Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BBS ACT, BBS FENG, BBS FE
End term Examination
BSM 2111: STATISTICAL INFERENCE

<u>DATE: August 2020</u>   TIME 2 hour

<u>Instructions:</u> Answer question ONE and any other TWO questions

<u>**Question 1 (30marks)**</u>

(a) What do we mean by an estimator being

   i. Unbiased   **(2 Marks)**
  ii. Efficient   **(2 Marks)**
 iii. Consistent   **(2 Marks)**

(b) The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between 0 and 20 minutes, inclusive

   i. What is the probability that a person waits less than 15 minutes?   **(2 Marks)**
  ii. Find expected value and the standard deviation of the amount of time a person must wait for the bus   **(4 Marks)**

(c) Suppose that $x_1, x_2, ..., x_n$ are independent and identically distributed whose common probability density function is given by

$$f(x) - \begin{cases} (\theta+1)x^\theta & \text{if } 0 < x < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

where $\theta (>0)$ is unknown parameter. Determine an estimator for θ by using method of moment

(6 Marks)

(a) Suppose that $y_1, y_2, ..., y_n$ are a random sample with a common density depending on a parameter θ. Suppose further that θ_1 and θ_2 are both unbiased estimators of θ based on $y_1, y_2, ..., y_n$ and that

$$Eff(\theta_1, \theta_2) = \frac{Var(\theta_2)}{Var(\theta_1)} = \frac{2n^3 + 5n + 1}{2n^3 + n^2 - 1}$$

Which estimator is preferred? **(2 Marks)**

(d) In order to estimate the proportion of students who graduate within six years, the administration at a private university examined the records of 600 randomly selected students who entered the university six years ago, and found that 312 had graduated. Assuming that the sample is sufficiently large, construct a 98% confidence interval for the six-year graduation rate **(5 Marks)**

(e) An economist wishes to estimate, with a 95% confidence interval, the yearly income of welders with at least five years experience to within $1,000. He estimates that the range of incomes is no more than $24,000, so using the Empirical Rule he estimates the population standard deviation to be about one-sixth as much, or about $4,000. Find the estimated minimum sample size required. **(3 Marks)**

(f) A two-sided test of H0:μ=0 yields a P-value of 0.04. Will the 99% confidence interval for μ include 0? Explain your reasoning. **(2 Marks)**

Question 2   (20 marks)

(a) State without proof the central limit theorem **(2 Marks)**

(b) A travelling salesman makes a sale on 55% of his calls on regular customers. He makes 5 sales calls each day.

   i. Construct the probability distribution of X, the number of sales made each day. **(2 Marks)**

   ii. Find the probability that, on a randomly selected day, the salesman will make a sale. **(4 Marks)**

   iii. Assuming that the salesman makes 20 sales calls per week, find the mean and standard deviation of the number of sales made per week **(4 Marks)**

(c) Suppose that the mean cost across the country of a week supply of a product is $36.6 with a standard deviation of $ 1.7.

i.   Find the probability that the cost of a randomly selected week supply is between $35 and $40. **(4 Marks)**

ii.   Find the probability that the mean cost of 20 randomly selected weeks' supply is between $37 and $40. **(4 Marks)**

Question 3 (20 marks)

(b) Suppose a random sample $x_1, x_2, ..., x_n$ of size n was collected from a Poisson population with unknown mean θ.

i.   Use the method of maximum likelihood (ML) to estimate θ. **(6 Marks)**

ii.   Determine if the ML estimator found in i above an unbiased estimator. **(3 Marks)**

(c) Suppose a random sample $x_1, x_2, ..., x_n$ are iid with the Uniform distribution on the interval (0, θ) Where θ ($\theta>0$) is the unknown parameter.

iii. Derive the method of moments estimator for θ **(4 Marks)**

iv.   Is the method of moment estimator derived above a consistent estimator? **(7 Marks)**

Question 4 (20 marks)

(a) Differentiate between type I error and type II error as used in hypothesis testing **(4 Marks)**

(b) In the past the average length of an outgoing telephone call from a business office has been 143 seconds. A manager claims that the average has decreased after the introduction of policy changes. A sample of 100 telephone calls produced a mean of 133 seconds, with a standard deviation of 35 seconds. At the 1% level of significance test the managers claim. Use the rejection region approach. **(7 Marks)**

(c) In the previous year the proportion of deposits in checking accounts at a certain bank that were made electronically was 45%. The bank wishes to determ1ine if the proportion is higher this year. It examined 20,000 deposit records and found that 9,217 were electronic. Determine, at the 1% level of significance, whether the data provide sufficient evidence to conclude that more than 45% of all deposits to checking accounts are now being made electronically. Use the p-value approach. **(9 Marks)**

Question 5 (20 marks)

(a) A software company markets a new computer game with two experimental packaging designs. Design 1 is sent to 11 stores; their average sales the first month is 52 units with sample standard deviation 12 units. Design 2 is sent to 6 stores; their average sales the first month is 46 units with sample standard deviation 10 units. Construct a point estimate and a 95% confidence interval for the difference in average monthly sales between the two package designs. Assume that the two populations have the equal variances. Interpret your answer **(10 Marks)**

(b) A business owner had been working to improve employee relations in his company. He predicted that he met his goal of increasing employee satisfaction from 65% to 80%. Employees from 3 departments were asked if they were satisfied with the working conditions of the company. The results are shown in the following table:

Level of satisfaction	Department		
	Finance	Sales	Technology
Satisfied	12	25	23
dissatisfied	6	8	7

Is there association between the level of satisfaction and the department in the company at 5% level of significance? **(10 Marks)**