Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
CYBER NETWORK SECURITY
END OF SEMESTER EXAMINATION
CNS 1105: DISCRETE MATHEMATICS

DATE:4th August 2023. Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION 1: (30 MARKS)

a) Define the following terms as used in logic theory

i) A proposition (1 marks)

ii) A truth table (1 marks)

b). Show that the following compound proposition is a tautology.
$[\neg p \wedge (p \vee q)] \rightarrow q$. (4 marks)

c) Given the matrices $A = \begin{pmatrix} 1 & 3 \\ 9 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 0 \\ 4 & -2 \end{pmatrix}$ find the matrix D where $D = 3A + AB - 5C$ (4 marks)

d) Suppose set A = {1,2,3,4} and Set B = {0,2,4,6} and relation $(a,b) \in R$ such that a < b.

i. List the elements of R (1 Mark)
ii. Domain and range of R (1 mark)
iii. Complement of R (1 mark)

f) Prove by induction that $7 + 10 + 13 + 16 + ... + (3n+4) = \frac{n}{2}(3n+11)$ (4 marks)

g) Let $\Sigma = \{1,2,3,4,5,6,7,8,9\}, A = \{1,2,3,4\}, B = \{2,4,6,8\}$. List the elements of the following sets. $(A \cap B) \cup (A - B)$ (3 marks)

h) Given the following functions $f(x) = 10x^2 - 6x + 7$ and $g(x) = 2x + 1$. Find the composite functions. $f(g(x))$
(2 marks)

j) i. Find the first five terms of the sequence defined by $a_n = a_{n-1} + 3a_{n-2}$, $a_0 = 1, a_1 = 2$ (2 marks)

ii. Is the sequence $a_n = 2^n$ a solution to the recurrence relations $a_n = 8a_{n-1} - 16a_{n-2}$. (3 marks)

iii. Solve the recurrence relations with $a_n = 7a_{n-1} - 10a_{n-2}$ with $a_0 = 2, a_1 = 1$

(3 marks)

QUESTION 2: (20 MARKS)

a) Determine whether the sequence $a_n = n^2(4)^n$ is a solution to the recurrence relations.

$a_n = -3a_{n-1} + 4a_{n-2}$. (5 marks)

b) Solve the recurrence relations $a_n = -a_{n-1} + 5a_{n-2} - 3a_{n-3}$, with initial conditions $a_0 = 0, a_1 = 1, a_2 = 2$ (8 marks)

c) Use mathematical induction to prove that $4 + 7 + 10 + \ldots\ldots\ldots + (3n+1) = \frac{n(3n+5)}{2}$ (7 marks)

QUESTION 3: (20 MARKS)

a) If $A = \{4,5,6,7\}$ and $B = \{a,b,c,d\}$, Draw an arrow diagram of each function, determine the type and state whether the function has an inverse or not.

i) $f = \{(4,a),(5,b),(6,b),(7,d)\}$ (3 marks)

ii) $g = \{(4,b),(5,d),(6,a),(7,a)\}$ (3 marks)

iii) $h = \{(4,d),(5,b),(6,a),(7,c)\}$ (3 marks)

b) Given the following functions $f(x) = 5x^2 - 3x + 20$ and $g(x) = 2x + 3$ .Find

i. $g^{-1}(f(x))$ (3 marks)

ii. $\frac{f(x+h) - f(x)}{h}$ (5 marks)

c) Find the largest interval *D* on which the formula $f(x) = x^2$ defines a one -to-one function.

(3 marks)

QUESTION 4 (20 MARKS)

a) Consider the following propositions:
p:Swimming at the New Jersey shore is allowed
q:Sharks have been spotted near the shore
Write the compound propositions symbolized by:
(i) $p \vee \neg q$ (1 marks)
(ii) $\neg(q \wedge p)$ (1 marks)

(iii) $\neg p \rightarrow q$ (1 marks)

(iv) $\neg p \leftrightarrow \neg q$ (1 marks)

b) Construct a truth table for $[(p \vee q) \wedge (\neg r)] \leftrightarrow (q \rightarrow r)$.Is this proposition a tautology? Justify your answer. (5 marks)

c) Let $P(x)$ be the statement "$x \leq 4$ ".If the universe of discourse is the set of integers, what is the truth values of the following?

(i) $P(0)$ (1 marks)

(ii) $\exists x P(x)$ (1 marks)

(iii) $\forall x P(x)$ (1 marks)

d) Write the converse, contrapositive and inverse of the following statement

If Jack plays his guitar then Sara will sing (3 marks)

e) Test the validity of the following argument:

If I study hard, I will pass my exams .If I pass my exams then I will climb Mt Kenya. I did not climb Mt Kenya. Therefore I did not pass my exams. (5 marks)

QUESTION FIVE (20 MARKS)

a) Let $A = \{1,2,3,4\}$.In the given relation below, determine whether the following Properties reflexive, symmetric, ant symmetric or transitive is satisfied or not. In each case justify your answer.

$R = \{(1,1),(2,2),(2,3),(2,4),(3,2)(3,3)(3,4),(4,2),(4,4)\}$ (4 marks)

b) Let $A = \{1,2,3\}$, $B = \{a,b,c\}$ and $C = \{x,y,z\}$.Consider the following relation R and S.

$R = \{(1,b),(2,a),(2,c)\}$ and $S = \{(a,y),(b,x),(c,y),(c,z)\}$

i. Find the composition relation RoS (3 marks)

ii. Find the matrices M_R, M_S and M_{RoS}. (3 marks)

iii. Find the elements of R^c and S^c. (2 marks)

c) In a class of 50 first year STRATHMORE University's Computer Science students studies subsidiary subjects: mathematics, Electronics and accounting. Given that 30 study mathematics, 18 study Electronics, 26 study accounting, 9 study mathematics and electronics, 16 study Mathematics and accounting, and 8 study electronics and accounting, 47 study at least one of the three subjects.

(i) How many students study none of the three subjects? (1marks)

(ii) How many students study all of the three subjects? (2 marks)

(iii) Use the value obtained in (ii) to represent the information in Venn-diagram (3 marks)

(iv) How many students study mathematics and electronics but not accounting

(1 marks)

(v) How many students study mathematics but neither electronics nor accounting? (1 marks)