



**SCHOOL OF COMPUTING AND ENGINEERING SCIENCES**

**BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING**

**END OF SEMESTER EXAMINATION**

**MAT 1101: MATHEMATICS I**

DATE: 28<sup>TH</sup> OCTOBER 2024

Time: 13:00 - 15:30

**Instructions**

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

**QUESTION ONE [30 MARKS]**

(a) Prove by mathematical induction that  $6^{n+2} + 7^{2n+1}$  is divisible by 43 for all integers  $n \geq 0$ . **[4 Marks]**

(b) Evaluate  $\lim_{\theta \rightarrow \pi/2} \left\{ \frac{\sin \theta - 1}{\ln \sin \theta} \right\}$  **[3 Marks]**

(c) The charge  $q$  on the plates of a capacitor is given by  $q = CVe^{-t/CR}$ , where  $t$  is the time,  $C$  is the capacitance and  $R$  the resistance. Determine

(i) The rate of change of charge, which is given by  $\frac{dq}{dt}$ . **[1 Mark]**

(ii) The rate of change of current, which is given by  $\frac{d^2q}{dt^2}$ . **[1 Mark]**

(d) Evaluate  $\int_0^{\pi/2} \sin^2 x \cos^3 x dx$  **[4 Marks]**

(e) Solve the equation  $2 \sin 2\phi \sin \phi = 0$  in the range of  $\phi = 0$  to  $\phi = 180^\circ$ . **[4 Marks]**

(f) Find the constant  $k$  such that the function  $f(x)$  is continuous at  $x = \pi$ .

$$f(x) = \begin{cases} k^3 - x^3 & \text{if } x \leq \pi \\ k \sin x & \text{if } x > \pi \end{cases}$$

[2 Marks]

(g) Given that  $f(x) = \sqrt{\sin(2x+7)}$ , find  $f'(x)$  from first principles.

[4 Marks]

(h) Find the area between the curves whose equation is  $y = 2x + \sin x + e^{-x}$ , the  $x$ -axis, and the lines  $x = 0$ ,  $x = \frac{\pi}{2}$ .

[3 Marks]

(i) Show that  $\int_0^2 x^2 e^{2x} dx = \frac{1}{4} [5e^4 - 1]$

[4 Marks]

### **QUESTION TWO [15 MARKS]**

(a) The extension,  $x$  metres of an undamped vibrating spring after  $t$  seconds is given

$$x = 0.54 \cos(0.3t - 0.15) + 3.2. \text{ Calculate the speed of the spring, given by } \frac{dx}{dt}, \text{ when}$$

(i)  $t = 0$

[1 Mark]

(ii)  $t = 2$  seconds

[1 Mark]

(b) Show that, if  $P$  and  $Q$  are constants and  $y = P \cos(\ln t) + Q \sin(\ln t)$ , then

$$t^2 \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + y = 0$$

[5 Marks]

(c) The electrostatic potential on all parts of a conducting circular disc of radius  $r$  is given by the equation

$$V = 2\pi\sigma \int_0^r \frac{R}{\sqrt{R^2 + r^2}} dR$$

Solve the equation by determining the integral.

[3 Marks]

(d) One hundred people were asked about three brands of soft drinks  $A$ ,  $B$  and  $C$ . 18 like  $A$  only, 23 like  $A$  but not  $B$ , 26 like  $A$ , 8 like  $B$  and  $C$ , 48 like  $C$ , 8 like  $A$  and  $C$ . 54 like one and only one of the drinks.

(i) Represent the information on a Venn diagram.

[3 Marks]

(ii) Find the number that like  $B$ .

[1 Mark]

(iii) How many do not like any of the three brands?

[1 Mark]

**QUESTION THREE [15 MARKS]**

(a) Prove that

$$\frac{\sin 4x - \sin 2x}{\cos 4x + \cos 2x} = \tan x$$

[3 Marks]

(b) Show that  $y = \frac{2}{3}(t-1)^3 + 2t(t-2)$  has a maximum value of  $\frac{2}{3}$  and a minimum value of  $-2$ .

[4 Marks]

(c) Evaluate  $\int_5^6 \frac{6x-5}{(x-4)(x^2+3)} dx$ .

[8 Marks]

**QUESTION FOUR [15 MARKS]**

(a) Express  $5.3\cos t - 7.2\sin t$  in the form  $R\sin(t + \alpha)$ . Hence solve the equation  $5.3\cos t - 7.2\sin t = 4.5$  in the range  $0 \leq t \leq 2\pi$ .

[5 Marks]

(b) A closed cylindrical container has a surface area of  $400 \text{ cm}^2$ . Determine the dimensions for maximum volume.

[5 Marks]

(c) Given that  $I_n = \int (\ln x)^n dx$  show that  $I_n = x(\ln x)^n - nI_{n-1}$  and use this result to evaluate

$$\int_1^e (\ln x)^3 dx.$$

[5 Marks]

**QUESTION FIVE [15 MARKS]**

(a) Find the finite area enclosed by the curves whose equations are  $y = 16 - x^2$  and

$$y = x^2 - 4x.$$

[5 Marks]

(b) Use Maclaurin's series to expand  $\ln(1+x)$  in ascending powers of  $x$  up to and including the term  $x^6$ . Hence determine

$$\int_0^1 \frac{\ln(1+x)}{x} dx$$

giving the answer correct to four decimal places.

[5 Marks]

(c) Determine the power series for  $\sin(a+h)$  and hence determine the value of  $30^0 - 30'$  correct to five decimal places.

[5 Marks]

**END**