Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

END OF SEMESTER EXAMINATION

MAT 1201: MATHEMATICS II

DATE: 8th March 2024 Time: 3 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE [30 MARKS]

(a) Given that $u = e^x\left[\sin(y+z) - y\cos(y+z)\right]$. Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + u = 2e^x \sin(x+y)$.

[5 Marks]

(b) A 2×2 matrix has eigenvalues $\lambda_1 = 3$ and $\lambda_2 = 2$ with corresponding eigenvectors $x_1 = [1 \quad 1]^T$ and $x_2 = [1 \quad 2]^T$. Determine the:

i. Modal matrix M and spectral matrix Λ. **[2 Marks]**
ii. Matrix A **[3 Marks]**

(c) Find the stationary points of the function $z = x^3 + y^2 - 2xy - 5y + 15$ and determine their nature.

[6 Marks]

(d) In a mass spring-damper system, the acceleration $\frac{d^2x}{dt^2}$ m/s^2, velocity $\frac{dx}{dt}$ m/s and displacement x m are related by the equations:

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 4x = 35$$
$$2\frac{d^2x}{dt^2} + \frac{dx}{dt} + 2x = 20$$
$$-3\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 24$$

By using Gauss Jordan method, determine the acceleration, velocity and displacement for the system.

[5 Marks]

(e) A solid of revolution is formed by rotating about the $x-axis$, the area between the $x-axis$, the lines $x=0$ and $x=1$ and a curve through the points with the following coordinates:

x	0.00	0.25	0.50	0.75	1.00
y	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the slid formed using Simpson's rule. **[4 Marks]**

(f) Given that $A = \begin{pmatrix} 2 & x \\ x & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$. AB is a singular matrix, determine the possible values of x.

[5 Marks]

<u>QUESTION TWO [15 MARKS]</u>

(a) Given that

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \\ 3 & -1 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & -1 & 1 \\ 4 & -10 & -8 \\ -1 & 7 & 5 \end{pmatrix}$$

i. Find the matrix product AB. **[2 Marks]**

ii. Hence solve the following system of simultaneous equations:

$$x + 2y + 3z = -6$$
$$-2x + y + 2z = 1$$
$$3x - y - z = 1$$

[3 Marks]

(b) Solve $\frac{\partial^2 u}{\partial x \partial y} = 4e^y \cos 2x$, given that at $y=0$, $\frac{\partial u}{\partial x} = \cos x$ and at $x = \pi$, $u = y^2$.

[4 Marks]

(c) A company makes three types of security locks A, B and C, each of which requires cutting, assembly, and finishing. Each unit of A requires 2 hours for cutting, 1 hour for assembly, and 3 hours for finishing. Each unit of B requires 1 hour for cutting, 2 hours for assembly, and 1 hour for finishing. Each unit of C requires 1 hour for cutting, 2 hours for assembly , and 3 hours for finishing.
Available machine resources provide exactly 10 hours for cutting, 14 hours for assembly, and 18 hours for finishing each week.

i. Use the information provided to form a system of simultaneous equations.
[2 Marks]

ii. Apply Cramer's rule to calculate the number of locks of each type produced given that there is optimum utilization of machine time at the factory in a week.
[4 Marks]

QUESTION THREE [15 MARKS]

(a) Find the possible percentage error in computing the parallel resistance r of the three resistance r_1, r_2 and r_3 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ if r_1, r_2, r_3 are each in error by $+1.2\%$.
[3 Marks]

(b) A wave equation is given by $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$. Show that the equation is satisfied by a solution $u = \left(A\cos\frac{\omega x}{C} + B\sin\frac{\omega x}{C}\right)\left(C\cos\omega t + D\sin\omega t\right)$.
[4 Marks]

(c) Solve the equation $\begin{vmatrix} 1 & -1 & -2 \\ 2 & 3x & 1 \\ 2 & 1 & x \end{vmatrix} = 2$. **[4 Marks]**

(d) Verify Euler's theorem for $u = \dfrac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}$. **[4 Marks]**

QUESTION FOUR [15 MARKS]

(a) Express the matrix $\begin{pmatrix} 2 & 1 & 3 \\ -1 & 4 & 1 \\ 0 & 2 & -2 \end{pmatrix}$ as the sum of a symmetric and skew-symmetric matrix. **[3 Marks]**

(b) Form the partial differential equation by eliminating the arbitrary function from $f\left(x+y+z,\ x^2+y^2+z^2\right)=0$. **[4 Marks]**

(c) A closed cylindrical container is to be made from a sheet of material in order to contain 100 m^3. Use Lagrange's method to determine the dimensions if the sheet of metal is minimum. **[4 Marks]**

(d) If $u=\sec^{-1}\left(\dfrac{x^3-y^3}{x-y}\right)$, evaluate $x^2\,u_{xx}+2xy\,u_{xy}+y^2\,u_{yy}$. **[4 Marks]**

QUESTION FIVE [15 MARKS]

(a) Determine the constants a and b in order that $w=x^2+ay^2-2xy+j\left(bx^2-y^2+2xy\right)$ Should satisfy the Cauchy-Riemann equations. Deduce the derivative of w.

[4 Marks]

(b) Show that $u=2xy+3y$ is harmonic and determine harmonic conjugate v. Express the functions $\dfrac{dw}{dz}$ and $w=u+jv$ in terms of z.

[4 Marks]

(c) Find the image in the $w-plane$ of the circle $|z|=2$ if $w=\dfrac{z-j}{z+j}$.

[7 Marks]

END