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Pay-As-You-Drive as a Pricing Alternative in Motor Insurance

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

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...12/03/2021... [*Date*]

This Research Project has been submitted for examination with my approval as the Supervisor.

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Table of Contents

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Abstract

A Pay-As-You-Drive (PAYD) pricing method simulating the replacement of the fixed costs of vehicle ownership and operation, with variable costs. The objective was to estimate the pricing effect of each strategy and deduce whether PAYD pricing can be employed as an alternative pricing method. The pricing effect is the focus of this paper. Consumers should be able to choose among various pricing structures. With a per-unit-of-distance alternative pricing scheme, drivers would be able to save money on car insurance by driving less and might even decide that some of the extra kilometers driven are not worth the cost. The data incorporated in this study is mileage data from private passenger vehicle data in the Commonwealth of Massachusetts. The data analyses over one million data points. The results showed that the traditional Generalised Linear Model (GLM) method of pricing was still more efficient. Even so, for insurers with the largest portfolios in the market, that have considerable participation in relation to the total, the Pay-As-You-Drive pricing method can be a simple and effective alternative pricing technique. The decision as to which pricing technique to employ is thus best left to the strategy of each insurer and should be analyzed in relation to the cost of each and how much this will impact positively to the result of the portfolio.

***Keywords*:** Pay As You Drive, Motor Insurance, Generalised Linear Model, Alternative Pricing Method.

1. Introduction

1.1 Background

Being the owner of a vehicle brings about two challenges. First, it is probably one of the most expensive things to own. Second, a driver is liable for pedestrians, the security of passengers and fellow drivers, other people's property, and themselves. Without some form of insurance, the risks associated with owning this asset has the potential of serious personal financial risk.

In today's world, car insurance is considered an essential investment. This insurance protects you against any untoward incident. It also covers third-party liability, where one is liable for damages to other vehicle owners. In Kenya, as quoted in the Insurance (Motor Vehicle Third Party Risk) act, obtaining third-party insurance is mandatory to drive on roads. Even so, according to the Insurance Research Council (IRC), nearly 13% of motorists, or about one in eight drivers, operate uninsured, usually due to the simple fact that vehicle insurance is costly to the majority.

Applying the law of large numbers in premium pricing, also known as the 'Law of Averages', motorists pay a fixed premium (expected loss average) and this happens to be a cover for unlimited mileage (distance driven). Therefore, it is observed that this conventional method of pricing results in low-risk motorists cross-subsidizing high-risk motorists, which is unfair and encourages high-risk driving (Butler et al. 1988). Also, with the rise in high-risk claims, premium averages then tend to shift upward, propelling the low-risk policies to also carry this cost.

Pay As You Drive (PAYD) is a unique concept in car insurance that provides benefits offered in comprehensive insurance for a reduced cost. PAYD pricing means that insurance premiums are based directly on the amount of distance a vehicle is driven during the policy term. It changes the exposure unit from the vehicle-year to the vehicle-kilometer. It incorporates other rating factors so lower-risk motorists pay less and higher-risk motorists to pay more per unit of travel. It gives motorists a new financial incentive that involves reducing annual mileage in order to reduce the chance of being involved in an accident.

We can summarize that PAYD pricing offers the following benefits: for insurers, improved actuarial accuracy; for consumers, an opportunity to save money by choosing to drive less; and for society, a reduction in automobile accidents and other negative externalities brought about by driving.

1.2 Problem statement

According to Bordoff and Noel (2008), when you dine at an all-you-can-eat restaurant, chances are you eat more than you would if you paid separately for a dish on a menu. All-you-can-drive as a pricing strategy might seem absurd, but if we think about it, auto insurance is priced this way today. Drivers who are similar in other aspects; age, gender, location and claims history pay nearly the same premium if they drive 8,000 or 80,000 kilometers a year. Just as an all-you-can-eat restaurant encourages more eating, traditional insurance pricing encourages more driving. The extra driving that results from this inefficient system leads to more accidents and therefore more motor insurance claims. There are two major consequences of the current all-you-can-drive method of pricing motor insurance.

First, unfortunately, drivers today cannot pay less for car insurance by consuming less. Without a per-unit-of-distance pricing scheme, drivers are not aware of the cost of driving each extra kilometer. The current system thus encourages an inefficient high level of driving, which imposes significant social costs in the form of more accidents. With a per-unit distance charge, drivers would be able to save money on car insurance by driving less and might even decide that some of the extra kilometers driven are not worth the cost. Second, the current pricing structure of auto insurance has the effect of forcing low-mileage drivers to subsidize the insurance cost of high-mileage drivers. This system is inequitable because low-income people drive fewer miles on the average and thus are liable for fewer accidents.

Consumers should be able to choose among various pricing structures. Consumers sometimes prefer a flat rate's predictability and low cost , and other times prefer the affordability that comes from paying a price consistent with the proportion of the product they use. For example, diners can often choose between all-you-can-eat and separate dish prices in a restaurant. Similarly, telephone service purchasers can choose between flat rates that allow unlimited use, or rates based on the number of calls or minutes used. In Kenya, electricity services also offer the option of a monthly rate and a usage-based rate that encourages consumers to be more efficient in their consumption. There is, for this reason, nothing radical or difficult about consumption-based pricing in general insurance.

According to Forbes, in recent times, insurance companies have been called upon to lower or refund premiums due to the COVID-19 related lockdown. On March13, 2020 Warren Buffet, CEO of Geico's parent company Berkshire Hathaway, told Yahoo Finance that the reduction in driving as a result of the COVID-19 outbreak is presenting itself in the form of fewer accidents, according to Geico's claim data. The effect has been coined a " coronavirus silver lining" for insurers.

Without the variety of pricing structures in motor insurance products, many people currently facing extraordinary financial stress during these times of COVID-19 opt out of their insurance plans. This creates significant financial and legal problems both for the motorists who drive uninsured, and other road users who must bear the cost of uninsured accidents. It is critical for the insurance industry to adjust to this need, to protect their customers from overpaying for their auto-insurance during this time of dramatically reduced driving.

1.3 Objectives

This study has two objectives.

a. To apply PAYD pricing to Motor Insurance Data.
b. To compare PAYD method of pricing with the conventional pricing method.

These objectives should direct us to a conclusion on the effect of PAYD pricing models as an alternative method of pricing motor insurance policies.

1.4 Responses to frequent questions and concerns about PAYD pricing.

Is the risk of accident correlated with mileage driven?

Study conducted in California, pursuant to their Insurance Code Section 1861.02 (a), the second most important factor in coming up with premiums is the number of kilometers driven annually. As such, the reduction in mileage should result in a reduction of premium charged. This is not a widely agreed upon actuarial standard and explains why it is not extensively employed in the insurance industry currently. However, it is easily understood to customers, and everyone seems comprehend that the likelihood of causing an accident will fall substantially if someone cuts their driving in half and/ is on the road much less.

Automobile insurance should focus on safety, affordability, and equity.

PAYD pricing helps achieve all of these goals: It significantly increases road safety as policyholders tend to drive less, it makes vehicle ownership more affordable and provides financial savings particularly for lower-income motorists, and it increases equity by making premiums more actuarially accurate.

Odometer fraud will be a major problem.

The growing use and development in telematics can help reduce fraud in insurance claims. Odometers are increasingly tamper-resistant and the financial incentive for fraud is relatively low.

PAYD pricing would be an invasion of privacy.

Auditing of an odometer does not necessarily provide information considered private. Already, odometer readings are collected during vehicle servicing, vehicle sales, and accident investigations. Some PAYD systems do track when and where a vehicle is driven, however, this would be a consumer option, and these systems can be designed with controls over how the data are processed, stored, and used, as with other personal data such as telephone calls and credit card transactions.

1.5 Significance of the study

The study will present a useful solution to the insurance industry. It illuminates areas in the industry that have the potential for innovation and the option for forward-thinking pricing alternatives for insurance products. The research aims to shed some light on the possibility for economic benefit of PAYD pricing in the insurance market.

2. Literature Review

Substantial research has gone into exploring the relation between accident rates and travel activities (Litman 2012 and 2017). The various studies indicate that all else being equal, changes in risk exposure causes similar changes in accident numbers. Even a somewhat "perfect" driver faces risks that are beyond their control.

Annual accident risk is seen to be the product of two factors: the accident risk per-kilometer multiplied by the individual annual mileage. High-risk drivers may crash every 80,000 kilometers, while lower-risk drivers may crash only every 800,000 kilometers. Reducing annual mileage should reduce annual collision risk.

Over the last few years, data on this relationship between travel activities and accident rates for individual vehicles have become available. Bordoff and Noel (2008) used Progressive Insurance Corporation data and found a strong positive relationship between mileage and claims. A study by Ferreira and Minike (2012), explains how data on annual mileage and insurance claims matched together found that all else being equal, as mileage increases so do the chance of having an accident. Boucher, Pérez-Marín, and Santolino (2013) used a Spanish data set and found a strong positive relationship between travel activity and the frequency of claims under 10,000 annual kilometers, but a weaker relationship for higher mileages. Overall, these studies validate the actuarial practicability of PAYD pricing.

Currently, it is possible to collect high-quality real-time data efficiently to model individual risk. In terms of how the data is collected, in most studies data is usually recorded either by the vehicles' on-board diagnostics (OBD) or the users' smartphone and transmitted to a central database for processing and analysis (Boquete et al. 2010, Iqbal and Lim, 2006). This facilitates the development of indicators for estimating driver's risk exposure.

Some studies show an on-board platform inside the vehicle which acquires, and processes data obtained from the Global Positioning System (GPS), and a mobile-telephone use detection circuit (Boquete et al. 2010). This system uses a mobile connection to transmit data between the on-board system (OS), and the control center (CC). Position, speed, and time are recorded and transmitted continuously. Other studies also portray light or weather sensors that communicate via infrared or Bluetooth reporting numerical values to the OS (Iqbal and Lim, 2006).

Barmpounakis et al. (2016) reason out that, since the technological obstacles that used to exist, are nowadays overtaken, these systems can also be employed for real-time traffic monitoring. Vehicle trajectories could be extracted from video recordings, using a trajectory extraction system, to collect traffic data (Barmpounakis et al., 2015). Although this is currently unavailable, it is very likely to be employed for this purpose any day now.

The PAYD model is a more simplistic approach using fewer parameters as risk indicators. This approach has the following advantages: it is easier to implement, the period for the development, and the verification of the model is significantly smaller, and it targets the vehicles that are not often driven. Despite the immense benefits and study that has gone into PAYD pricing, there are currently barriers limiting its widespread adoption and loopholes that need discovering.

This study aims to consider a PAYD pricing model with clearly defined risk measures for a well-informed and accurate per-kilometer rate. This will give the stakeholders a better understanding of how the PAYD pricing system offers economic benefits to the customers, the company, and society.

Smartphone and technology-based hybrid approaches play a big role in driving the expansion of the usage-based insurance market. The inclusion of technology makes the processes simpler, with a reduced service cost. Furthermore, the increasing adoption of telematics and connected cars is predicted to extend the expansion of the usage-based insurance market size.

Usage-based insurance reduces the danger of accidents and theft of vehicles because it tracks driving behavior, supplementing business growth. It can detect the missing or stolen vehicles in the telematics system linked to the vehicle through GPS. These factors are anticipated to extend the demand for Usage-based insurance market size. According to Valuates Reports, the global Usage-based Insurance market size is projected to reach USD 77.25 Billion by 2026, from USD 25.46 Billion in 2020, at a compound annual growth rate (CAGR) of 20.32% during 2020-2026.

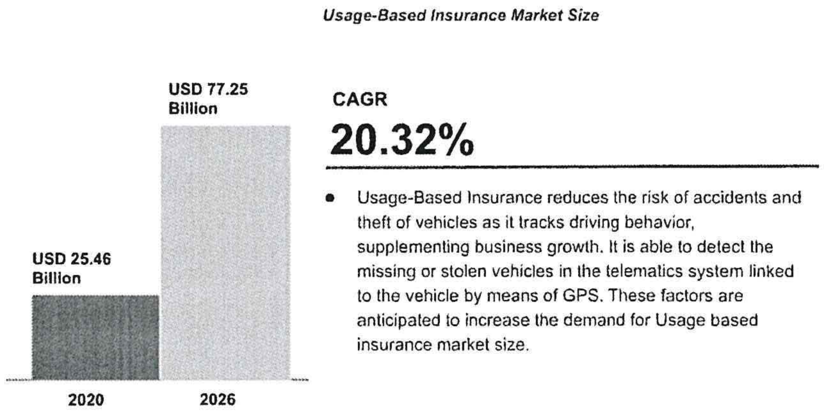

Figure 1: UBI Projections

The COVID-19 pandemic has affected the lives of many globally. Given the present situation, most individuals do their best to save their money. In America, more and more drivers are switching to the new trend of usage-based insurance programs. The surge in popularity for usage-based automobile insurance policies during this period is not surprising.

Overall, it is clear the automotive insurance market is changing and changing rapidly. The insurance providers that choose to become involved with usage-based insurance programs early, will have a definite advantage over those companies who wait and risk losing valuable customers to their more innovative competitors.

3. Research Methodology

3.1 Introduction

This section begins by defining some common terms used in general insurance and a brief description of the major requirements to consider when pricing. The section also introduces Generalized Linear Models GLMs and the role of these models in pricing non-life insurance business. The concept of these models will be utilized in our study.

3.1.1 Key definitions

Name	Definition
Rate	Price for a unit of risk exposure covered under an insurance contract.
Premium	The rate multiplied by the number of exposure units.
Pure Premium	A premium equal to the expected value of the loss to a large number of policyholders, without loading for profit.
Earned premium	The premium collected by an insurance company for the portion of the policy that has expired.
Loading	The process used to adjust the pure premium to consider all other administrative costs and to eventually provide profit.

3.1.2 Pricing in general insurance

To price, actuaries need to have a proper understanding of the product they will have to price. There are some major considerations that a pricing actuary should be keen about:

a. *Knowledge of level of risk underlying a product.*
b. *Information on technicalities of the policy and the impact it has on the underlying risk.*
c. *The environment in which a certain line of business or product will operate in.*

The knowledge and pricing idea that we incorporate in this study is heavily influenced by Pietro Parodi in his book, Pricing in General Insurance (2014).

3.2 Background

Conventional methods for pricing vehicle insurance are generally reliant on time periods, which includes months and years. An applicant's data: age, sex, location of residence, and driving record are combined with other factors to make an actuarial class, which is then used to price. This price is then related to a unit of exposure. Below are the main driving indicators used in literature so far for calculating drivers' risk under PAYD pricing:

1. Total distance driven by the policyholder. (the higher the mileage the higher the risk)
2. Type of road network. (higher accident frequency in the city and higher severity out of the city)
3. Vehicle type.

The unit of exposure is the period of time also referred to as a term. Since the insurance contract is defined based on the exposure unit, conventional insurance contracts are principally defined by the term. However, we observe that such conventional insurance mixes a fixed cost with a variable usage pattern. This approach penalizes low mileage customers. What is needed is an improved system and method for addressing such issues, which this research aims to convey.

Distance-based insurance enables variable use to be paired with variable pricing, unlike conventional insurance, where a set cost is paired with variable usage. The tactic enables distance-based insurance to be purchased and used in the form of a utility product by:

a. Allowing costs to reflect usage.

b. Eliminating inefficient pricing.

c. Creating consumer alternatives that are actuarially accurately.

3.3 Generalized Linear Models (GLMs)

The implementation of Generalized Linear Model in both statistics and actuarial science is accredited to the work published by John Nelder and Robert Wedderburn (1972). GLMs are an extension of the Gaussian linear models' framework.

3.3.1 Estimating claim frequency using the Poisson model

In actuarial literature, the Poisson model is considered the main tool used for estimating claim frequency / observed number of claims in non- life insurance. The random discrete variable Y_i, which represents observed number of claims, conditioned by the vector of explanatory variables X_i, which represents the characteristics of the insured, is assumed to be Poisson distributed, given by the density function:

$$f(Y_i = y_i | x_i) = \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!}$$

The standard estimator for this model is the Maximum Likelihood Estimator (MLE), defined as:

$$\mathcal{L}(\beta) = \prod_{i=1}^{n} \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!} = \prod_{i=1}^{n} \frac{e^{-e^{x_i\beta}} (e^{x_i\beta y_i})}{y_i!}$$

The main limitation of this model is that it may sometimes underestimate regression parameters which means some risk factors could appear to be significant when they have no considerable influence on the variation of claim frequency.

3.3.2 Estimating claim severity using the Gamma model

Claim severity is usually more challenging to predict as compared to claim frequency. Pinquet, describes a realistic model based on the gamma distribution that represents another generalization of the exponential family. If $c_1, c_2, \dots, c_i$ represents the costs of claims used by insured i and that they are independently Gamma distributed, the density function is as follows.

$$f(c_i) = \frac{1}{\Gamma(v)} (\frac{vc_i}{\mu_i})^v \exp\left(-\frac{vc_i}{\mu_i}\right), \qquad c_i > 0,$$

The log-likelihood function is as follows:

$$\mathcal{L}(\beta) = \prod_{i|y_i>0} \cdot \prod_{k=1}^{y_i} \left(\frac{1}{\Gamma(v)} (\frac{vc_{ik}}{\mu_i})^v \exp\left(-\frac{vc_{ik}}{\mu_i}\right)\frac{1}{c_{ik}}\right)$$

The above equation allows for the estimation of the required parameters. The main advantage of the Gamma model is flexibility obtained while estimating the cost of claims due to parameters v and μ.

3.3.3 Pure Premium Calculation

The pure premium in general insurance represents the expected cost of all claims declared by policyholders during the term of insurance. It is obtained by multiplying the estimated frequency and the estimated severity. This represents the mathematical expectation of the annual cost of claims declared by the policyholders, given below as:

$$E\left[\sum_{i=1}^{N} C_i\right] = E[Y] \times E[C_i]$$

for the claim amounts $c_1, c_2, \dots$ independent of their number (Y)

As observed above, the separate evaluation of severity and frequency of claims is relevant since the risk factors that influence each of these components of insurance premium, are usually different.

3.4 Description of Data

The data that will be incorporated in this study is mileage data from private passenger vehicle data in the Commonwealth of Massachusetts. The data analyses over one million data points. Public notice of the data's availability is posted on the EOEEA website at:

https://www.mass.gov/?pageID=eoeeaterminal&L=3&L0=Home&L1=Grants+%26+Technical+Assistance&L2=Data+Resources&sid=Eoeea&b=terminalcontent&f=eea_data-resources_2005-08-auto-insur-data&csid=Eoeea

The data is grouped into five classes and six territories as shown below.

Class	Average Annual Mileage	Territory	Average Annual Mileage
Adult	12,398	1	12,456
Business	14,173	2	12,149
< 3 yrs exp	12,911	3	12,262
3 – 6 yrs exp	13,207	4	11,798
Senior citizen	7,519	5	10,702
		6	10,523
ALL	11,695	ALL	11,695

Also available from the Paid Analytic Dataset is automobile mileage, fuel economy and claim costs used in the PAYD study. Considerable analytical work went into converting the raw policy transaction data to exposure records that match individual policy holders to an appropriate rating class, territory and mileage estimate.

3.5 Data limitations

This study has potential limitations. The research objectives were based on the current insurance environment in East Africa, particularly influenced by the Kenyan general insurance market. However, data from this region is not easily accessible and those available do not incorporate important factors used in this study, such as mileage. The findings are therefore subject to market adjustments when the appropriate data is employed for this market.

4. Research Findings

4.1 Analysis

This section presents analysis of the study. Computation of premium prices using both the traditional and the PAYD pricing method will be highlighted. Evaluation of the pricing effect will thereafter be presented and conclusions for PAYD pricing as an alternative method of pricing will be deduced.

4.1.1 Computing Pure Premium

Software Package

All simulations in the study are done with the R software (R Core Team, 2017). Estimating parameters for GLMs is done by use of the glm function from the stats package and the glm.nb function from the mass package (Venables and Ripley, 2002). The lme4 package was chosen.

Traditional Pure Premium

The first step in modelling is to clean the data and choose what features to use. After this is completed, we can fit a GLM to the data.

First, we read the data into R and split it into training/testing sets:

```
Frequency_data <- read.csv("C:/Users/Jasmine
Waithera/Desktop/Project2020/Data/Frequency_data.csv")set.seed(123)
#Set seed for reproducibility
sample <- sample.split(Frequency_data, SplitRatio = 0.80) #Split
data into 80% training 20% test
train = subset(Frequency_data, sample==TRUE)
test = subset(Frequency_data, sample==FALSE)
```

We then fit the model using the glm package. After fitting the model, we test the model using the following metrics: mean squared error (MSE) and mean absolute error (MAE).

```
pred <- exp(predict(model, test))
MSE <- sqrt(mean((test$no_claim_not_null - pred*test$EXP_COV)^2))
#Mean squared error
MAE <- sum(abs(test$ no_claim_not_null -
pred*test$EXP_COV))/nrow(test)
#Mean absolute error

> MSE = 0.182480104096452
> MAE = 0.0582112950956896
```

With this model we can now similarly build a severity model, from which we arrive at a complete or mature pricing model under the frequency-severity independence assumption.

Pay-As-You-Drive Pure Premium

To price Pay-As-You-Drive auto insurance, we define a reduced collective risk model while the entire number of claims has non-homogeneous Poisson distribution. By applying non-homogeneous Poisson distribution, we will enter the mileage of the discounted collective risk model to the premiums for Pay-As-You-Drive in Automobile insurances. We apply the double Stochastic Poisson Process for modeling the DCRM. The Double Stochastic Poisson Process provides flexibility by letting the intensity not only depend upon time but also by allowing it to be a model .

Table 1: Classification of Annual Mileage

Classification	Interval	Frequency
1	0-1000	5155
2	1001-10000	128866
3	10001-30000	469072
4	300001-47000	252577
5	>47000	144330

4.1.2 Calculating the price effect

Pay as you drive can be viewed as a pricing alternative that aims at reducing policyholder mileage by incorporating a per-mile price. It therefore follows that we need to estimate the reduction of price due to a reduction in mileage. This can be described as follows.

Let us assume state for traditional or conventional pricing method and state for the Pay-as-you-drive alternative. We can then assign binary values '1' for conventional state and '0' otherwise. The expected outcomes can now be represented as Z_1, the outcome from the treatment and Z_0, the outcome from the untreated state. The benefit of the alternative insurance pricing method can thereafter be defined as:

$$\Delta = Z_1 - Z_0$$

For our data analysis, an aggregate approach was used to evaluate the expected outcome for the two states of interest so as to determine the average effect of the pricing alternative.

4.2 Study Findings

The table presents a comparative summary of the results performed between in the study. The Efficiency variable measures the necessary increase in price for there to be a reduction of one percentage point in relations to the actual price. The Reduction p.p. corresponds to the reduction in percentage points in relation to claims estimated. Thus, it is possible to measure the efficiency of each model in a standardized manner. The smaller the value found for this variable, the more efficient is the technique.

Pay-As-You-Drive pricing model showed the second-best efficiency. It is believed that the GLM shows greater variability mainly because of performing the adjustment to a hypothetical distribution, which is not suitable for all actual data. Pricing by historical pure premium presented a comparative advantage, because the automobile insurance portfolio is large and quite stable over time, allowing proper pricing considering only the average expected claims.

Table 2: Author's Calculations for efficiency

Technique	S/P (%)	Reduction p.p	Increase (%)	Standard Deviation	Standard Error	Efficiency (%)
Pure Prem. GLM	62.36	4.72	7.56	109.54	5.26	1.60
Pure Prem. PAYD	61.38	5.69	9.27	502.66	49.98	1.63

5. Summary and Conclusion

The results showed that the traditional GLM method of pricing was still more efficient. This result can be explained by several factors. First, the study considered the total database of insured vehicles over the years studied. In addition, in an automobile portfolio, the risks are not exposed to disasters or high severity. In this way, historical data of the portfolio is a fairly accurate estimator for future periods. Taking all of this in context, the historical pure premium pricing structure proved to be advantageous in comparison with the aggregate methods and other methods.

We can also conclude, for insurers with the largest portfolios in the market, that have considerable participation in relation to the total, the Pay-As-You-Drive pricing method can be a simple and effective alternative pricing technique. However, for the portfolios with a smaller market share and a low number of insured vehicles, this technique is less efficient, which decreases the credibility of the sample data and may distort the results, resulting in misguided pricing strategies, reduced competitiveness and financial losses to the company.

There are costs that need to be considered when implementing this PAYD pricing method. First, it is necessary to have a team with expertise in statistics, actuarial science or a related field, to accomplish the formulation, interpretation and appropriate updating of the models. Second, there are the high costs involved in installation and maintenance of computer programs associated with this method. The decision as to which pricing technique to employ is thus best left to the strategy of each insurer and should be analyzed in relation to the cost of each and how much this will impact positively on the result of the portfolio.

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