SCHOOL INSTITUTE OF MATHEMATICAL SCIENCES

BACHELOR OF BUSINESS SCIENCE ACTUARIAL SCIENCE/FINANCIAL ENGINEERING/FINANCIAL ECONOMICS

END OF SEMESTER EXAMINATION

BSM 2111: STATISTICAL INFERENCES

Date: 25th July 2024 Time: 2hrs

INSTRUCTIONS

1. This examination consists of FIVE questions.
2. Answer Question ONE (COMPULSORY) and any other TWO questions.

QUESTION ONE(30 Marks)

a)Explain the meaning of the term "random sample" [1 mark]

b)Define the term "statistic" [1mark]

c)Let

$$X = \begin{cases} 1, \text{ with prob } 0.5 \\ 0, \text{ with prob } 0.5 \end{cases}$$

Find $\mathbb{E}[X]$ [2marks]

d)Let X be a random variable with mean zero and variance 2. Define another variable by $Y = 2 + 3X$.
i)Find $\mathbb{E}[Y]$ and
ii)$Var[Y]$ [4marks]

e)Let $X_1, ..., X_n$ be a random sample from a Poisson distribution with parameter $\lambda > 0$. Show that both $\frac{1}{n}\sum_{i=1}^{n} X_i$ and $\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2$ are the method of moments estimators of mean and variance, respectively. [4marks]

f)Let $X_1, X_2, ..., X_n$ be a random sample of size n from a population with mean μ & variance σ^2. Show that $\mathbb{E}[\bar{X}] = \mu$. [2marks]

g)Let $X_1, X_2, ..., X_n$ be independent χ^2 random variables with $n_1, ..., n_k$ d.f respectively. Show that $V = \sum_{i=1}^{k} X_i \sim \chi^2$ distribution with $n_1 + n_2 + ... + n_k$ degrees of freedom.[3marks]

h)The following is a random sample data from a normal population: 7.2, 5.7, 4.9, 6.2, 8.5, 2.8.Construct a 95% confidence interval for the population mean μ. [4marks]

i)Let X be a binomial random variable. We wish to test the hypothesis $H_0 : p = 0.8$ versus $H_1 : p = 0.6$. Let the probability of type I error, $\alpha = 0.03$, be fixed. Find the probability of type II error, β, for $n = 20$. [5marks]

j)Let $\tilde{\theta}$ be a statistic that is normally distributed with mean θ and standard deviation $\sigma_{\tilde{\theta}}$ where σ is assumed to be known. Find the confidence interval for θ that possesses a confidence coefficient equal to $1 - \alpha$.[4marks]

QUESTION TWO(2O marks)

a)Distinguish between type I error and type II error as used in test of hypothesis. [2marks]

b)A toy store chain claims that at least 80% of girls under 8 years old prefer dolls over other types of toys. After observing the buying pattern of many girls under 8 years old, we feel that this claim is inflated. In an attempt to dispose of this claim, we observe the buying pattern of 20 randomly selected girls under 8 years old, and we observe X, the number of girls who buy stuffed toys or dolls. We wish to test the hypothesis $H_0 : p = 0.8$ against $H_1 : p < 0.8$. Suppose we decide to accept the H_0 if $X > 12$. This means that if $X \leq 12$ we will reject H_0.

i)Find α. [3 Marks]

ii)Find β for $p = 0.6$. [3 Marks]

iii)Find β for $p = 0.4$. [3 Marks]

c)The management of a local health club claims that its members lose on the average 15 pounds or more within the first 3 months after joining the club. To check this claim, a consumer agency took a random sample of 45 members of this health club and found that they lost an average of 13.8 pounds within the first 3 months of membership, with a sample standard deviation of 4.2 pounds.

(i) Find the p-value for this test. [3 Marks]

(ii) Based on the p-value in (i), would you reject the null hypothesis at $\alpha = 0.01$?[1 Mark]

d)In attempting to control the strength of the wastes discharged into a nearby river, an industrial firm has taken a number of restorative measures. The firm believes that they have lowered the oxygen consuming power of their wastes from a previous mean of 450 manganate in parts per million. To test this belief, readings are taken on $n = 20$ successive days. A sample mean of 312.5 and the sample standard deviation 106.23 are obtained. Assume that these 20 values can be treated as a random sample from a normal population. Test the appropriate hypothesis. Use $\alpha = 0.05$. [5 Marks]

QUESTION THREE(2O Marks)

a)The following table gives a classification according to religious affiliation and marital status for 500 randomly selected individuals. For $\alpha = 0.01$, test the null hypothesis that marital status and religious affiliation

		Religious affiliation					
		A	B	C	D	None	**Total**
Marital status	Single	39	19	12	28	18	116
	With spouse	172	61	44	70	37	384
	Total	211	80	56	98	55	**500**

are independent. [10 Marks]

b)(i)Consider two independent random samples $X_1, ..., X_n$ from an $N(\mu_1, \sigma_1^2)$ distribution and $Y_1, ..., Y_n$ from an $N(\mu_2, \sigma_2^2)$ distribution. Test $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 \neq \sigma_2^2$ for the following basic statistics: $n_1 = 25$,

$\tilde{x_1} = 410$, $S^2 = 95$, and $n_2 = 16$, $\tilde{x_2} = 390$, $S_2^2 = 300$. Use $\alpha = 0.05$ [4marks]

(ii)Because of the impact of the global economy on a high-wage country such as the United States, it is claimed that the domestic content in manufacturing industries fell between 1977 and 1997. A survey of 36 randomly picked U.S. companies gave the proportion of domestic content total manufacturing in 1977 as 0.37 and in 1997 as 0.36. At the 1% level of significance, test the claim that the domestic content really fell during the period 1977–1997 [3marks]

(iii)In a salary equity study of faculty at a certain university, sample salaries of 50 male assistant professors and 50 female assistant professors yielded the following basic statistics.

	Sample mean Salary	Sample standard deviation
Male assistant professor	$36,400	360
Male assistant professor	$34,200	200

Test the hypothesis that the mean salary of male assistant professors is more than the mean salary of female assistant professors at this university. Use $\alpha = 0.05$. [3marks]

QUESTION FOUR(20 marks)

a)Let $X_1, ..., X_n$ be a random sample from a gamma probability distribution with parameters α and β. Find the method of moment estimators for the unknown parameters α and β. [6marks]

b)Suppose $X_1, ..., X_n$ are random samples from a Poisson distribution with parameter λ.Find MLE $\tilde{\lambda}$ [9marks]

c)If S^2 is the variance of a random sample from an infinite population with finite variance σ^2, show that S^2 is an unbiased estimator for σ^2 [5marks]

QUESTION FIVE(20 marks)

a)Let S_1^2 denote the sample variance for a random sample of size 10 from Population I and let S_2^2 denote the sample variance for a random sample of size 8 from Population II. The variance of Population I is assumed to be three times the variance of Population II. Find two numbers a and b such that $P(a \leq S_1^2/S_2^2 \leq b) = 0.90$ assuming S_1^2 to be independent of S_2^2 [7 marks]

b)If Y & Z are independent random variables, Y has a χ^2-distribution with n d.f, and $Z \sim N(0, 1)$, show that $T = \frac{Z}{\sqrt{Y/n}}$ has a t-distribution with n degrees of freedom, i.e $T \sim T_n$ [4 marks]

c(i)Suppose $X_1, ..., X_n$ are a random sample from a geometric distribution with parameter $p, 0 \leq p \leq 1$. Find MLE $\tilde{p}$. [6 Marks]

(ii)Let the random variables $X_1, X_2, ..., X_n$ be from an $N[5, 1]$ distribution. Find a number a such that $P\Big(\sum_{i=1}^{5}(X_i - 5)^2 \leq a\Big) = 0.90$ [3 Marks]