measures

Let (x_j, y_j) and (x_k, y_k) be two observations from the random vector (X, Y). We say X and Y are concordant if $x_j < x_k$ and $y_j < y_k$ or $x_j > x_k$ and $y_j > y_k$ that is $(x_j - x_k)(y_j - y_k) > 0$.Similarly, X and Y are discordant if $x_j < x_k$ and $y_j > y_k$ or $x_j > x_k$ and $y_j < y_k$ i.e $(x_j - x_k)(y_j - y_k) < 0$
Therefore, two random variables X and Y are said to be concordant if small(large) values of X tend to be associated with small (large) values of Y.

Definition 3.4 (Kendall's Tau) *Let two i.i.d random vectors* (X_i, Y_i) *and* (X_j, Y_j) *each with joint distribution function* H*, Kendall's tau is defined as the difference between the probability of concordance and the probability of discordance of the two random vectors.*

$$\tau = \tau_{X,Y} = P[(X_i - X_j)(Y_i - Y_j) > 0] - P[(X_i - X_j)(Y_i - Y_j) < 0] \qquad (3.15)$$

Theorem 3.3 *Let* $(X, Y)^T$ *be a vector of a continuous random variable with copula* C*. Then Kendall's tau is given by;*

$$\tau(X, Y) = 4 \int\int_{[0,1]^2} C(u, v) dC(u, v) - 1 \qquad (3.16)$$

Definition 3.5 (Spearman's rho) *Let* $(X_i, Y_i), (X_j, Y_j)$ *and* (X_k, Y_k) *be independent random vectors with same joint distribution function* H *whose margins are* F *and* G*. Spearman's rho is the difference between probabilities of concordance and discordance of the vectors* (X_i, Y_i) *and* (X_j, Y_k)*.*

$$\rho = \rho_{X,Y} = 3\{P[(X_i - X_j)(Y_i - Y_k) > 0] - P[(X_i - X_j)(Y_i - Y_k) < 0]\} \qquad (3.17)$$

If X and Y are two random variables with F and G as their marginal distribution functions respectively, Spearman's rho can be said to ordinary linear correlation coefficient of the transformed random variables F(X) and G(X).

Theorem 3.4 *Let $(X,Y)^T$ be a vector of a continuous random variable with copula* C. *Then Spearman's rho is given by;*

$$\rho(X,Y) = 12\int\int_{[0,1]^2} C(u,v)duv - 3 \quad (3.18)$$

Kendall's tau and Spearman's rho are both symmetric measures of dependency-taking values in the interval [-1,1].

3.4.3 Tail Dependence

The tail dependence concept is associated with the measure of dependence of random variables X and Y in the lower joint tail or upper joint tail of a bivariate distribution.

Definition 3.6 *The upper tail dependence is defined as*

$$\lambda_U \lim_{q\to 1} Pr[X \geq F^{-1}(q)/Y \geq G^{-1}(q)] \quad (3.19)$$

where X and Y are two continuous variables with distribution functions F and G respectively and copula C

Nelsen (2006) shows that the coefficient of upper tail dependence can be defined from the copula C using

$$\lambda_U = \lim_{q\to 1} \frac{1-2q+C(q,q)}{1-q} \quad (3.20)$$

Definition 3.7 *The lower tail dependence is defined as*

$$\lambda_L \lim_{q\to 0} Pr[X \leq F^{-1}(q)/Y \leq G^{-1}(q)] \quad (3.21)$$

where X and Y are two continuous variables with distribution functions F and G respectively and copula C

Nelsen (2006) shows that the coefficient of lower tail dependence can be defined from the copula C using

$$\lambda_L = \lim_{q\to 0} \frac{C(q,q)}{q} \quad (3.22)$$

3.5 Copula Families

We aim to measure the dependence between Kenyan equity and foreign exchange returns; therefore, we consider bivariate copula specifications. We use parametric copulas; two elliptical copula (Gaussian and Student-t copula), three Archimedean copula (Frank, Gumbel,and Clayton). Each of these copulas describes a different dependence structure. The time-varying versions of the student-t copula and Gaussian copula are also considered.

Elliptical Copulas

Elliptical copulas are copulas of elliptical distribution. These copulas share various properties of multivariate normal distribution and are used to model multivariate non-normal dependencies and extreme events. They have an elliptical form and therefore symmetry in the tails. These copulas include Gaussian and the Student-t copula

Normal(Gaussian) Copula

It is derived from the bivariate normal distribution and is written as:

$$C(u, v; \rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} exp\left(\frac{2\rho rs - r^2 - s^2}{2(1-\rho^2)}\right) dsdt \qquad (3.23)$$

where ρ is the linear correlation coefficient and Φ^{-1} is the inverse of standard normal cumulative distribution function.

Patton (2006b) evolution equation will be considered to allow for time-varying dependence. The equation is written as;

$$\rho_t = \widetilde{\Lambda}(\omega_\rho + \beta_\rho \cdot \rho_{t-1} + \alpha \cdot \frac{1}{10}\sum_{j=1}^{10} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j}) \qquad (3.24)$$

where $\widetilde{\Lambda}(x) \equiv (1-e^{-x})(1+e^{-x})^{-1} = \tanh(x/2)$ is the modified logistic transformation to keep ρ_t in (-1,1) interval.

The student-t Copula

Student t copula allows for joint fat tails and exhibits upper and lower tail dependence. It is defined as

$$C^t(u,v;\rho,\mathcal{V}) = \int_{-\infty}^{t^{-1}(u;\mathcal{V})} \int_{-\infty}^{t^{-1}(v;\mathcal{V})} \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1 + \frac{s^2+t^2-2\rho st}{\mathcal{V}(1-\rho^2)}\right)^{\frac{-\mathcal{V}+2}{2}} dsdt \tag{3.25}$$

where ρ denotes the linear correlation coefficient and $\mathcal{V}$ is the degree of freedom. The time-varying t-copula function evolution equation is

$$\rho_t = \widetilde{\Lambda}(\omega_\rho + \beta_\rho \cdot \rho_{t-1} + \alpha_\rho \cdot \frac{1}{10}\sum_{j=1}^{10} \Phi^{-1}(u_{t-j};v) \cdot \Phi^{-1}(v_{t-j};v) \tag{3.26}$$

The relationship between upper and lower tail correlation coefficient and the parameter is given as follows,

$$\lambda^U = \lambda^L = 2(1 - t_{v+1}\frac{\sqrt{v+1}\sqrt{1-P_t}}{\sqrt{1-P_t}}) \tag{3.27}$$

$$\widetilde{\Lambda}(x) = 1 + x^2$$

3.5.1 Archimedean Copulas

Archimedean copulas are commonly applied, because of their ease of construction. In comparison to Elliptical copulas, Archimedean copulas have only one dependency parameter and offer a wide range of forms. In contrast to elliptical copulas, most of the Archimedean copulas have closed-form solutions and are not derived from the multivariate distribution functions using Sklar's Theorem

Definition 3.8 (Nelsen (2006)) *let ψ be a continuous strictly decreasing function from [0, 1] to $[0,\infty]$ such that $\psi(1) = 0$.We define the pseudo-inverse of ψ as the function $\psi[\hat{a}'1]$ with $Dom\psi[\hat{a}'1] = [0,\infty]$ and $Ran\psi[\hat{a}'1] = [0,1]$ given by;*

$$\psi^{[-1]}(u) = \begin{cases} \psi^{-1}(u) & 0 \le u \le \psi(0) \\ 0 & \psi(0) \le u \le +\infty \end{cases} \tag{3.28}$$

The different forms of Archimedean copulas have nice properties and can be easily constructed.

Clayton Copula

Clayton (1978) copula is an asymmetric copula that exhibits higher dependence in the lower tail than in the upper tail. Its distribution function is defined as;

$$C_\theta(u,v) = \max([u^{-\theta} + v^{-\theta} - 1]^{-\frac{1}{\theta}}, 0) \tag{3.29}$$

and its generator is

$$\varphi(x) = \frac{1}{\theta}(x^{-\theta} - 1) \tag{3.30}$$

where $\theta \in [-1, \infty)\backslash\{0\}$.

The relationship between the Clayton copula parameter and Kendall's tau is given by

$$\tilde{\theta} = \frac{2\tau}{1-\tau} \tag{3.31}$$

For $\theta = 0$ in the Clayton copula, the random variables are statistically independent. The coefficient of lower tail dependence is given by $\lambda_L = 2^{-\frac{1}{\theta}}$.

Gumbel Copula

Gumbel copula exhibits higher dependence in the upper tail than in the lower tail (**?**). Its distribution function is defined as;

$$C(u,v;\theta) = exp(-((-\ln u)^\theta + (-\ln v)^\theta)^{\frac{1}{\theta}} \tag{3.32}$$

and its generator is

$$\theta(x) = (-\ln x)^\theta \tag{3.33}$$

where $\theta \geq 1$.

The relationship between the Gumbel copula parameter and Kendall's tau is given by

$$\tilde{\theta} = \frac{1}{1-\tau} \tag{3.34}$$

The coefficient of upper tail dependence is given by $\lambda_u = 2 - 2^{\frac{1}{\theta}}$

Frank Copula

The Frank Copula (1979) does not include tail dependence and belongs to the symmetric copula family. Its distribution function is defined as;

$$C(u,v) = -\theta^{-1} log\{1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{[e^{-\theta} - 1]}\} \tag{3.35}$$

and its generator is

$$\varphi(u) = -log[\frac{e^{-\theta u} - 1}{e^{-\theta} - 1}] \tag{3.36}$$

3.6 Parameter Estimation

The bivariate dynamics of the returns x and y are determined by three functions $f(x|w)$, $g(y|w)$, and $c(u, v|w)$. When separate parameters denoted as θ_x, θ_y and θ_c are used in the functions *f,g, and c* respectively, parameter estimation is straight forward (Bartram et al., 2007). Therefore the log-likelihood is given as;

$$\log h(x, y|w, \theta) = \log c(u, v|w, \theta_c) + \log f(x|w, \theta_x) + \log g(y|w, \theta_y) \tag{3.37}$$

with $\theta = [\theta_x, \theta_y, \theta_c]$.
Equation 3.37 can therefore be stated as

$$\mathcal{L}_{x,y}(\theta) = \mathcal{L}_{u,v}(\theta_c) + \mathcal{L}_x(\theta_x) + \mathcal{L}_y(\theta_y) \tag{3.38}$$

The log-likelihood function 3.37 is maximized by the maximum likelihood function simultaneously over all parameters to yield parameter estimates. However, the maximum likelihood estimation method, in some situations may be difficult to implement because of the complexity of the model or a large number of unknown parameters. Therefore we will use the *Inference function of margins*
In the first step, parameters pertaining to the marginal distribution are estimated from univariate time series as

$$\hat{\theta_x} \equiv \arg\max \sum \log f(x|w, \theta_x) \tag{3.39}$$

$$\hat{\theta_y} \equiv \arg\max \sum \log g(y|w, \theta_y) \tag{3.40}$$

In the second step parameters pertaining to the copula density are estimated by

$$\hat{\theta_c} \equiv \arg\max \sum \log c(u, v|w, \theta_c, \hat{\theta_x}, \hat{\theta_y}) \tag{3.41}$$

Thus the estimate of the dependent parameter $\hat{\theta_c}$. Patton (2006b) demonstrated that the IFM method yields asymptotically normal and efficient parameter estimates.

3.7 Model Selection

The best copula model is selected by using the AIC and BIC. AIC is calculated as,

$$AIC = 2(negativelog - likelihood) + 2k \tag{3.42}$$

where k is the number of parameters. AIC must consider the information lost as well as the model's complexities. The best copula model is determined by the lowest AIC value.

The BIC formula is given as;

$$BIC = -2log(likelihood) + k \cdot log(n) \tag{3.43}$$

where k denotes the number of parameters used in the given model and n is the sample size. The formulation of AIC and BIC is very similar. However, BIC penalizes model complexity more severely. The lowest value of BIC indicates a better fit of the model.

3.8 Portfolio VaR Estimation

VaR is a metric that describes how a portfolio of assets is likely to decline over a specific period. We define portfolio VaR at a time t with a confidence level $(1 - \alpha)$, where $\alpha \in (0, 1)$ as;

$$VaR_t(\alpha) = inf\{s : F_t(s) \geq \alpha\} \tag{3.44}$$

where F_t is the distribution function of the portfolio return at time t. Let $Z_{p,t}$ be our portfolio return composed of a two-asset return denoted as $z_{1,t}$ and $z_{2,t}$. The portfolio return is therefore calculated as follows;

$$Z_{p,t} = wz_{1,t} + (1 - w)z_{2,t} \tag{3.45}$$

where w and $(1 - w)$ represent the portfolio weights of assets 1 and 2. We therefore define the portfolio VaR as follows;

$$\begin{aligned} P(Z_{p,t} \leq VaR_t(\alpha) \mid \Omega_{t-1} &= P(wz_{1,t} + (1 - w)z_{2,t} \leq VaR_t(\alpha) \mid \Omega_{t-1} \\ &= P(z_{1,t} \leq \frac{VaR_t(\alpha)}{w} - \frac{1 - w}{w} z_{2,t} \mid \Omega_{t-1} = \alpha \end{aligned} \tag{3.46}$$

This means we have $100(1 - \alpha)\%$ confidence that the loss will be less than VaR in the period Δt, where Ω_{t-1} represents the information set at time t-1.

In our study, we consider the 2 assets to be equal in weight. We introduce Sklar's theorem in the VaR estimation as follows

$$P(Z_{p,t} \leq VaR_t(\alpha) \mid \Omega_{t-1} = \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{VaR_t}{w} - z_{2,t}} f(z_{1,t}, z_{2,t} \mid \Omega_{t-1}) dz_{1,t} dz_{2,t}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{VaR_t}{w} - z_{2,t}} c(F(z_{1,t}), F(z_{2,t} \mid \Omega_{t-1}), f(z_{1,t} \mid \Omega_{t-1}) \times f(z_{2,t} \mid \Omega_{t-1}) dz_{1,t} dz_{2,t} \tag{3.47}$$

3.8.1 VaR Estimation Procedure

Understanding the multivariate distribution of assets is crucial when estimating portfolio Value at Risk (VaR^{α}), which is why copulas are employed. We will utilize the Monte Carlo simulation method to estimate VaR^{α} for the portfolio. By employing the copula methods presented, we estimate VaR^{α} as follows:

1. Fit an ARMA(1,1)-GARCH(1,1) model with a Student-t distribution to both datasets and acquire the standardized residuals.
2. Transform the standardized residuals into uniformly distributed samples
3. Estimate the dependence structure of two sets of uniformly distributed residuals using copula modeling. Several copulas are estimated for each set of transformed data vectors, and the suitable copula is selected based on model selection criteria.
4. Generate N random samples using the most appropriate copula model obtained through estimation by Monte Carlo simulation.
5. Obtain the (simulated) standardized residuals by applying probability integral transformation. Subsequently, calculate the (simulated) asset returns by using the standardized residuals and the forecasted means and variance from step 1.
6. Given the simulated returns, we calculate the VaR of different portfolios. The estimation of VaR could be done at different levels of confidence intervals.

CHAPTER FOUR

RESULTS AND FINDINGS

4.1 Data

In this paper, we investigated the dependence structure of the stock and foreign exchange markets in Kenya. Specifically, we will analyze the daily stock prices of the NSE 20 index and foreign exchange rates (KES/USD) from January 3, 2011 to December 31, 2022. The total number of observations is 2982.

4.2 Preliminary Study

Figure ?? represents time series plots for stock prices and exchange rates. From the plot, the stock market generally showed a downward trend while the exchange rate market showed an upward trend. We empirically investigated the dependence between Kenyan

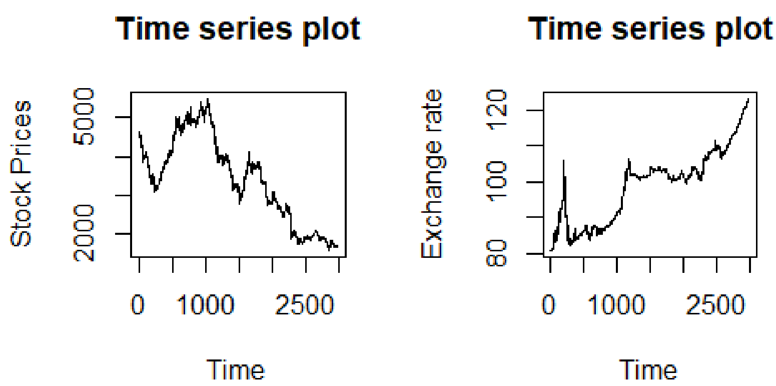

Figure 4.1: Daily prices of Stock and Foreign Exchange markets

stock market returns and foreign exchange rate returns. The stock and exchange rate returns were calculated by subtracting the logarithm of two consecutive trading days The daily return is defined as,

$$r_t = log\left(\frac{P_t}{P_{t-1}}\right) = logP_t - logP_{t-1} \tag{4.1}$$

where P_t is the closing price at time t.

As observed in Figure 4.2, it is clear that there was an emergence of a cluster of return

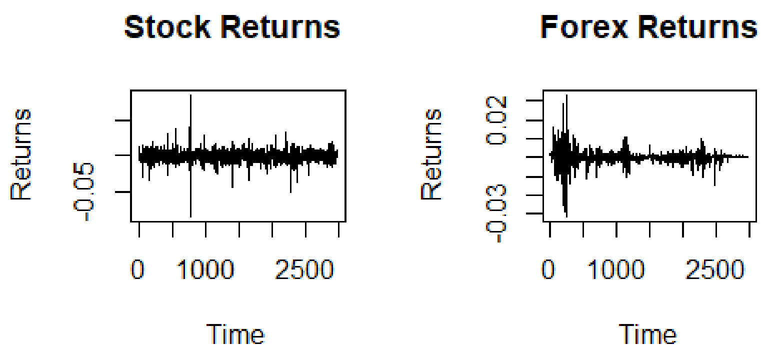

Figure 4.2: Daily Returns of Stock and Foreign Exchange markets

volatility in Kenyan stock returns and exchange rate returns. From Figure 4.2 we realize that the log returns fluctuate around their means. This implies that the time series of the log returns of these assets might appear to be stationary. We employed the Augmented Dickey-Fuller statistical test (ADF test) to check whether our log returns series are stationary. The results in Table 4.1 show that all return variables are stationary.

Table 4.1: Test of Stationery by Dickey-Fuller

Returns	t-statistics	Lag	P-Value
Stock	-13.059	14	0.01
Forex	-13.226	14	0.01

Table 4.2: Descriptive Statistics

	Min	**Max**	**Mean**	**Std.Dev**	**Skewness**	**Kurtosis**
Stock	-0.0860	0.08634	-0.0003310	0.0069	-0.3627	19.8076
Forex	-0.0312	0.03297	0.0001420	0.0027	-0.4333	31.3525

We first look at the summary statistics of the returns displayed in Table 4.2. The mean of both returns is very close to zero as observed in Figure 4.2 and the mean of forex return is slightly higher than that of stock returns. It is also observed that the standard deviation, the minimum, and maximum returns of the assets are relatively similar in size and significantly smaller and close to zero indicating that the values of the dataset are clustered around the mean, signifying low variability. As discussed in Section 1, asset returns tend to display excess kurtosis and non-zero skewness. Kurtosis is a measure of whether the asset returns exhibit light or heavy tails when compared to a normal distribution which has a kurtosis of 3. Skewness measures asymmetry. A skewness of zero denotes symmetry, as seen in normal distribution. Positive skewness suggests a distribution with a longer right tail relative to the left, while negative skewness shows the reverse. We observe that the assets' returns have all estimated kurtosis that are greater than 3. Hence, this implies that the distributions of the assets' returns have heavier tails than those of a normal distribution. However, when extreme outliers are present in a dataset, the estimates of kurtosis have the potential to become exceedingly large. As a result, providing a meaningful interpretation for these elevated kurtosis values is difficult, as it is unclear whether the true measure of kurtosis is substantial or if the inflated values are primarily influenced by the presence of outliers. The summary statistics also suggest that the returns of both assets are negatively skewed showing that the left tails are fatter than the right tails. The estimates of kurtosis and skewness tend to suggest that the returns do not follow normal distribution. As a result, the Jarque-Bera test is implemented to check for the normality of the assets' returns. From Table 4.3,

Table 4.3: Jarque Bera Statistics

	Jarqbera.test	**df**	**P Value**
Stock	48873	2	$< 2.2e - 16$
Forex	122368	2	$< 2.2e - 16$

we observed that all statistical p-values are significantly less than 0.05. Therefore, the Jarque-Bera test rejects the null hypothesis of normality for the daily log-returns stock and foreign exchange markets.

4.2.1 Correlation Analysis

Next, we turned to examining the relationship between the stock and foreign exchange returns by examining the linear correlation and the empirical rank correlations, Spearman's rho, and Kendall's Tau. From Table 4.4, we see that the correlation between the Kenyan stock and foreign exchange returns was significantly negative for Kendall's tau and Spearman's rho while for linear correlation, it was not significant. The non-

Table 4.4: Correlation between Kenyan Stock and Foreign Exchange Returns

	Linear Correlation		Kendall's Tau		Spearman's Rho	
	Stock	Forex	Stock	Forex	Stock	Forex
Stock	1	-0.0336	1	-0.0245	1	-0.0361
Forex	-0.0336	1	-0.0245	1	-0.0361	1
P-Values	0.06638		0.04537		0.04855	

significant linear correlation suggests a nonlinear relationship between the variables hence linear model does not provide a good fit for the data. We therefore consider the rank correlation results. Kendall's tau and Spearman's rho measure the strength and direction of the monotonic relationship between two variables. The negative values indicate that as one variable increases in rank, the other tends to decrease. Therefore negative results indicated that the decrease in Kenyan stock was the depreciation of the Kenyan shilling against the USD. Overall, the negative values for Kendall's Tau and Spearman's Rho suggest a weak negative relationship between the equity and forex return series, with potential implications for diversification and risk management strategies.

4.3 Arch Effects Test

A consistent empirical finding in financial log returns is that while they exhibit a low correlation between successive periods, indicating a lack of linear dependence, there is notable serial dependence observed. This is evident in the high correlation observed in the squared returns, highlighting a distinct level of serial dependence in the volatility of returns over time. This phenomenon is termed the ARCH (autoregressive conditional heteroskedastic) effect, signifying the presence of conditional heteroskedasticity or autocorrelation in the squared series of an otherwise uncorrelated time series. The primary rationale for employing the GARCH(1,1) model in finance is to identify and address conditional heteroskedasticity within time series data. Hence, it is essential to evaluate the significance of ARCH effects to ascertain the appropriateness of applying the GARCH(1,1) model to a given time series. The ACF is used to test for linear dependence at different lags.

Figure 4.3 shows the autocorrelation of the daily log returns of stock and foreign

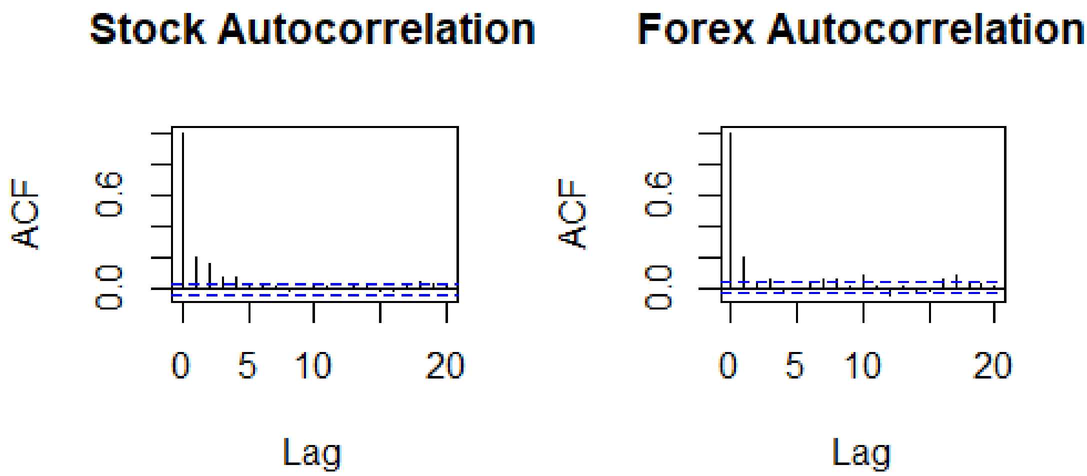

Figure 4.3: Autocorrelation Plot for Log-Returns of the Assets

exchange markets. On the stock graph, we observe that the spikes are very close to zero except for the spikes at lag 1,2,3 and 4 which rise above the dashed blue line and this suggests that most of the spikes are not statistically significant. In other words, this suggests that the serial correlation of the daily stock log returns is negligible. On the Foreign exchange graph, we observe that the spikes are also close to zero with several spikes rising above the dashed blue line in several lags compared to stock. However, the serial correlation is not significant. All the returns appear to have arch effects during the first few lags but the serial correlation decreases with an increase in lags. The decay in serial dependence might be related to the decline in volatility, particularly driven by reduced speculation. In other words, investors may aim to secure long-term gains through strategic buying or selling of particular assets. This behavior contributes to a stabilization of prices, resulting in diminished fluctuations and a subsequent reduction in overall market volatility. Consequently, this decline in volatility is associated with a decrease in correlation among asset returns at different time lags.

Figure 4.4 shows the ACF plots for the squared daily log returns of stock prices and foreign exchange rates. Specifically, we observe that all the spikes exceed the blue dashed line on the foreign exchange graph. This indicates that the spikes hold statistical significance, or in simpler terms, the squared returns of foreign exchange rates exhibit correlation with one another. However, the spikes in the stock price graph are very close

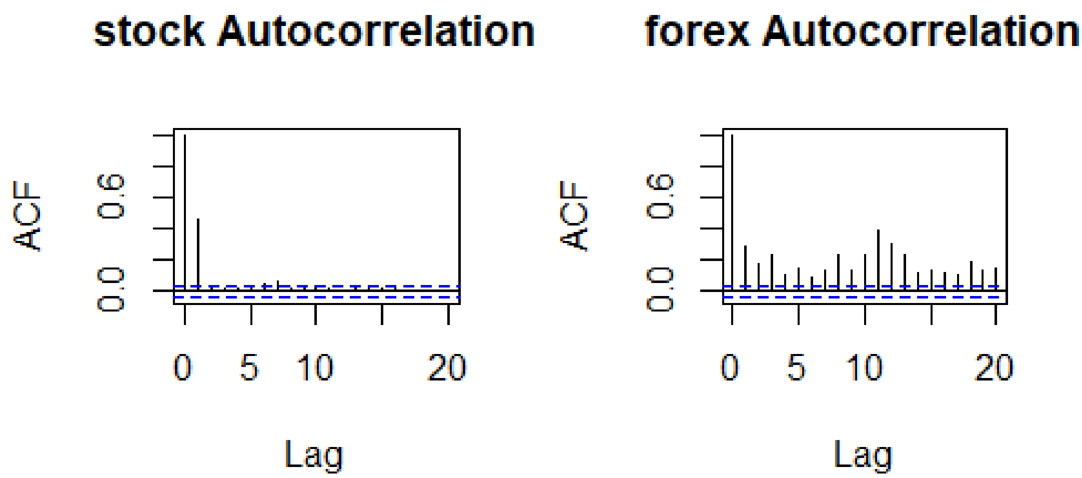

Figure 4.4: Autocorrelation Plot for Squared Log-Returns of the Assets

to zero with spikes at lag 1,6, and 7 slightly exceeding the blue dashed line. The serial correlation in the case of squared returns of stock prices is negligible. We, therefore, used Lagrange multiplier tests to test for the arch effect in the mean of daily returns. The results from Table 4.5 indicate statistically significant findings. The p-values are

Table 4.5: Langrage multiplier test for Arch Effects

	ARCH-LM	**df**	**P-value**
Stock	4409.7	1	$< 2.2e-16$
Forex	11396	1	$< 2.2e-16$

below 0.05 which supports the alternative hypothesis that the returns are heteroscedastic, implying the presence of conditional heteroskedasticity in the data. Based on these findings, we opted to employ ARMA-GARCH models to adequately model the marginal distributions of both returns. We considered three different innovation distributions for these ARMA-GARCH models.

4.4 ARMA (1,1)-GARCH (1,1)

When configuring a copula model, accurate marginal distribution specification may seem unnecessary. However, it's essential to conduct tests for marginal distribution models as they are crucial for constructing a copula model. If the model for marginal distribution is misspecified, the probability integral transformations wouldn't be independent and identically distributed (i.i.d.), leading to inaccuracies in any copula model. Before copula model estimation, the marginal distributions are separately estimated and the results are used to get the probability integral transforms, u and v. To model the marginal distribution, ARMA is used to address the conditional mean while GARCH is the conditional variance. Hence we develop an ARMA-GARCH model. The choice of ARMA(1,1)-GARCH(1,1) models for the two sets of return series was made considering their effectiveness in capturing the typical volatility patterns found in financial time series. Following this, the parameters of the GARCH models were determined using the Maximum Likelihood method, and this estimation process involved three distinct innovation distributions.

Table 4.6: Estimates of ARMA(1,1)-GARCH(1,1) parameters under various innovation distributions for stock returns

	Normal		t		Skewed t	
Parameters	Estimates	P-value	Estimates	P-value	Estimates	P-values
μ	-0.0002	0.3375	-0002	0.1651	-0.0003	0.1071
ϕ	0.7062	0.0000	0.6632	0.0000	0.6641	0.0000
θ	-0.4975	0.0000	-0.4551	0.0000	-0.4560	0.0000
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
α	0.1577	0.0000	0.1797	0.0000	0.18085	0.0000
β	0.7644	0.0000	0.5920	0.0000	0.5884	0.0000
ν			5.8825	0.0000	5.8691	0.0000
ξ					0.9824	0.0000
LL	10968.28		11126.46		11126.69	
AIC	-7.3548		-7.4602		-7.4597	
BIC	-7.3427		-7.4461		-7.4436	

ARMA(1,1)-GARCH(1,1) models were applied to model the volatility of both stock and forex returns. The comparison of different GARCH model specifications for each market return was based on the use of AIC and BIC. The model selection process involved choosing the specification with the lower AIC or BIC values. Based on the results from Table 4.6 and 4.7, models utilizing t and skewed t distributions offer better

Table 4.7: Estimates of ARMA(1,1)-GARCH(1,1) parameters under various innovation distributions for Forex returns

	Normal		t		Skewed t	
Parameters	Estimates	P-value	Estimates	P-value	Estimates	P-values
μ	-0.0002	0.0000	-0003	0.0000	-0.0003	0.0000
ϕ	0.2459	0.1436	0.3529	0.0000	0.3580	0.0000
θ	-0.0095	0.9574	-0.0620	0.4481	-0.0693	0.3974
ω	0.0000	0.9662	0.0000	0.9954	0.0000	0.9949
α	0.1519	0.0000	0.0900	0.0000	0.0945	0.0000
β	0.8447	0.0000	0.8878	0.0000	0.8827	0.0000
ν			5.8825	0.0000	4.0810	0.0000
ξ					1.0270	0.0000
LLF	15457.42		15903.6		15907.23	
AIC	-10.367		-10.665		-10.667	
BIC	-10.355		-10.651		-10.651	

fits compared to the model assuming a normal distribution. This is evident in the higher likelihoods and lower AIC and BIC values associated with the t and skewed t distribution models. We, therefore, select ARMA (1,1)-GARCH(1,1) with t distributions as it fits better than models with the other distributions.

The summary of margin estimation results in Table 4.6 and 4.7 reveals that the estimated GARCH coefficient β and ARCH coefficient α are predominantly statistically significant at the 5% level across all assets' returns. This indicates the existence of volatility clustering phenomena in stock returns and foreign exchange returns. Furthermore, it is observed that the conditional variance equations $\alpha + \beta$ are consistently less than 1 for all instances. This suggests that the series of market returns satisfies the stationary condition. Additionally, it is noteworthy that the sum of $(\alpha + \beta)$ is very close to 1. Volatility exhibits persistence when $\alpha + \beta = 1$, shows less persistence when $\alpha + \beta < 1$, and demonstrates explosiveness when $\alpha + \beta > 1$. Forex returns have high volatility persistence $\alpha + \beta = 0.9778$ compared to stock returns which have a lower volatility persistence $\alpha + \beta = 0.7717$. Moreover, the degrees of freedom parameter ν for both returns are strongly significant, implying that the residual distribution deviates from normality and that employing the t distribution is appropriate for our ARMA(1,1)-GARCH (1,1) model.

4.4.1 Goodness of Fit Tests for Marginal Models

In the context of constructing a copula model, accurate specification of the marginal distributions is essential. Using an incorrectly specified model for the marginals could lead to non-i.i.d. probability transforms, resulting in a misrepresentation of the copula. Assessing the goodness-of-fit for the marginal model is crucial in ensuring the reliability and accuracy of the copula model.
Following the estimation of the ARMA-GARCH models, we computed the standardized residuals. To assess the adequacy of the fit of these standardized residuals, we employed the Ljung-Box test to investigate the autocorrelation at lags 1, 5, and 9 for the ARMA(1,1)-GARCH(1,1)-t model, utilizing a significance level of 5%. Engle's ARCH-LM test was used to assess heteroscedasticity problems. Table 4.8 provides

Table 4.8: Weighted Ljung-Box Test and Arch LM Test on the Marginal Distribution Models

	Stock		Forex	
Lags	Statistics	P-Value	Statistics	P-Value
Ljung-Box				
1	0.003273	0.9544	4.242	0.0394
5	2.803279	0.5977	19.572	0.0000
9	4.1437	0.6570	33.362	0.0000
Arch LM				
3	0.1506	0.6980	0.5225	0.4698
5	0.3173	0.9356	0.6407	0.8415
7	0.8356	0.9388	1.5915	0.8028

the results for Arch LM and Ljung-Box tests. We observed that the p-values of the Ljung-Box test exceed 5% for the stock residuals. This indicates that at lags 1, 5, and 9, the null hypothesis of no autocorrelation cannot be rejected. However, in the case of exchange rate residuals, the p-values of the Ljung-Box test remained below 5% at all lags. This suggests that we reject the null hypothesis of no autocorrelation at all lags. The p-value from the Ljung-Box test for residuals in the exchange rate data suggests the existence of serial dependence in the residuals. According to Engle's ARCH-LM test results, the p-values consistently exceed the 5% threshold for all the residuals. This suggests that there is no evidence to reject the null hypothesis, suggesting the absence of an ARCH effect at any lag. We opt to utilize the bivariate standardized residuals assuming they are independent and identically distributed.

4.5 Copula Modeling

After obtaining standardized residuals from the ARMA(1,1)-GARCH(1,1)-t model, the subsequent stage was to model the marginal distributions using copula. Each variable within the copula model exhibits a marginal probability distribution that follows a uniform distribution over the interval (0,1). Hence, the marginal distribution needs to be transformed to be uniform (0,1). To transform the standardized residuals, we used the cumulative student t function. The series of univariate probability integral transforms u_t is obtained using the formula:

$$u_t = F_t(\varepsilon_t, \nu) \tag{4.2}$$

where u_t represents independent and identically Uniform (0,1) distributed variables and F_t denotes the cumulative distribution function (CDF) of the t distribution. Patton (2006a) introduced the Lagrange Multiplier test for assessing serial independence in the probability integral transforms. More specifically, u_t and v_t are the probability transforms derived from the cumulative distribution functions of the standardized marginal distributions of the two returns. The Kolmogorov-Smirnov tests were conducted to examine whether the transformed series conforms to a uniform distribution over the interval (0,1). The results in Table 4.9 show that the transforms are independent and identically distributed. The results also show that the values are within the intervals (0,1). As a result, we conclude that the marginal distributions adequately account for each market return.

After the estimation of the parameters for the marginal distribution F_i, the subsequent step involves copula parameters estimation, as detailed earlier. In our analysis, we

Table 4.9: Caption

	LM	df	P-value	Min	Mean	Max	K-S test
u_t	-432.5	1	1	0.00015	0.4996	0.9999	$< 2.2e - 16$
v_t	-400.1	1	1	0.001897	0.4857	0.9990	$< 2.2e - 16$

employ five different copula functions.Using Inference Function for Margins (IFM) methods, the chosen copula functions are fitted to the respective residuals series. The results of the copula modeling process are presented in Table 4. This table displays the outcomes derived from the IFM method. AIC and BIC functions were used to determine the best-fit copula. The results for the estimated copula parameters are presented in

Table 4.10: Copula estimates of Kenyan stock-exchange rate

	$\hat{\theta}_1$	$\hat{\theta}_2$	Kendall's Tau	Lower Tail	Upper Tail	AIC	BIC	LLF
Gaussian	-0.05		-0.03	0	0	-0.7	5.3	1.35
Student's t	-0.05	5.22	-0.03	0.04	0.04	-41.17	-29.17	22.58
Clayton	0.01		0.01	0	0	1.96	7.96	0.02
Gumbel	1.01		0 .01	0	0.01	1.66	7.66	0.17
Frank	-0.29		-0.03	0	0	-1.5	4.5	1.75

Table 4.10. The best fitting copula for the Kenyan stock-forex pair is the student-t copula. These results are similar to those in (?). It is crucial to emphasize that the copula function models the dependence structure between market returns. Therefore, the choice of the most appropriate copula should be based on the observed characteristics of dependence between these returns. To get a better picture of this dependence, Kendall's tau and tail dependence were computed as shown in Table 4.10. The Kendall's tau based on the best-estimated copula was negative. The negative value was close to the value estimated in Table 4.4. This indicates that there is a negative correlation between Kenya's stock returns and exchange rate returns. The parameter estimates for the symmetric tail dependence in the Student's t copula are consistently low, with the value being 0.04. This suggests that there are limited extreme dependencies in the stock and exchange rate pair. Specifically, it implies that extreme movements in the exchange market have a relatively modest impact on the extreme movements of the stock markets.

For copulas that account for the asymmetric dependence tail dependence, the upper tail dependence value for the Gumbel copula was very weak and the lower tail dependence value for the Clayton copula was zero.

4.6 Time Varying dependence and Conditional Copula estimation

We estimated time-varying dependence parameters utilizing dynamic copulas. Table 4.11 shows the parameters associated with the time-varying dependencies of the selected copula. In this context, the parameter $\omega 0$ signifies the levels of dependence, ω_1 denotes the degree of persistence, and ω_2 captures the adjustments in the dependence process. The Gaussian copula was selected highlighting the symmetry in the co-movements of these returns. On the other hand, the t copula was chosen suggesting that the connection between the two variables is more pronounced during extreme market conditions. As shown in Table 4.11, the dynamics of the dependence parameter demonstrated an enhancement in the performance compared to the corresponding time-invariant models. This improvement was evident when considering the Akaike Information Criterion (AIC). Notably, the results indicated that a time-varying student-t copula emerged as the most effective dependence model, providing a better description of the dependence structure between Kenyan stocks and the exchange rate.

4.7 Portfolio VaR Estimation

To assess the portfolio risk, VaR was introduced as the most widely used method. We considered an investor who can only invest in the equities market and the forex market. The student's t-copula is selected as the most suitable copula. Using student t copula, we quantified the portfolio risk in three portfolios, the equally weighted portfolio, the global minimum variance portfolio, and the mean-variance portfolio. The equally weighted portfolio assumes the weight of the assets is the same. The equally weighted portfolio can be defined as follows;

$$w_1 = w_2 = \frac{1}{2} \tag{4.3}$$

In the Global Minimum Variance portfolio, the investor aims to minimize portfolio risk without imposing any constraints on the portfolio return. The Global Minimum

Table 4.11: Time Varying Copula estimates of Kenyan stock-exchange rate

Copulas		stock-exchange rate
Gaussian	ω	-0.1087
	β	1.5539
	α	0.1994
	AIC	-0.7376
Student's t	ω	-0.1766
	β	0.6507
	α	0.3059
	ω_2	-1.9276
	β_2	-0.3280
	α_2	1.1223
	AIC	-44.7251

Variance Portfolio is defined as follows;

$$\begin{aligned} &\text{Min}_w && w'\Sigma w \\ &\text{Subject to} && w'1 = 1 \end{aligned} \tag{4.4}$$

For a portfolio containing two securities

$$\begin{aligned} &\text{Min}\{w_x^2 Var(x) + w_y^2 Var(y) + 2w_x w_y \rho xy\sigma x\sigma y\} \\ &\text{Subject to:} \quad w_x + w_y = 1 \\ &\qquad w \geq 0 \end{aligned} \tag{4.5}$$

Markowitz (1952) established the Mean-Variance portfolio where one assumes that a rational investor seeks to maximize the expected return for a given level of risk or minimize risk for a given level of return. In our case, we sought to minimize the risk and maximize the expected return. We, therefore, represent the mean-variance portfolio optimization problem as follows;

$$\begin{aligned} &\text{Max}_w && w'\mu - \lambda \cdot w'\Sigma w \\ &\text{Subject to} && w'1 = 1 \\ & && w \geq 0 \\ & && \lambda = 0.01 \end{aligned} \tag{4.6}$$

where λ is the risk aversion parameter and $w'\Sigma w$ represent variance-covariance where Σ is a covariance estimate of the assets.

This optimization problem aims to find the optimal portfolio weights w that maximizes the expected return $w'\mu$ while considering the trade-off between return and risk, controlled by the risk aversion parameter λ.

Table 4.12: VaR estimated by student's t copula

Level,%	Equally-weighted	Global Minimum Variance	Mean Variance
99.9	3.89	3.89	4.01
99	2.10	2.12	2.29
95	1.36	1.36	1.41
90	1.01	1.01	1.05
Weights			
Stock	0.5	0.5058	0.6255
Forex	0.5	0.4942	0.3745

The obtained results for VaR^{α} are shown in Table 4.12. Lower quantiles indicate a more conservative estimate of potential losses, while higher quantiles represent more extreme scenarios. We obtained VaR^{α} under different confidence levels $1-\alpha$. The VaR^{α} of the equally weighted portfolio is similar to the Global Minimum Variance except at confidence level $1-\alpha = 99\%$. The optimal portfolio weights under the Global Minimum Variance and the Mean Variance portfolio are presented in Table 4.12. Based on the table, if the investor's goal is to maximize the expected portfolio return while minimizing risk, allocating more weight to stocks seems advisable. However, if the investor aims to manage a portfolio containing both assets, investing proportionately in both, as indicated in the table, is recommended to minimize overall portfolio risk.

CHAPTER FIVE
CONCLUSION

5.1 Conclusion

In this thesis, we investigated the dependence structure between the Kenyan stock market (NSE 20) and the foreign exchange market (KES/USD) and utilized it to estimate the portfolio risk. To study the dependence between these two markets, we specified the marginal models for the assets' returns and a joint model for the dependence. We employed ARMA(1,1)-GARCH(1,1) with t distributions to model the marginal distribution for each asset's return series. For the joint model, we chose eight copulas with different dependence structures. Using AIC values, we found that the student-t copula was the best-fitting copula when compared to the other static copulas. However, the student-t copula was slightly outperformed by the time-varying student-t copula. Several studies have found symmetric tail dependence between stock and foreign exchange markets (Ning, 2010; Sewe et al., 2014; Kumar et al., 2019). The student t copula assumes symmetric tail dependence indicating that the tail dependence between stock and foreign exchange markets are alike when both markets experience simultaneous booms and crashes. The negative dependence in the bivariate return series aligns with market expectations. This aligns with the idea that investors tend to withdraw from a country when the investment climate in a country worsens, resulting in a downturn in equity prices, equity index levels, and depreciation of the country's currency against the global currencies.

VaR was utilized to measure portfolio risk. The correlation between price movements of financial assets within a portfolio significantly impacts the accuracy of VaR estimation. However, constraints on the joint distributions of these assets may hinder VaR performance (Huang et al., 2009). It is therefore crucial for the joint distribution of the portfolio to be independent of normality assumptions. The assumption of linear correlation and normality of asset returns is insufficient in capturing the complexity of dependence structures. This thesis utilized the student-t copula to estimate portfolio VaR. Copula models can extract the dependence structure from the joint distribution function while also isolating this dependence structure from the univariate marginal distribution. We used Monte Carlo simulations to build a joint distribution by combining marginal modeling with copula functions. One can choose a portfolio that maximizes returns while maintaining a specific level of risk measured by VaR. Likewise, one can minimize risk while targeting a desired level of return. We identified the global minimum variance portfolio and mean-variance portfolio at different confidence levels. We noted that when minimizing portfolio risk, optimal portfolio weights tend to be almost similar. Significant tail dependence suggests an increased likelihood of extreme events, resulting in a higher VaR estimate compared to what a bivariate normal distribution would imply (Wang et al., 2013). Neglecting the importance of tail dependence could result in underestimated risks. Accurate tail dependence estimates are therefore essential for effective risk management and for obtaining a more accurate VaR estimation.

5.2 RECOMMENDATION

The results of this study could contribute to a more accurate evaluation of the dependence between stock and foreign exchange markets, potentially enhancing risk management. These findings may assist policymakers in better designing their foreign exchange management policy and help investors more effectively diversify their portfolios and manage risks. Our findings hold significant practical implications for cross-market risk management and asset valuation. In times of stock and foreign exchange market crashes, investors should explore alternative asset classes, as investments solely in these markets could result in losses. Put simply, when these markets face downturns, investors should diversify their portfolios away from the two markets. For instance, international investors engage in trading equities in foreign markets to achieve global portfolio diversification. These investors, who hold foreign equities, face exposure to both equity and exchange rate risks. Additionally, they must factor in the correlation between equity and exchange rates to effectively manage the risks stemming from this interdependence.

We have examined the dependence structure between the two markets and estimated the portfolio VaR of the two assets. We suggest that additional variables such as bonds be included in the study of dependence structure in the Kenyan financial markets. The estimation of portfolio risk could be extended by comparing the performance of copula-based methods with the traditional VaR estimation methods, estimating portfolio VaR using time-varying copulas, and using alternative risk measures other than VaR e.g. expected shortfall, lower partial moment, etc.

REFERENCES

Aloui, R. and Aïssa, M. S. B. (2016). Relationship between oil, stock prices and exchange rates: A vine copula based garch method. *The North American Journal of Economics and Finance*, 37:458–471.

Aloui, R., Aïssa, M. S. B., and Nguyen, D. K. (2013). Conditional dependence structure between oil prices and exchange rates: A copula-garch approach. *Journal of International Money and Finance*, 32:719–738.

Andersen, T. G., Bollerslev, T., Christoffersen, P. F., Diebold, F. X., et al. (2006). Practical volatility and correlation modeling for financial market risk management. *The Risks of Financial Institutions, University of Chicago Press for NBER*, pages 513–548.

Ang, A. and Chen, J. (2002). Asymmetric correlations of equity portfolios. *Journal of financial Economics*, 63(3):443–494.

Arbel, J., Crispino, M., and Girard, S. (2019). Dependence properties and bayesian inference for asymmetric multivariate copulas. *Journal of Multivariate Analysis*, 174:104530.

Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D. (1999). Coherent measures of risk. *Mathematical finance*, 9(3):203–228.

Bartram, S. M., Taylor, S. J., and Wang, Y.-H. (2007). The euro and european financial market dependence. *Journal of Banking & Finance*, 31(5):1461–1481.

Bashir, U., Yu, Y., Hussain, M., and Zebende, G. F. (2016). Do foreign exchange and equity markets co-move in latin american region? detrended cross-correlation approach. *Physica A: Statistical Mechanics and Its Applications*, 462:889–897.

BenSaïda, A. (2018). The contagion effect in european sovereign debt markets: A regime-switching vine copula approach. *International Review of Financial Analysis*, 58:153–165.

Berg, D. and Bakken, H. (2006). A copula goodness-of-fit test based on the probability integral transform. *The Norwegian Computing Center and University of Oslo, Oslo*.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3):307–327.

Branson, W. H. (1983). A model of exchange-rate determination with policy reaction: evidence from monthly data.

Chen, X. and Fan, Y. (2006). Estimation of copula-based semiparametric time series models. *Journal of Econometrics*, 130(2):307–335.

Chen, X., Fan, Y., and Patton, A. J. (2004). Simple tests for models of dependence between multiple financial time series, with applications to us equity returns and exchange rates. *London Economics Financial Markets Group Working Paper*, (483).

Cherubini, U., Luciano, E., and Vecchiato, W. (2004). *Copula methods in finance*. John Wiley & Sons.

Chiang, T. C., Jeon, B. N., and Li, H. (2007). Dynamic correlation analysis of financial contagion: Evidence from asian markets. *Journal of International Money and finance*, 26(7):1206–1228.

Chollete, L., Heinen, A., and Valdesogo, A. (2009). Modeling international financial returns with a multivariate regime-switching copula. *Journal of financial econometrics*, 7(4):437–480.

Clayton, D. G. (1978). A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. *Biometrika*, 65(1):141–151.

Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics*, 28(5):777–795.

Cuñado, J. and de Gracia, F. P. (2003). Do oil price shocks matter? evidence for some european countries. *Energy economics*, 25(2):137–154.

Cytonn (2021). Review of kenyan shilling performance. https://cytonnreport.com/topicals/review-of-the-kenyan-shilling-performance: :text=Dec

da Silva Filho, O. C., Ziegelmann, F. A., and Dueker, M. J. (2012). Modeling dependence dynamics through copulas with regime switching. *Insurance: Mathematics and Economics*, 50(3):346–356.

Dahir, A. M., Mahat, F., Ab Razak, N. H., and Bany-Ariffin, A. (2018). Revisiting the dynamic relationship between exchange rates and stock prices in brics countries: A wavelet analysis. *Borsa Istanbul Review*, 18(2):101–113.

Delgado, N. A. B., Delgado, E. B., and Saucedo, E. (2018). The relationship between oil prices, the stock market and the exchange rate: Evidence from mexico. *The North American Journal of Economics and Finance*, 45:266–275.

Diamandis, P. F. and Drakos, A. A. (2011). Financial liberalization, exchange rates and stock prices: Exogenous shocks in four latin america countries. *Journal of Policy Modeling*, 33(3):381–394.

Dimitrova, D. (2005). The relationship between exchange rates and stock prices: Studied in a multivariate model. *Issues in political Economy*, 14(1):3–9.

Dornbusch, R. and Fischer, S. (1980). Exchange rates and the current account. *The American Economic Review*, 70(5):960–971.

Embrechts, P., Lindskog, F., and McNeil, A. (2001). Modelling dependence with copulas. *Rapport technique, Département de mathématiques, Institut Fédéral de Technologie de Zurich, Zurich*, 14:1–50.

Embrechts, P., McNeil, A., and Straumann, D. (2002). Correlation and dependence in risk management: properties and pitfalls. *Risk management: value at risk and beyond*, 1:176–223.

Engle, R. F. (1982). *Autoregressive conditional heteroscedasticity with estimates of the variance of inflationary expectations*. School of Economics.

Exchange, N. S. (2023). Nse-annual-report-2022-integrated-report. https://www.nse.co.ke/wp-content/uploads/NSE-Annual-Report-2022-Integrated-Report.pdf.

Fan, Y. and Patton, A. J. (2014). Copulas in econometrics. *Annu. Rev. Econ.*, 6(1):179–200.

Fermanian, J.-D. (2005). Goodness-of-fit tests for copulas. *Journal of multivariate analysis*, 95(1):119–152.

Frank, M. J. (1979). On the simultaneous associativity of f (x, y) and x+ y- f (x, y). *Aequationes mathematicae*, 19:194–226.

Garcia, R. and Tsafack, G. (2011). Dependence structure and extreme comovements in international equity and bond markets. *Journal of Banking & Finance*, 35(8):1954–1970.

Genest, C., Quessy, J.-F., and Rémillard, B. (2006). Goodness-of-fit procedures for copula models based on the probability integral transformation. *Scandinavian Journal of Statistics*, 33(2):337–366.

Genest, C., Rémillard, B., and Beaudoin, D. (2009). Goodness-of-fit tests for copulas: A review and a power study. *Insurance: Mathematics and economics*, 44(2):199–213.

Genest, C. and Rivest, L.-P. (1993). Statistical inference procedures for bivariate archimedean copulas. *Journal of the American statistical Association*, 88(423):1034–1043.

Gumbel, E. J. (1960). Distributions des valeurs extrêmes en plusieurs dimensions. In *Annales de l'ISUP*, volume 9, pages 171–173.

Gurgul, H. and Machno, A. (2016). Modeling dependence structure among european markets and among asian-pacific markets: a regime switching regular vine copula approach. *Central European Journal of Operations Research*, 24(3):763–786.

Hafner, C. M. and Manner, H. (2012). Dynamic stochastic copula models: Estimation, inference and applications. *Journal of Applied Econometrics*, 27(2):269–295.

Hau, H. and Rey, H. (2006). Exchange rates, equity prices, and capital flows. *The Review of Financial Studies*, 19(1):273–317.

Hu, L. (2006). Dependence patterns across financial markets: a mixed copula approach. *Applied financial economics*, 16(10):717–729.

Huang, J.-J., Lee, K.-J., Liang, H., and Lin, W.-F. (2009). Estimating value at risk of portfolio by conditional copula-garch method. *Insurance: Mathematics and economics*, 45(3):315–324.

Jain, A. and Biswal, P. C. (2016). Dynamic linkages among oil price, gold price, exchange rate, and stock market in india. *Resources Policy*, 49:179–185.

Joe, H. (1997). *Multivariate models and multivariate dependence concepts*. CRC press.

Joe, H. (2005). Asymptotic efficiency of the two-stage estimation method for copula-based models. *Journal of multivariate Analysis*, 94(2):401–419.

Joe, H. (2014). *Dependence modeling with copulas*. CRC press.

Joe, H. and Xu, J. J. (1996). The estimation method of inference functions for margins for multivariate models.

Jondeau, E. and Rockinger, M. (2006). The copula-garch model of conditional dependencies: An international stock market application. *Journal of international money and finance*, 25(5):827–853.

Kamal, J. B. and Haque, A. E. (2016). Dependence between stock market and foreign exchange market in south asia: A copula-garch approach. *The Journal of Developing Areas*, pages 175–194.

Karmakar, M. (2017). Dependence structure and portfolio risk in indian foreign exchange market: A garch-evt-copula approach. *The Quarterly Review of Economics and Finance*, 64:275–291.

Kim, K.-h. (2003). Dollar exchange rate and stock price: evidence from multivariate cointegration and error correction model. *Review of Financial economics*, 12(3):301–313.

Kole, E., Koedijk, K., and Verbeek, M. (2007). Selecting copulas for risk management. *Journal of Banking & Finance*, 31(8):2405–2423.

Kolev, N., Anjos, U. d., and Mendes, B. V. d. M. (2006). Copulas: a review and recent developments. *Stochastic models*, 22(4):617–660.

Kumar, S., Tiwari, A. K., Chauhan, Y., and Ji, Q. (2019). Dependence structure between the brics foreign exchange and stock markets using the dependence-switching copula approach. *International Review of Financial Analysis*, 63:273–284.

Lee, C.-H., Doong, S.-C., and Chou, P.-I. (2011). Dynamic correlation between stock prices and exchange rates. *Applied financial economics*, 21(11):789–800.

Li, R., Hu, Z., Li, S., and Yu, K. (2019). Dynamic dependence structure between chinese stock market returns and rmb exchange rates. *Emerging Markets Finance and Trade*, 55(15):3553–3574.

Longin, F. and Solnik, B. (2001). Extreme correlation of international equity markets. *The journal of finance*, 56(2):649–676.

Maneejuk, P. and Yamaka, W. (2019). Predicting contagion from the us financial crisis to international stock markets using dynamic copula with google trends. *Mathematics*, 7(11):1032.

Markowitz, H. M. (1952). Portfolio selection. *The journal of finance*, 7(1):77–91.

McNeil, A. J., Frey, R., and Embrechts, P. (2015). *Quantitative risk management: concepts, techniques and tools-revised edition*. Princeton university press.

Michelis, L. and Ning, C. (2010). The dependence structure between the canadian stock market and the usd/cad exchange rate: a copula approach. *Canadian Journal of Economics/Revue canadienne d'économique*, 43(3):1016–1039.

Nelsen, R. B. (2005). Copulas and quasi-copulas: an introduction to their properties and applications. In *Logical, algebraic, analytic and probabilistic aspects of triangular norms*, pages 391–413. Elsevier.

Nelsen, R. B. (2006). *An introduction to copulas*. Springer Science & Business Media.

Ning, C. (2010). Dependence structure between the equity market and the foreign exchange market–a copula approach. *Journal of International Money and Finance*, 29(5):743–759.

Oh, D. H. and Patton, A. J. (2017). Modeling dependence in high dimensions with factor copulas. *Journal of Business & Economic Statistics*, 35(1):139–154.

Okhrin, O., Okhrin, Y., and Schmid, W. (2013). On the structure and estimation of hierarchical archimedean copulas. *Journal of Econometrics*, 173(2):189–204.

Patton, A. (2013). Copula methods for forecasting multivariate time series. *Handbook of economic forecasting*, 2:899–960.

Patton, A. J. (2006a). Estimation of multivariate models for time series of possibly different lengths. *Journal of applied econometrics*, 21(2):147–173.

Patton, A. J. (2006b). Modelling asymmetric exchange rate dependence. *International economic review*, 47(2):527–556.

Patton, A. J. (2009). Copula–based models for financial time series. In *Handbook of financial time series*, pages 767–785. Springer.

Patton, A. J. (2012). A review of copula models for economic time series. *Journal of Multivariate Analysis*, 110:4–18.

Poon, S.-H. and Granger, C. W. (2003). Forecasting volatility in financial markets: A review. *Journal of economic literature*, 41(2):478–539.

Poon, S.-H., Rockinger, M., and Tawn, J. (2004). Extreme value dependence in financial markets: Diagnostics, models, and financial implications. *The Review of Financial Studies*, 17(2):581–610.

Rémillard, B. (2017). Goodness-of-fit tests for copulas of multivariate time series. *Econometrics*, 5(1):13.

Rosenberg, J. V. and Schuermann, T. (2006). A general approach to integrated risk management with skewed, fat-tailed risks. *Journal of Financial economics*, 79(3):569–614.

Savu, C. and Trede, M. (2008). Goodness-of-fit tests for parametric families of archimedean copulas. *Quantitative Finance*, 8(2):109–116.

Scaillet, O. (2007). Kernel-based goodness-of-fit tests for copulas with fixed smoothing parameters. *Journal of Multivariate Analysis*, 98(3):533–543.

Sewe, S. O., Weke, P. G., and Mungâ€™atu, J. K. (2014). Modelling dependence between the equity and foreign exchange markets using copulas. *Applied Mathematical Sciences*, 8(117):5813–5822.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, 19(3):425–442.

Sklar, M. (1959). Fonctions de repartition an dimensions et leurs marges. *Publ. inst. statist. univ. Paris*, 8:229–231.

Smith, M. S., Gan, Q., and Kohn, R. J. (2012). Modelling dependence using skew t copulas: Bayesian inference and applications. *Journal of Applied Econometrics*, 27(3):500–522.

Tastan, H. (2006). Estimating time-varying conditional correlations between stock and foreign exchange markets. *Physica A: statistical Mechanics and its Applications*, 360(2):445–458.

Tsai, I.-C. (2012). The relationship between stock price index and exchange rate in asian markets: A quantile regression approach. *Journal of International Financial Markets, Institutions and Money*, 22(3):609–621.

Wamalwa, P. S. and Were, M. (2021). Is it export-or import-led growth? the case of kenya. *Journal of African Trade*, 8(1):33–50.

Wang, A. (2010). Goodness-of-fit tests for archimedean copula models. *Statistica Sinica*, pages 441–453.

Wang, Y. C., Wu, J.-L., and Lai, Y.-H. (2013). A revisit to the dependence structure between the stock and foreign exchange markets: A dependence-switching copula approach. *Journal of Banking & Finance*, 37(5):1706–1719.

Wang, Z.-R., Chen, X.-H., Jin, Y.-B., and Zhou, Y.-J. (2010). Estimating risk of foreign exchange portfolio: Using var and cvar based on garch–evt-copula model. *Physica A: Statistical Mechanics and its Applications*, 389(21):4918–4928.

APPENDICES

APPENDIX A: Ethical Clearance Confirmation

Strathmore University Institutional Scientific and Ethical Review Committee (SU-ISERC)

Strathmore UNIVERSITY

Final Decision

This document certifies that the study:

\\\"\\\"

Principal Investigator: Mr. Lanya, Jacob Owino
Reference number: SU-IERC1365/22

Was reviewed and received the following status:
\\\"done\\\"
Additional Comments: Final decision: **approved**
Comments sent:

Reviewer #1:
'The study is well conceptualised.
'

04 July 2022 04:18:14

APPENDIX B: Similarity Report

Modeling the dependence between equity returns and foreign exchange market rates in Kenyan Financial markets A copula approach.pdf

ORIGINALITY REPORT

25%	26%	22%	9%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	www.diva-portal.org Internet Source	2%
2	ir.jkuat.ac.ke Internet Source	1%
3	lnu.diva-portal.org Internet Source	1%
4	www.mdpi.com Internet Source	1%
5	cris.maastrichtuniversity.nl Internet Source	1%
6	lup.lub.lu.se Internet Source	1%
7	Submitted to Universitat Mannheim Student Paper	1%
8	su-plus.strathmore.edu Internet Source	1%
	vdoc.pub	