



**SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
END OF SEMESTER EXAMINATION**

EMT 2211: Engineering Mathematics II

Instructions

Time: 3 Hours

Date: 19th March, 2026

1. This examination consists of **FIVE** questions.
 2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.
-

QUESTION ONE (30 MARKS)

- (a) Use the Laplace transform of the second derivative to derive [3 Marks]

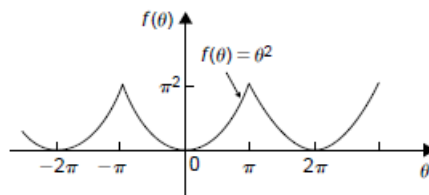
$$\mathcal{L}\{\cos at\} = \frac{s}{s^2 + a^2}$$

- (b) The final value theorem states that:

$$\lim_{t \rightarrow \infty} [f(t)] = \lim_{s \rightarrow 0} [s\mathcal{L}\{f(t)\}]$$

Verify the final value theorem for the function $(2 + 3e^{-2t} \sin 4t)\text{cm}$, which represents the displacement of a particle. State its final steady value. [3 Marks]

- (c) Determine the Fourier series for the function $f(\theta) = \theta^2$ in the range $-\pi < \theta < \pi$. The function has a period of 2π . [5 Marks]



- (d) A transformation in three dimensional space is defined by the following 3×3 matrix, where x is a scalar constant.

$$C = \begin{pmatrix} 2 & -2 & 4 \\ 5 & x-2 & 2 \\ -1 & 3 & x \end{pmatrix}.$$

Show that C is non singular for all values of x .

[4 Marks]

- (e) The velocity of a car, accelerating at uniform acceleration a between two points, is given by $v = u + at$, where u is its velocity when passing the first point and t is the time taken to pass between the two points. If $v = 21\text{m/s}$ when $t = 3.5\text{s}$ and $v = 33\text{m/s}$ when $t = 6.1\text{s}$, use matrix inverse to find the values of u and a , each correct to 4 significant figures. [5 Marks]
- (f) Find the particular solution of the equation: $x \frac{dy}{dx} = \frac{x^2 + y^2}{y}$, given the boundary conditions that $y = 4$ when $x = 1$. [5 Marks]
- (g) The equation of motion of a train is given by:

$$m \frac{dv}{dt} = mk(1 - e^{-t}) - mcv,$$

where v is the speed, t is the time and m, k and c are constants. Determine the speed v given $v = 0$ at $t = 0$. [5 Marks]

QUESTION TWO (15 MARKS)

- (a) In determining a Fourier series to represent $f(x) = x$, in the range $-\pi$ to π , Fourier coefficients are given by:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos nx \, dx$$

and $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx \, dx$

where n is a positive integer. Show by using integration by parts that $a_n = 0$ [5 Marks]

- (b) (i) Obtain a Fourier series for the function defined by: [7 Marks]

$$f(x) = \begin{cases} x, & \text{when } 0 < x < \pi \\ 0, & \text{when } \pi < x < 2\pi \end{cases}$$

- (ii) For the Fourier series in part (i), let $x = 0$ and deduce a series for $\frac{\pi^2}{8}$. [3 Marks]

QUESTION THREE (15 MARKS)

- (a) Use Euler's method to obtain a numerical solution of the differential equation $\frac{dy}{dx} + y = 2x$, given the initial conditions that at $x = 0, y = 1$, for the range $0(0.2)1.0$. Draw the graph of the solution in this range. [11 Marks]
- (b) Solve the differential equation $\frac{dy}{d\theta} = \sec \theta + y \tan \theta$ given the boundary conditions $y = 1$ when $\theta = 0$. [4 Marks]

QUESTION FOUR (15 MARKS)

- (a) A direct current (d.c) circuit comprises three closed loops. Applying Kirchhoff's laws to the closed loops gives the following equations for current flow in milliamperes:

$$2I_1 + 3I_2 - 4I_3 = 26$$

$$I_1 - 5I_2 - 3I_3 = -87$$

$$-7I_1 + 2I_2 + 6I_3 = 12$$

Use determinants to solve for I_1 , I_2 and I_3

[6 Marks]

- (b) The 2×2 matrix A is given below.

$$A = \begin{pmatrix} 1 & 8 \\ 8 & -11 \end{pmatrix}$$

- (i) Find the eigenvalues of A . [3 Marks]
(ii) Determine an eigenvector for each of the corresponding eigenvalues of A . [4 Marks]
(iii) Find a 2×2 matrix P , so that [2 Marks]

$$P^T A P = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

where λ_1 and λ_2 are the eigenvalues of A , with $\lambda_1 < \lambda_2$.

QUESTION FIVE (15 MARKS)

- (a) Find each of the following Laplace transforms or inverse Laplace transforms, showing, where appropriate, the technique used.

(i) $\mathcal{L} \left\{ 3e^{-\frac{1}{2}x} \sin^2 x \right\}$ [3 Marks]

(ii) $\mathcal{L}^{-1} \left\{ \frac{3s^3 + s^2 + 12s + 2}{(s-3)(s+1)^3} \right\}$ [4 Marks]

- (b) Use Laplace transform to solve the differential equation: [8 Marks]

$$\frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} + 10y = e^{2x} + 20,$$

given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = -\frac{1}{3}$.

END OF PAPER

Formula sheet

Function, $f(t)$	Laplace transforms $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$
1	$\frac{1}{s}$
k	$\frac{k}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$t^n (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
Function, $e^{at} f(t)$	Laplace transforms $\mathcal{L}\{e^{at} f(t)\}$
$e^{at} t^n$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2 - \omega^2}$
$e^{at} \cosh \omega t$	$\frac{s-a}{(s-a)^2 - \omega^2}$

Fourier series for periodic functions	
Over period 2π	$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ where $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$; $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$ and $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$; $(n = 1, 2, 3, \dots)$
Over period L	$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos \left(\frac{2\pi nx}{L}\right) + b_n \sin \left(\frac{2\pi nx}{L}\right)]$ where $a_0 = \frac{1}{L} \int_{-L/2}^{L/2} f(x) dx$; $a_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \cos \left(\frac{2\pi nx}{L}\right) dx$ and $b_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \sin \left(\frac{2\pi nx}{L}\right) dx$; $(n = 1, 2, 3, \dots)$