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Analysis of risk measures in portfolio optimization for the Uganda Securities Exchange.

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Analysis of Risk Measures in Portfolio Optimization for the Uganda Securities Exchange

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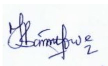
Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

Signature: 
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Abstract

For the most recent years, risk has become one of the essential parameters in portfolio optimization problems. Today most practitioners and researchers in portfolio optimization have used variance as a standard risk measure. This approach has been found subjective. The Markowitz (1952) mean-variance model considered variance as an adequate portfolio risk measure, and asset returns are multivariate normally distributed and that investors have a quadratic utility function which is subjective too. Other risk measures have been suggested to overcome the limitations of the mean-variance model. This paper analyzes which portfolio optimization models can better explain the optimal portfolio performance (high return, low risk) for the Uganda Security Exchange(USE). We compare Mean-Variance (MV), Mean Absolute Deviation (MAD), Robust Portfolios and Covariance Estimation Models(The Shrinked Mean-Variance (SMV) Models & Alternative Covariance Estimator (ACE) Models) and Mean-Conditional Value-at-Risk (Mean-CVaR) models in terms of the risk and performance. Portfolios were developed by employing the MV, MAD, SMV, ACE and Mean-CVaR models. For the computed monthly returns and price data (February 2010 to January 2021) for USE selected stocks, we considered the results show that Mean-CVaR and ACE portfolios have the highest performance ratio compared to other models. We find that VaR is the best risk measure for portfolio optimization for the USE since it has lower values across all models than other risk measures. It is vital to consider all the available risk measures for a regulator or practitioner to make a good decision since using one can be subjective; as seen in our results, different risk measures yield different results.

Keywords: Portfolio Optimization, Risk measures, Conditional Value-at-Risk, Variance, Uganda Security Exchange.

Contents

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List of abbreviations

MV- Mean-Variance
MAD- Mean Absolute Deviation
MM- Minimax
VaR- Value at risk
CVaR- Conditional Value-at-Risk
RMVE- Robust Mean and Covariance Estimators
SMV- Shrinked Mean Variance
ACE- Alternative Covariance Estimator
USE- Uganda Securities Exchange
BoU- Bank of Uganda
UCL- Uganda Clays Ltd
BATU- British American Tobacco Uganda Ltd
BOBU- Bank of Baroda Ltd
DFCU- Development Finance Company of Uganda Ltd
NVL- New Vision Printing and Publishing Company Ltd
SBU- Stanbic Bank Uganda
NIC- National Insurance Corporation
UMEM- UMEME Ltd
KCB- Kenya Commerical Bank Ltd
UGA- Stanbic Bank Uganda(UGA) DEAD - DUPL
LSI- Local Share Index

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Chapter 1

Introduction

1.1 Background of the study

A portfolio is a collection of financial instruments. Portfolio optimization is the process of selecting the best portfolio (asset distribution) by the investor from a list of portfolios that will give him or her the highest level of return given an investment objective. Risk measures are tools used to assess the performance of an investment basing on the exposure and degree at which the associated prices move (Volatility). The question is which risk measure to be selected for the appropriate portfolio investment management.

1.1.1 Modern Portfolio Theory (MPT)

Markowitz(1959) laid the groundwork for MPT defining an investor's portfolio selection problem regarding expected return and variance of return. He postulates that an investor should maximize expected portfolio return while minimizing portfolio variance of return. Since the introduction of Markowitz (1952) [19] Mean-Variance (MV) model, variance has become the most common risk measure in portfolio optimization. However, this model relies strictly on the assumption that the returns of assets are multivariate normally distributed or the investor's utility function is quadratic , [7]. The most recent literature has shown that the Markowitz framework and MV formulation based on these two assumptions seem not to hold in the real market. Markowitz argues that given estimates of the returns, volatilities, and correlations of a set of investments and constraints on investment choices [21]. From this, it is possible to perform an optimization that results in the risk/return, or mean-variance efficient frontier, according to him, this frontier is efficient because every portfolio on this frontier is a portfolio that results in the greatest possible expected return for that level of risk. However, it is observed that in postwar US data, the slope of the mean-standard deviation frontier is much higher than reasonable risk aversion and consumption volatility estimates suggest. Brooks and Kat (2002) [15] also show that hedge funds returns are not normally distributed.

Therefore due to issues raised and limitations of Markowitz's (1952) Mean-Variance (MV) model, different measures of risk measures in portfolio optimization have been proposed, such as Mean Absolute Deviation (MAD) by Konno and Yamazaki (1991), Minimax (MM) attributed to Young (1998), Albuquerque (2009) Beta models and Silva et al. [2017] Beta-Conditional Value-at-Risk (CVaR) models. There have been several studies in Portfolio optimization using different risk measures by employing portfolio optimization models. Mayanja (2011) [6] urge that to carry out portfolio optimization, one needs software, which must have inbuilt algorithms. This means that for one to think about portfolio optimization, one should first be assured of a portfolio that exists, then the next step would be which percentages in terms of allocation should be allocated to each portfolio. Such portfolios are available in Stock (Securities) Exchanges where they are traded; in these exchange markets, buyers and sellers hold securities and make transactions decisions when maturity(can be short or long term) reaches to exchange these assets. Literature shows stock markets play a vital role in developing economies in generating capital, especially where they are organized and efficient. Besides, there are some of the new trends in portfolio optimization, such as diversification methods, risk-parity portfolios, the mixing of different sources of alpha, and practical multi-period portfolio optimization Kolm et al. (2014), [3].

1.1.2 The Uganda Securities Exchange (USE)

The Uganda Securities Exchange is one such stock market where portfolios are traded. The formal operation started in 1997 after USE's license by the Capital Markets Authority of Uganda. The USE began formal trading operations in January 1998 following the listing of its first instrument, the 4-year East African Development Bank (EADB) Bond. In January 2000, USE listed its first equity, Uganda Clays Ltd [[6], [20]]. Since 2000, USE has been growing with more individual investors, and financial institutions listed hold shares of these companies at USE. Among the Securities currently traded at the Exchange include Government Bonds, Corporate Bonds, Commodities, and Ordinary Shares. However, Literature shows that risk measurement in terms of market analysis at USE has become more complex as more companies are listed. Individual investors and financial institutions prefer investment portfolios that are stable and with less volatility. Several finance models exist which can be used to determine returns and return volatility [5].

There exist portfolio optimization studies in the Uganda Security Exchange.

Most of the studies conducted on portfolio optimization in Uganda Securities Exchange have concentrated on the tests for the stock performance of the models, generating more risk measures like volatility, Sharpe Ratio (SR), Risk Parity (RP), Expected Shortfall (ES) or CVaR which they used to assess stock performance [2]. These studies conducted were on how the portfolio performs, but not on the investigation of which portfolio optimization models can better explain the portfolio performance in terms of return and risk for the Uganda Security Exchange. Therefore this justifies the purpose of our study on analysis of which portfolio optimization models Mean-Variance (MV), Mean Absolute Deviation (MAD), Robust Portfolios and Covariance Estimation Models(The Shrinked Mean-Variance (SMV) Models and Alternative Covariance Estimator (ACE) Models and Mean-Conditional Value-at-Risk (CVaR) models, can better explain the optimal portfolio performance in terms of returns and risk for the Uganda Security Exchange(USE).

1.2 The Problem statement

One way of selecting optimal portfolios at USE is through market surveillance and speculation. However, USE is not mature enough to facilitate investors' investment choices, leading to the low utility of the same to the investors. Literature shows that the approach of market surveillance and speculation is insufficient and subjective. One way of facilitating optimal investment portfolios in the market is by ensuring that the models used in portfolio optimization are as close to the actual market scenario as possible. Therefore, we want to answer the central question: Can we get an alternative method of selecting optimal portfolios at USE? To answer this question, we compared traditional portfolio optimization models to select optimal portfolios with the proposed risk models. We used variance, Covariance, Value-at-Risk(VaR) and Conditional Value-at-Risk(CVaR) as risk measures to find out which model is efficient(high return, low risk) for USE.

1.3 Objectives of the study

1.3.1 Main Objective

This study's main objective is to establish the best risk measure for Portfolio Optimization for the USE.

1.3.2 Specific Objectives

The specific objectives of this research are;

(i) To develop optimal portfolios using the traditional portfolio optimization models and the proposed risk models at USE.

(ii) To determine the performance of the portfolios in terms of returns at USE.

(iii) To determine the risk associated with each portfolio using different risk measures at USE.

1.4 Significance of the study

The study will:

(i) To provide an alternative method of selecting optimal portfolios at USE.

(ii) Establish the best risk measure for Portfolio Optimization for the USE.

(iii) Develop portfolio optimization models to select an optimal portfolio for investment at Uganda Securities Market and other Ugandan financial institutions with interest in portfolio investment.

(iv) Add on the foundation and further research portfolio optimization on Securities Exchanges especially developing markets like USE.

We structured the rest of this paper as follows. The next chapter (2) literature review discusses the theoretical framework of traditional portfolio optimization models and the risk models together with empirical literature on some studies done in portfolio optimization. Chapter (3) the methodology discusses the mathematical models, mainly traditional portfolio optimization models and the proposed risk models, Parameter estimation, portfolio compositions and portfolio formations. In the later chapter (4) we discuss the formed portfolios' computational and performance results by employing the five optimization models mentioned using the proposed data on chapter (3) of the Uganda Securities Exchange.

Chapter 2

Literature Review

2.1 Theoretical Literature

2.1.1 Methods used in determining Optimal Portfolio

Markowitz(1959) proposed the Mean-Variance (MV) method to select optimal portfolio defining an investor's portfolio selection problem in terms of expected return and variance of return, but this was later found to rely on two assumptions that the returns of assets are multivariate normally distributed or investor's utility function is quadratic, [7]. Markowitz's view on the selection of optimal portfolio was challenged by limitations associated with this method for selecting an optimal portfolio, for example, the objective function of the Mean-Variance (MV) model is associated with covariance matrix in which, through the process of optimization, may result in a quadratic optimization problem (literature shows that this is quite a challenge to obtain optimal solutions). Brooks and Kat (2002) also show that returns from hedge funds are not normally distributed, which challenges Markowitz's assumption of the normality of returns. Also, Mean-Variance (MV) assumes that the standard deviation is an adequate portfolio risk measure (Kroll et al. 1984, [16]). Markowitz's MV model further assumption that investors' perception of risk is symmetrical is not valid in the real market. Therefore due to some of these drawbacks of Markowitz's MV model for optimal portfolio selection, other methods for portfolio optimization selection were proposed.

To overcome Markowitz's assumption of normality of assets returns, Young (1998) proposed the Minimax model as an alternative method for optimal portfolio selection. Young (1998) method is based on game theory that involves trade-offs between choices and payoffs, meaning if each player(investor(s)) behaves rationally. The game theory asserts that a solution for every situation can be determined by assuming that players (investors) seek to maximize their expected minimum returns (maximum criteria) or minimize their maximum expected losses (Minimax criteria). In modelling decision-making under risky conditions, Young assumed that nature is the other player playing against the investor. Young's method for optimal portfolio selection is also based on the idea that the investor(s) makes decisions to protect themselves from the worst possible outcome. Young's optimal portfolio selection method has been proved to have logical advantages over other optimal portfolio models like Markowitz's MV model, especially when prices or returns are not normally distributed. However, it generates similar results when prices are normally distributed.

To overcome Markowitz's challenges that may be faced during the optimization process of the MV model's objective function, the dimension of the covariance matrix that may result in a quadratic optimization problem may be a challenge to obtain optimal solutions. Konno and Yamazaki (1991) proposed Mean Absolute Deviation (MAD) Model as another alternative method to determine an optimal portfolio by retaining the advantages of the MV model while eliminating some of its shortcomings. Konno and Yamazaki (1991) model framework seeks to minimize the return's average absolute deviation subject to a given level of mean return restriction. In this regard, this formulation employs linear functions that can be solved pretty quickly. This model's construction involves creating a non-linear formulation that approximates a linear one. Further, the non-linear formulation is the absolute deviation from the portfolio's mean return. Silva, L. P., Alem, D., & Carvalho, F. L. (2017) proposed alternative models (Beta-CVaR) that combine the mean absolute deviation (MAD) and the conditional value at risk (CVaR), the results obtained with the proposed models were satisfactory. They generated an optimal portfolio with a higher effective return than the other models in all test groups. Therefore using these frameworks to select optimal portfolios, this study uses some of these approaches to develop optimal portfolios using secondary data collected from the Uganda Securities Exchange, including information on securities and value of the USE Local Share Index. Data collection sheet together with average monthly stock prices, monthly closing stock index values were collected from the monthly bulletin compiled by the USE. The data include computed monthly returns and prices for the period from February 2010 to January 2021. Therefore a total of 132 months were used for the analysis.

2.1.2 Methods used in determining Portfolio Performance

In this section, we give a review of how portfolio performance is determined. Basing on past researches, portfolio performance has been calculated by using the reward per risk equation as shown in Equation (2.1), i.e. dividing portfolio means to return with the is associated risk that portfolio is,

$$Portfolio\ Performance = \frac{Portfolio\ Mean\ Return}{Portfolio\ risk}. \tag{2.1}$$

Portfolio Performances is usually calculated based on the mean return of the portfolio with its associated risk. The optimal portfolios generated are Mean-Variance (MV), Mean Absolute Deviation(MAD), Robust Portfolios and Covariance Estimation Models(The Shrinked Mean-Variance (SMV) Models & Alternative Covariance Estimator (ACE) Models) and Mean-Conditional Value-at-Risk (CVaR) models. We employ this technique to form the five portfolios (MV, MAD, Mean-CVaR, SMV and ACE). We shall use these generated optimal portfolios to determine the mean return of each portfolio and its associated risk to obtain the overall portfolio performance of each portfolio. We will compare each model's overall performance to assess the one with the highest and lowest performance. Therefore, as different risk measures (Portfolio Optimization Models) were proposed, this study seeks to compare these Portfolio Optimization Models' performance in terms of returns. We shall be using Equation (2.1) to obtain portfolio performance associated with each portfolio at Uganda Securities Exchange(USE).

2.1.3 Risk tools used in Portfolio Optimization

Markowitz's Mean-Variance (MV) model in 1959 assumed that the standard deviation is an adequate portfolio risk measure. This approach was a limitation of Markowitz's view of how risks can be measured in portfolio optimization. In actual market data taking standard deviation or variance as a measure of risk may be wrong since this is just a measure of dispersion in the considered data. In response to this, in 1992, Hiroshi Konno and Hiroaki Yamazaki provided an alternative to Markowitz's Mean-Variance model through their proposed Mean-Absolute Deviation (MAD) model. The MAD model framework defines risk by using the mean absolute deviation. The earlier computational advantages of MAD over MV discussed in section (2.2)are preferred in the portfolio optimization process. The computational advantages for MAD allowed Konno to solve optimization problems involving over 1,000 stocks within a fraction of the time of the mean-variance model [17]. Advancements in computing have removed many of the concerns surrounding large-scale portfolio optimization. [1]. This discussion of the computational advantages and disadvantages of MV and MAD is sometimes ignored, and the emphasis is on the performance of the model.

Young (1998) proposed the Minimax (MM) model as an alternative risk measure. The Minimax (MM) model is based on game theory that involves trade-offs between choices and payoffs. Young's view was based on the criterion that investors seek to minimize their maximum expected losses

(Minimax criteria) and always make decisions to protect themselves from the worst possible outcome. At Uganda Securities Exchange, there are various ways risk is measured, such as Value at Risk (VaR), which is used to measure the risk of loss for investments or associated portfolios. Much as VaR estimates the maximum loss of an investment(s) or portfolios at a given confidence interval and time horizon, this risk measure is not also as perfect. Therefore in this study, we shall use both Variance, Covariance, Value-at-Risk(VaR) and Conditional Value-at-Risk(CVaR) as risk measures

2.2 Empirical Literature

Table 2.1: Review of studies on portfolio optimization.

Year	References	The Study Summary
2004	Byrne et al. [14]	Byrne et al.[2004] compared the portfolio holdings produced by different risk measures, rather than the risk-return trade-off. The results indicated that the portfolio compositions produced by different risk measures vary quite markedly from measure to measure. They suggest the choice of model depends very much on the individual's attitude to risk rather than any theoretical and practical advantages of one model over another.
2010	Hoe et al. [7]	Hoe et al. [2010] discussed and compared the portfolio compositions and performances of four different portfolio optimization models employing different risk measures, specifically the variance, absolute deviation, minimax and semi-variance. Results of this study show that the minimax model outperforms the other models.

(Continued)

Table 2.1 – continued.

Year	References	The Study Summary
2017	Silva et al. [3]	Silva et al. [2017] discussed the efficiency of traditional portfolio optimization models with alternative optimization models that combine the mean absolute deviation (MAD) and the conditional value at risk (CVaR)when the returns of financial assets are highly volatile. They find that the traditional models provide portfolios with higher returns, but alternative optimization models that combine the mean absolute deviation (MAD) and the conditional value at risk (CVaR) were able to generate lower risk portfolios.
2017	Kasenbacher et al. [1]	Kasenbacher et al. [2017] compared Markowitz's Mean-Variance model and Konno's Mean-Absolute Deviation model. On data (S&P 500) considered , they find that Mean Absolute Deviation the model outperforms the Mean-Variance model..

2.3 Conclusions

Chapters 2 (2.1.1, 2.1.2 and 2.1.3) gives a review on different methods used in determining optimal portfolio, methods used in determining portfolio performance and risk tools used in portfolio optimization. Most of the proposed risk measures are modifying Markowitz(1952) Mean-Variance (MV) model. Most Literature on these studies on portfolio optimization is or was done on developed securities market like Kuala Lumpur Composite Index (KLCI), German Securities market, and New York Stock Exchange (NYSE) where the result shown on these studies seems robust, especially on the performance of portfolio optimization models. The most financial securities market in developing economies like USE are still developing and has different factors from these other financial markets. This, together with differences in weight which is in most cases due to the non-normality of the data displayed by these financial markets, yields different results for these risk measures. Market dynamics make it a different case for these risk measures when we look at them in different financial markets. Much as there have been efforts proposed to alternative risk measures and models in portfolio optimization, according to Silva et al. [2017] [3], the accurate estimation of financial asset returns remains a significant challenge in portfolio optimization.This study uses traditional optimal portfolio models and alternative risk models to come up with optimal portfolios. We will use secondary data collected from the Uganda Stock Exchange (USE) to form these portfolios, we go ahead and analyze their performances in terms of their returns and then risk associated with each portfolio using the methods discussed in section 2.1.3. The models' formulation is discussed in section 3.1 of chapter 3 of this study.

Chapter 3

Methodology

3.1 The mathematical formulation for optimal portfolio formation

The data collected was used in testing and analysis of different risk measures in portfolio optimization for the Uganda Securities Exchange. Portfolios were developed by employing the MV (3.1), MAD (3.2), SMV (3.1.4), ACE (3.1.4) and Mean-CVaR (3.7) models to compare the portfolio performance in terms of returns and risk associated with each optimal portfolio. The data include computed monthly returns and prices for the period from February 2010 to January 2021. We employed different portfolio optimization models to form these optimal portfolios.

3.1.1 Mean-Variance (MV) model

As proposed by Markowitz(1952), [19], the MV model seeks to minimize Variance VR) at a given level of expected returns E(R) shown in Equation (3.1). Equation (3.1) , together with its constraints, is the famous Markowitz's Mean-Variance Model, also commonly known as the Modern Portfolio Theory (MPT) model, [19]. Markowitz laid the groundwork for the modern portfolio theory. The MV model's objective is to find the weight of assets that will minimize the portfolio variance at a level of the required rate of return. This model is a quadratic programming model [7]. We formulate the mathematical model as follows:

$$Minimize\ \{Var(R) = \sum_{j=1}^{N} \sum_{k=1}^{N} w_j w_k \sigma_{jk}\}. \tag{3.1}$$

Subject to;

1. $E(R)=\sum_{j=1}^{N} w_j y_j \geq \rho W$, ρ is a parameter representing the minimal rate of return required by an investor. Mean return, $E(R)$ of a portfolio exceeds some minimum (ρW)

2. $\sum_{j=1}^{N} w_j \leq W$, the total allocations to the portfolio do not exceed the budget (W),

3. $0 \leq w_j \leq u_j$, for j=1,..., n and =1,..., N, maximum budget share that can be invested in assets j is u_j.

4. $\sigma_{jk} = \frac{1}{(T-N)} \sum_{t=1}^{T} (y_{jt} - y_j)(y_{kt} - y_k)$, is the covariance between assets j and k.

Parameters,
N is the number of assets, T is time, y_{jt} is return of asset j at time t, y_j is the mean return of asset j, y_{kt} is the return of asset k at time t, y_k is the mean return of asset k, w_j is the portfolio allocation for asset j, u_j is the maximum budget share that can be invested in assets j. w_k is the portfolio allocation for asset k.

The Markowitz model's simplicity has made it popular, with only two summary statistics, i.e. mean and variance, to compute. We will employ Markowitz view that given estimates of the returns, volatilities, and correlations of a set of investments and constraints on investment choices, to perform an optimization that results in the risk/return or mean-variance efficient frontier that is efficient, i.e. every portfolio on this frontier is a portfolio that results in the greatest possible expected return for that level of risk. From Model (3.1),

$$\mu_p = \sum_{j=1}^{N} w_j y_j, \quad to\ denote\ portfolio\ average\ return,$$

$$\sigma_p^2 = \sum_{j,k=1}^{N} w_j w_k \sigma_{jk}, \quad to\ denote\ portfolio\ variance.$$

Then μ_p and σ_p^2 will be the desired level of the expected return on the portfolio and ts variance, respectively.

3.1.2 Mean-Absolute Deviation (MAD) model

Konno and Yamazaki (1991) proposed the Mean Absolute Deviation (MAD) model as a risk measure to overcome the Mean-Variance (MV) model's weakness. The MAD model assumes that the standard deviation is a satisfactory portfolio risk measure. MAD employs a mean's absolute deviation for measuring risk instead of the variance. Literature shows that if returns are normally distributed, both MAD, MV and MM yield the same results. Basing on [17], MAD is mathematically formulated as follows,

$$Minimize\ w(y) = E\left[\left(\left|\sum_{j=1}^{N} y_j w_j - E\left[\sum_{j=1}^{N} y_j w_j\right]\right)\right|\right], \tag{3.2}$$

Subject to

1. $\sum_{j=1}^{N} w_j y_j \geq \rho W$, ρ is a parameter representing the minimal rate of return required by an investor. This constraint means portfolio expected return exceeds some minimum (ρW),

2. $\sum_{j=1}^{N} w_j \leq W$, the total allocations to the portfolio do not exceed the budget (W),

3. $0 \leq w_j \leq u_j$, for j=1,..., n and =1,..., N, maximum budget share that can be invested in assets j is u_j.

Parameters,
N is the number of assets, T to be used later is the time, y_j, is a random variable representing return per period for asset j , w_j, is portfolio allocation to asset j, u_j, is the maximum budget share that can be invested in asset j, ρW is the minimum level of return, W is the total allocation. Konno and Yamazaki (1991) [17] assume that the expected value can be approximated by the average over time so that,,

$$r_t = E[y_j] = \frac{1}{T}\sum_{j=1}^{N} y_{jt} \tag{3.3}$$

Here, y_{jt} is the realization of random variable y_j during period j. Substituting Equation(3.3) into Equation (3.2) we have Equation (3.4)

$$w(y) = E\left[\left|\left(\sum_{j=1}^{N} y_j w_j - E\left[\sum_{j=1}^{N} y_j w_j\right]\right)\right|\right] = \frac{1}{T}\sum_{j=1}^{N}\left[\left|\left(\sum_{j=1}^{N}(y_{jt} - y_j)w_j\right)\right|\right] \tag{3.4}$$

From Equation (3.4) by letting $z_{jt} = y_{jt} - y_j$ and simplifying (3.4) we have Equation (3.5) and now the optimization problem in (3.2) can be written as,

$$Minimize\ \ w(y) = \frac{1}{T}\sum_{j=1}^{N} z_{jt} w_j \tag{3.5}$$

Subject to the same constraints in Equation(3.2). By simplifying Equation (3.5) further that is $z_{jt}w_j = b_t \; for \;\; t = 1, \cdots, T$ we have,

$$Minimize \;\; w(y) = \frac{1}{T}\sum_{j=1}^{N} b_t, \tag{3.6}$$

Subject to

1. $b_t \pm \sum_{j=1}^{N} z_{jt}w_j \geq 0, \quad t = 1, \cdots T,$ b_t is a linear form to represent returns on asset j at time t with respective portfolio allocation w_j. This constrain account for the deviation of the values below and above the expected value of the portfolio.
2. $\sum_{j=1}^{N} w_j Y_j \geq \rho W$, ρ is a parameter representing the minimal rate of return required by an investor. This constraint means portfolio expected return exceeds some minimum (ρW),
3. $\sum_{j=1}^{N} w_j \leq W$, the total allocations to the portfolio do not exceed the budget (W),
4. $0 \leq w_j \leq u_j, \quad j = 1, ..., N$, maximum budget share that can be invested in assets j is u_j.

Equation(3.6) , together with its constraints, becomes our linear optimization problem. [7] show that there is no need to calculate the covariance matrix for this linear problem. Furthermore, it is a linear program and Equation(3.6) penalizes both negative and positive deviations. Literature shows that investors prefer higher positive deviations and avoid lower negative deviations in portfolio return [7].

3.1.3 Mean-CVaR Models

Most literature shows that traditional optimization models fail to provide efficient portfolios, especially when financial assets' returns are highly volatile, Silva et al. [2017], [3]. Various risk measures have been proposed as an alternative to variance. VaR is one of such proposed risk measures. VaR is the maximum value one stands to lose for a given period at a given confidence level. VaR's weakness is that it does not tell the amount or magnitude of the actual loss after VaR estimate, which occurs with probability $(1 - \alpha)$. For example, if the 99% VaR is, say, 2 million Kshs, we would expect to lose not more than 2 million Kshs with 99% confidence, but we do not know what amount the actual loss would be after $(1 - \alpha)$. CVaR, also referred to as Expected shortfall (ES), was proposed to overcome such a challenge. The Expected shortfall (ES) estimate is the expected loss given that the portfolio return already lies below the pre-specified worst-case quantile return. This approach is fundamental, especially if we experience a catastrophic event; this can tell us the expected loss in our financial position. Würtz et al. [2015] propose the Mean-CVaR Model where covariance risk now replaced by the CVaR as the risk measure. MV model (3.1) considered variance as a satisfactory portfolio risk measure; asset returns are multivariate normally distributed, investors have a quadratic utility function which is subjective. This model no longer restricts the set of assets to have a multivariate elliptically contoured distribution, reducing distribution bias and improving computational efficiency. Basing on Würtz et al. [2015] Mean-CVaR model is mathematically formulated as follows,

$$Min_w \quad CVaR_\alpha(w), \tag{3.7}$$

$$\begin{aligned} s.t \quad w^T\widehat{\mu} &= \overline{r}, \\ w^T\widehat{1} &= 1. \end{aligned}$$

Where,

$$CVaR_\alpha(w) = \frac{1}{1-\alpha}\int_{f(w,r)\leq VaR_\alpha(w)} f(w,r)p(r)dr,$$

$CVaR_\alpha$ is the Conditional Value at Risk associated with portfolio W, $f(w,r)$ denote the loss function when we choose the portfolio W from a set X of feasible portfolios, r is the realization of the random events with a probability density function denoted by $p(r)$.

$$VaR_\alpha(w) = min\{\gamma \in \mathcal{R} : \Psi(w,\gamma) \geq \alpha\},$$

VaR_α is the Value at Risk associated with portfolio W, with a given confidence level α,

$$\Psi(w,\gamma) = \int_{f(w,r)\leq\gamma} p(r)dr,$$

$\Psi(w,\gamma)$ is the cumulative distribution function of the loss associated with a fixed decision vector w.

Since Equation (3.7) is an optimization problem, the author proposes minimizing $CVaR_\alpha$ and VaR_α are not equivalent. They, therefore, consider the following more straightforward auxiliary function,

$$F_\alpha(w,\gamma) = \gamma + \frac{1}{1-\alpha}\int_{f(w,r)\leq\gamma}(f(w,r)-\gamma)p(r)dr. \tag{3.8}$$

The $F_\alpha(w,\gamma)$ function in (3.8) has the important properties that make it useful for the computation of $CVaR_\alpha$ and $VaR_\alpha(w)$, for example $F_\alpha(w,\gamma)$ is a convex function of γ, $VaR_\alpha(w)$ is a minimizer of $F(w,\gamma)$ and the minimum value of the function $F_\alpha(w,\gamma)$ is $CVaR_\alpha$. The latter follows performing an optimization following (3.7).

3.1.4 Robust Portfolios and Covariance Estimation Models

Würtz et al. [2015] proposed Robust Portfolios and Covariance Estimation Models to compute the mean and covariance matrix of the set of financial assets to achieve better stability properties compared to traditional minimum variance portfolios. We use two different approaches implemented by Würtz et al. [2015] that is robust mean and covariance estimators, and the shrinkage estimator.

(a) **The Shrinked Mean Variance (SMV) Models**; Würtz et al. [2015] considered a convex combination of the empirical estimator with some suitable chosen target. According to the Authors, a mixing parameter was selected to maximize the expected accuracy of the shrinked estimator; this was done by using an analytic estimate of the shrinkage in-tensity. Unlike the computational cost required in the MV model (3.1), Shrinked Mean-Variance Models increases and in terms of boundness, shrinkage estimate is always positive definite and well-conditioned, which is advantageous in terms of convergence.

(b) **Alternative Covariance Estimator (ACE) Models**; Würtz et al. [2015] provide an alternative to estimate covariance from an R's recommended packages, such as MASS, which has inbuilt functions to generate optimal portfolios. We apply this method on the 11 years' historical price data (132 months from $2010-02-26$ to $2021-01-26$ for nine stocks listed on the LSI USE indexes to compare the performance of this model with other models.

3.2 Parameter Estimation

3.2.1 Parameters in the Mean-CVaR and Robust Portfolios and Covariance Estimation models

Different R packages under *library(fPortfolio)* were used to estimate most parameters for the computed monthly returns data for each of the selected stocks using Equation (3.9) while performing the optimization of portfolios. The $((1-\alpha)$ is the confidence level.

3.3 Data collection

[1] This study uses secondary data (computed monthly returns and prices for the selected stocks from February 2010 to January 2021) from USE Local Share Index (LSI), which tracks only the USE's local companies. This period was not randomly selected; we chose this period because the Ugandan economy reported solid economic growth, especially from 2016 to 2019, estimated at 6.3%, the expansion of services drove this. We considered nine stocks, Uganda Clays Ltd (UCL), British American Tobacco Uganda Ltd (BATU), Bank of Baroda Ltd (BOBU), Development Finance Company of Uganda Ltd (DFCU), New Vision Printing and Publishing Company Ltd (NVL), Stanbic Bank Uganda (SBU), National Insurance Corporation (NIC), UMEME Ltd (UMEM) and Stanbic Bank Uganda(UGA) DEAD stock listed at USE. The nine stocks we considered were randomly selected for the analysis. There are also cross border companies listed on the Uganda Securities Exchange, which are East African Breweries Ltd, Kenya Airways, Jubilee Holdings Ltd, Equity Bank Ltd, Kenya Commercial Bank Ltd, Nation Media Group, Centum, UCHUMI) among others. Mathematically the nine stocks out of seventeen stocks by then we consider will be an excellent sample to represent the whole stocks listed at USE for this analysis to study the portfolio composition and risk measures. Information on the trading dates, opening price, closing price, Stock names, low/high prices, the volume traded was collected from the USE website. Using a data collection sheet, we only considered stock names, trading dates, and closing prices since we are interested in calculating expected values, standard deviation, and correlations of stock returns to calculate expected returns and volatilities of these stocks. The monthly returns were computed using Equation (3.9) from February 2010 to January 2021.

3.4 Data Analysis

We used the 11 years' historical price data (132 months from $2010-02-26$ to $2021-01-26$) for nine stocks listed on the LSI USE indexes from the USE website (https://www.use.or.ug/). We computed the returns for each stock at the monthly price for the monthly prices of the selected stocks at the monthly price, which we used for analysis. For analysis purposes, we assigned zeros(0 prices) where the stock was not traded, for example, the first two months of NIC stock and the first 35 months of UMEM stock. The sample of the first 12 months of 132 months for the selected stock prices are as shown on Figure 3.1 below,

[1]Communicated by criscent.birungi@strathmore.edu

	BOBU	BATU	DFCU	NIC	NVL	SBU	UCL	UMEM	UGA
2010-02-26	67.68	250	310.00	NA	464.00	16.50	50.00	NA	82.5
2010-03-26	66.24	295	340.00	NA	480.00	18.60	65.00	NA	93.0
2010-04-26	74.40	320	353.00	22.38	540.00	21.00	76.00	NA	105.0
2010-05-26	105.60	320	400.00	22.38	720.00	20.00	51.00	NA	100.0
2010-06-26	102.00	320	400.00	24.24	730.00	20.50	55.00	NA	102.5
2010-07-26	102.00	330	400.00	26.11	765.00	22.50	60.00	NA	112.5
2010-08-26	103.20	330	402.50	26.11	791.00	23.50	60.00	NA	117.5
2010-09-26	106.80	330	405.00	26.11	817.00	24.50	55.00	NA	122.5
2010-10-26	111.60	1010	407.50	26.11	765.00	26.20	53.00	NA	131.0
2010-11-26	115.20	1250	407.50	26.11	645.00	26.50	52.00	NA	132.5
2010-12-26	120.00	1740	410.00	26.11	610.00	26.50	45.00	NA	132.5
2011-01-26	130.80	1740	425.00	26.11	525.00	27.50	50.00	NA	137.5

Figure 3.1: Sample of the first 12 months of 132 months of the stock prices

Figure 3.1 only exemplifies the sample of the first 12 months of 132 months for stock prices we considered for this study. We then computed the monthly returns for each of the selected stocks using Equation, (3.9) below,

$$r_{i,t+1} = ln\left(\frac{P_{i,t+1}}{P_{i,t}}\right) \quad for \quad i = 1, \cdots, 9 \; and \; t = 0, \cdots, 131. \tag{3.9}$$

Where, i denote the stock number, t denotes the period in months,, $P_{i,t+1}$ denote the stock i price at month $t+1$, $P_{i,t}$ denotes the stock i price at month t and $r_{i,t+1}$ denote the stock i return at month $t+1$. We used Equation (3.9) to compute the stock returns from $2010-02-26$ to $2021-01-26$. The logs of non-numerical numbers were assigned zeros for easy analysis. Again, the sample of the first 12 months of 132 months for the selected stock returns are as shown on 3.2 below,

	BOBU	BATU	DFCU	NIC	NVL	SBU	UCL	UMEM	UGA
2013-01-26	0.113328685	-0.0105356550	0.0000000000	0.000000000	0.000000000	0.2231435513	0.154150680	0.0357180826	0
2013-02-26	0.292136423	-0.0004414037	0.0295588022	0.000000000	0.000000000	0.0000000000	0.000000000	0.0512932944	0
2013-03-26	0.000000000	0.0004414037	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.0000000000	0
2013-04-26	-0.223143551	0.0000000000	0.0000000000	0.000000000	0.000000000	0.1823215568	0.000000000	0.0487901642	0
2013-05-26	0.080042708	-0.0004414037	0.0000000000	0.000000000	0.000000000	-0.0689928715	0.000000000	0.1609303668	0
2013-06-26	-0.080042708	0.0022050726	0.0000000000	0.000000000	0.000000000	0.0689928715	-0.154150680	-0.0699585886	0
2013-07-26	0.000000000	0.1123842495	0.0058083416	0.000000000	0.000000000	-0.1823215568	0.000000000	0.0425596144	0
2013-08-26	-0.042559614	0.0000000000	-0.0009657171	0.000000000	0.008298803	0.0000000000	0.000000000	0.0000000000	0
2013-09-26	0.122602322	0.4541302801	-0.0048426245	0.000000000	0.000000000	0.0000000000	0.000000000	0.0137933221	0
2013-10-26	0.000000000	0.0124225200	0.0048426245	0.000000000	0.040491361	0.1823215568	0.000000000	0.0000000000	0
2013-11-26	-0.080042708	0.0000000000	0.1395518804	-1.822100186	0.000000000	0.0000000000	0.000000000	0.0270286724	0
2013-12-26	-0.042559614	0.0000000000	0.0000000000	1.789386989	0.000000000	-0.1823215568	0.000000000	-0.0270286724	0

Figure 3.2: Sample of the some of 12 months of 132 months of the stock returns.

Figure 3.2 only exemplifies the sample of the first 12 months of 132 months for stock returns we considered for this study. We show from $2013-01-26$ since the values before this trading date has infinite numbers. Figure 3.3 shows the USE LSI selected stock returns movements over $2010-02-26$ to $2021-01-26$ in terms of performance and volatility.

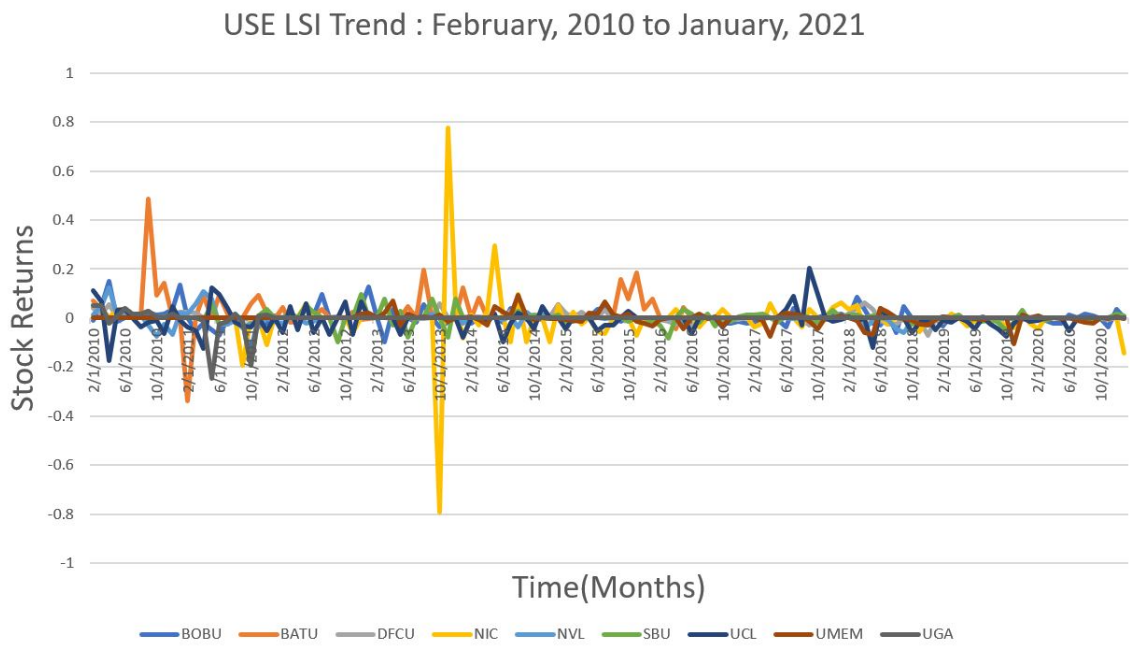
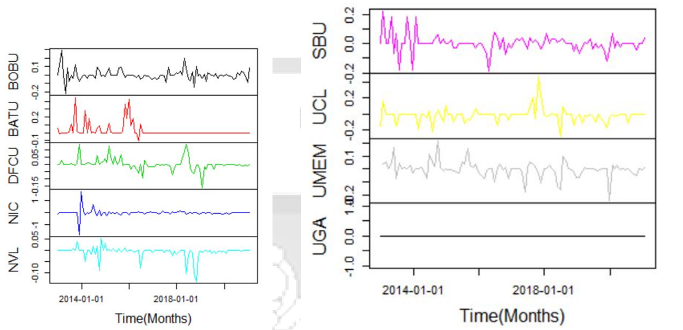


Figure 3.3: Source: (https://www.use.or.ug/).

The time-series graph on Figure 3.3 shows the logarithmic returns of nine assets (stocks) included in the USE Local Share index. Figure 3.3 illustrates the stock returns movements throughout $2010-02-26$ to $2021-01-26$ (132 months) for stock returns we considered for this study. NIC stock being a highly volatile stock compared to others. BATU was also highly volatile between$2010-08-01$ to $2011-11-01$ and $2013-01-01$ to $2016-02-01$ and lowered later. UCL's returns were highly volatile between $2017-01-01$ to $2018-08-01$. Other stocks' volatility is skewed around zero(0) especially after 2020. This can be explained by the effects of the lockdown/COVID-19. This period can be handled by using non stationality properties.



(a) First USE LSI Trend : Feb, 2010 to Jan, 2021 (b) Second USE LSI Trend : Feb, 2010 to Jan, 2021

3.4.1 Portfolio Compositions

Basing on the mathematical framework of MV (3.1), MAD (3.2), Mean-CVaR (3.7) models and Robust Portfolios and Covariance Estimation Models in subsection, (3.1.4), Portfolios were developed. Weights were assigned on the selected stocks using different techniques, for example, equal weights feasible portfolio with "LongOnly" constraints and others we consider optimal portfolio allocation using $setWeights()$ function, which is the default case. With the collected data that is, Computed monthly returns and prices for the period February 2010 to January 2021 for nine stocks Uganda Clays Ltd (UCL), British American Tobacco Uganda Ltd (BATU), Bank of Baroda Ltd (BOBU), Development Finance Company of Uganda Ltd (DFCU), New Vision Printing and Publishing Company Ltd (NVL), Stanbic Bank Uganda (SBU), National Insurance Corporation (NIC), UMEME Ltd (UMEM), and Stanbic Bank Uganda(UGA) DEAD stock. We then allocated these weights in percentages of each stock based on the five optimization models to develop five optimal portfolios. The codes for generating these portfolios are in appendix A.3. The portfolio compositions are shown on table 3.1,

The results in Table 3.1 shows portfolios generated by the five portfolio optimization models and their compositions. Due to differences on the weight

Portfolio Weights					
Stocks	Mean-CVaR (3.7)(%)	MAD (3.6)(%)	MV (3.1)(%)	SMV (3.1.4)(%)	ACE (3.1.4)(%)
UCL	0.00	2.03	5.07	15.48	0.48
BATU	24.27	9.69	1.53	00.00	2.47
BOBU	0.39	3.39	3.00	4.11	0.49
UGA	74.83	-	-	21.63	81.32
NVL	0.00	42.58	55.82	28.83	7.93
UMEM	0.15	6.19	5.71	-	-
NIC	0.02	1.67	1.39	1.80	00.52
SBU	0.34	9.65	6.45	8.07	00.17
DFCU	0.00	24.80	21.03	20.09	6.62

Table 3.1: Optimal portfolio compositions of five different models.

of stocks, results from portfolio compositions normally differ. Difference in weight may be probably due to the non-normality displayed by data [14]. For example, different stocks have different performance over time. We used different optimal asset allocations techniques to attach weights on all assets. In chapter (4), we use $Variance(Sigma)$, $Covariance$, $Value\ at\ Risk(VaR)$ and $Conditional\ Value\ at\ Risk(CVaR)$ as risk measures for all the five optimal portfolios to compare which model is efficient(high return, low risk) for USE.

3.4.2 Portfolio formation

All the models were coded in R. The optimization was run in R under the use of different R functions under the $library(fPortfolio)$, check appendix A.3 for details. We used the techniques of portfolio formation in *Rmetrics* to form our optimal portfolios. The data set (computed monthly returns and prices for the selected stocks at USE from February 2010 to January 2021) was read in a time-series format. For portfolio diversification purposes, all our five portfolios are diversified enough to contain all the stocks that we considered for this study: each portfolio contains (6-9) stocks. We made sure all the nine stocks (UCL, BATU, BOBU, DFCU, NVL, SBU, NIC, UMEM and UGA) that we considered are included at least in all five portfolios. We allocated portfolio weights using different methods, wherein some portfolios, we used equal-weighted methods to make sure there is equal allocation of assets in some portfolios and others we used portfolio formation in *Rmetrics* to allocate the portfolio weights. The detail of each portfolio formation is discussed as below, and portfolio formation codes are shown in appendix A.3,

(a) **Mean-Variance (MV) Portfolio** From the previous discussion, in 1.1.1 basing on Markowitz (1952) [19] Mean-Variance (MV) model frame, that given estimates of the returns, volatilities, and correlations of a set of investments and constraints on investment choices [21], we performed an optimization that resulted in the risk/return or mean-variance efficient frontier illustrated on Figure 3.5, which is efficient. Figure 3.10 shows that every portfolio portfolios in this frontier give the highest possible return at a given level of risk. We employ Würtz et al. [2015] framework to come up with the MV Portfolio Frontier, and we present this result on Efficient frontier and MV Portfolio - Long Only Constraints to display, portfolio weights, the weighted returns and the covariance risk budgets.

```
Title:
 MV Portfolio Frontier
 Estimator:         covEstimator
 Solver:            solveRquadprog
 Optimize:          minRisk
 Constraints:       LongOnly
 Portfolio Points:  4 of 4

Portfolio Weights:
    BOBU   BATU   DFCU    NIC    NVL    SBU    UCL   UMEM
1 0.0300 0.0153 0.2103 0.0139 0.5582 0.0645 0.0507 0.0571
2 0.0335 0.2512 0.3294 0.0211 0.1525 0.1487 0.0000 0.0636
3 0.0000 0.5917 0.2481 0.0100 0.0000 0.1502 0.0000 0.0000
4 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

Covariance Risk Budgets:
     BOBU    BATU    DFCU     NIC     NVL     SBU     UCL    UMEM
1  0.0261  0.0042  0.1810  0.0152  0.5995  0.0557  0.0629  0.0554
2  0.0230  0.5968  0.2339  0.0014  0.0153  0.1043  0.0000  0.0253
3  0.0000  0.9326  0.0449 -0.0036  0.0000  0.0261  0.0000  0.0000
4  0.0000  1.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000

Target Returns and Risks:
     mean     Cov    CVaR     VaR
1 -0.3553  1.8332  5.4863  3.4717
2  0.6382  2.7809  4.4081  2.7274
3  1.6317  5.2220  3.9954  1.7252
4  2.6251  8.5201  2.4103  0.0441
```


Figure 3.5: MV Portfolio

Figure 3.5 shows the MV Portfolio we obtained for the *eight* stocks we included in this portfolio listed on USE. The optimal results, show that NVL has the highest weight of 0.5582 and the highest covariance risk budget of 0.5995 followed by DFCU with 0.2103 and 0.1810 receptively. In terms of Target Returns and Risks of the MV Portfolio Frontier, Cov has the highest values with [8.5201 highest, 1.8332 lowest] followed by CVaR risk measure is has the highest values with [5.4863 highest, 2.4103 lowest] and then VaR with [3.4717 highest, 0.0441 lowest]. The mean return from the MV Portfolio Frontier is [2.6251 highest, -0.3553 lowest]. Figure 3.6 shows the Efficient frontier for the MV Portfolio, and Figure 3.7 shows MV portfolio weights, the weighted returns and the covariance risk budgets. Considering the MV Portfolio, $VaR's$ optimal results would be the best risk measure since it has the lowest values in this portfolio than other risk measures ($CVaR$ & Cov) available.

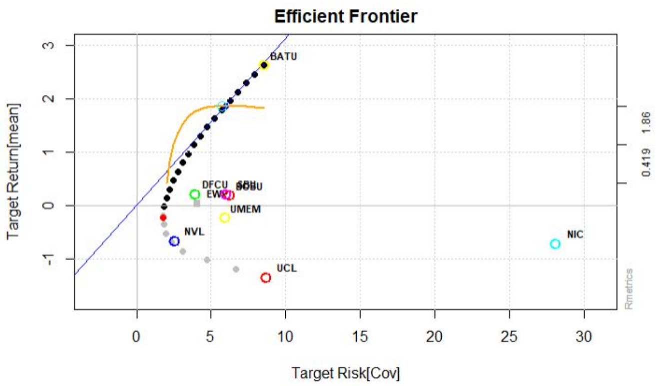


Figure 3.6: MV portfolio Frontier and Sharpe ratios.

Figure 3.6 is the efficient frontier of a long-only constrained mean-variance portfolio in MV (3.1). Figure 3.6 includes the efficient frontier, the tangency line and tangency point for a zero risk-free rate, the equal weights portfolio, all single assets risk vs return points. The yellow line shows the line of Sharpe ratios. The maximum of the line of Sharpe ratios coincides with the tangency portfolio point. The right-hand side of Figure 3.10 shows the range of the Sharpe ratio.

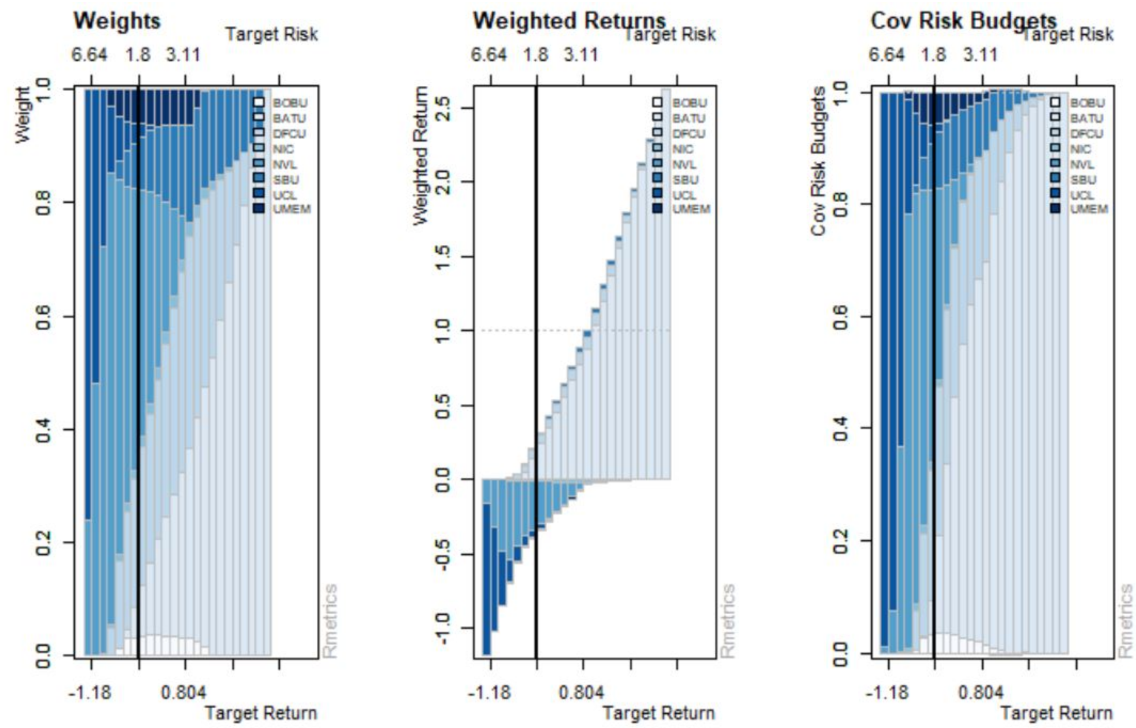


Figure 3.7: MV Portfolio - Long Only Constraints.

Figure 3.7 shows portfolio weights, the weighted returns and the covariance risk budgets for *eight* stocks. Both weights and covariance risk budgets for all the stocks increase at a decreasing rate. Weighted returns increase with a target return for BOBU, BATU, DFCU, NIC, NVL and SBU stocks and are increasing at a decreasing rate for UMEM & UCL stocks where the weighted return is negative.

(b) **Mean-Absolute Deviation (MAD) Portfolio** For the MAD Portfolio we applied R. Baganzi et al., 2017, [2] setup and used the default settings, we applied the code for portfolio optimization for our data set and obtained the following MAD Efficient portfolio,

```
Title:
 MAD Efficient Portfolio
 Estimator:         covEstimator
 Solver:            solveRquadprog
 Optimize:          minRisk
 Constraints:       LongOnly

Portfolio Weights:
  BOBU   BATU   DFCU    NIC    NVL    SBU    UCL   UMEM
0.0339 0.0969 0.2480 0.0167 0.4258 0.0965 0.0203 0.0619

Covariance Risk Budgets:
  BOBU   BATU   DFCU    NIC    NVL    SBU    UCL   UMEM
0.0362 0.1911 0.2681 0.0123 0.3216 0.1039 0.0102 0.0565

Target Returns and Risks:
  mean    Cov   CVaR    VaR
0.0002 1.8806 4.5954 2.6995
```

Figure 3.8: MAD Efficient Portfolio

Figure 3.8 shows the MAD Efficient Portfolio we obtained; for the *eight* stocks we included in this portfolio listed on USE, the results show that NVL has the highest portfolio weight of 0.4258 and covariance risk budget of 0.3216 followed by DFCU with 0.2480 and 0.2681 receptively. In terms of Target Returns and Risks of the MAD portfolio, $CVaR$ risk measure is highest with 4.5954, followed by VaR with 2.6995 and then covariance with 1.8806. The mean return from the MAD portfolio is 0.0002. Considering the optimal results from MAD Efficient Portfolio Cov would be the best risk measure since it has the lowest values in this portfolio than other risk measures ($CVaR$ & VaR) available.

(c) **Mean-CVaR Portfolio**

Mean-CVaR Portfolio

We applied Würtz et al. [2015] approach to develop the CVAR Feasible Portfolio. As discussed earlier, we applied the code for portfolio optimization for our data set and obtained the following CVaR Feasible Portfolio as shown below,

```
Title:
 CVaR Portfolio Frontier
 Estimator:         covEstimator
 Solver:            solveRglpk.CVAR
 Optimize:          minRisk
 Constraints:       LongOnly
 Portfolio Points:  5 of 5
 VaR Alpha:         0.05

Portfolio Weights:
    BOBU   BATU   DFCU    NIC    NVL    SBU    UCL   UMEM    UGA
1 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000 0.0000 0.0000
2 0.0000 0.0000 0.0155 0.0179 0.2903 0.0000 0.1143 0.0000 0.5621
3 0.0039 0.2427 0.0000 0.0002 0.0000 0.0034 0.0000 0.0015 0.7483
4 0.0100 0.6206 0.0000 0.0004 0.0000 0.0086 0.0000 0.0038 0.3566
5 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

Covariance Risk Budgets:
     BOBU    BATU    DFCU     NIC     NVL     SBU     UCL    UMEM     UGA
1  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  1.0000  0.0000  0.0000
2  0.0000  0.0000 -0.0077  0.1478  0.3020  0.0000  0.5579  0.0000  0.0000
3  0.0014  0.9989  0.0000 -0.0001  0.0000 -0.0003  0.0000  0.0000  0.0000
4  0.0014  0.9989  0.0000 -0.0001  0.0000 -0.0003  0.0000  0.0000  0.0000
5  0.0000  1.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000

Target Returns and Risks:
     mean     Cov    CVaR     VaR
1 -1.3487  8.6776 20.3442 15.4151
2 -0.3553  1.3631  3.5415  3.0383
3  0.6382  2.0701  0.5816  0.0550
4  1.6317  5.2925  1.4868  0.1406
5  2.6251  8.5201  2.4103  0.0441
```

Figure 3.9: Mean-CVaR Portfolio

Figure 3.9 shows the CVaR Feasible Portfolio we obtained for the selected stocks we included in this portfolio listed on USE. On this portfolio, we considered the optimal weights that are at least most assets are represented in the portfolio(check line 3 of portfolio weights). The optimal results show that UGA has the highest weight of 0.7483, followed by BATU with 0.2427. BATU has the highest Covariance Risk budget of 0.9989. In terms of Target Returns and Risks of the Mean-CVaR Portfolio, CV aR risk measure has the highest values with [20.3442 highest, 0.5816 lowest] followed by VaR with[15.4151 highest, 0.0441 lowest] and then covariance with [8.6776 highest, 1.3631 lowest]. The highest mean return from the CVaR Feasible Portfolio is [2.6251 highest, 0.6382 lowest]. We excluded the negative values as losses. We show these results in Figure 3.11; weights, performance attribution, and the risk attribution expressed by the covariance risk budgets. Considering the optimal results from Mean-CVaR, Portfolio Cov would be the best risk measure since it has the lowest values in this portfolio than other risk measures ($CVaR$ & VaR) available. Figure 3.10 shows the Efficient frontier for the CVaR Portfolio.

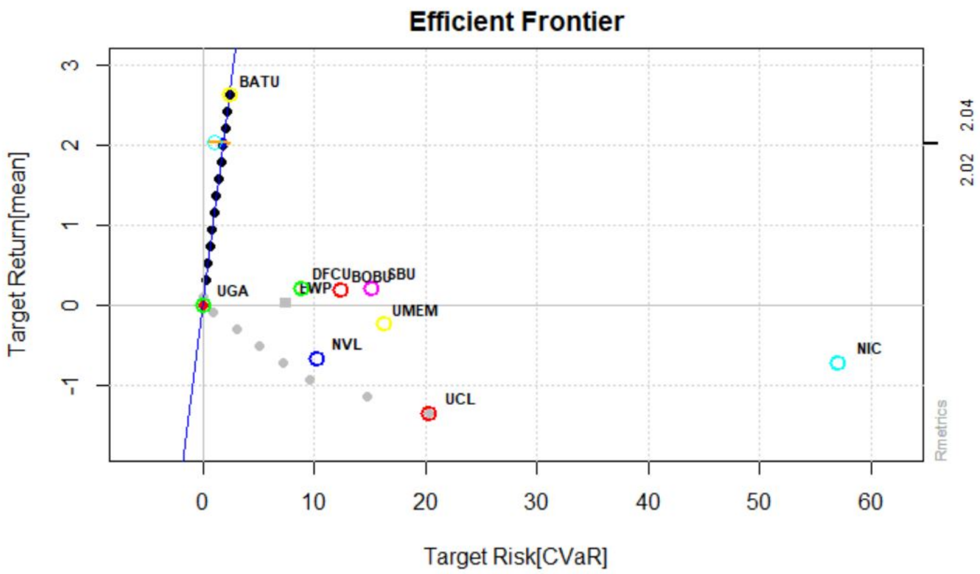


Figure 3.10: Mean-CVaR Efficient Frontier.

The efficient frontier on Figure 3.10 shows a long-only constrained Mean-CVaR portfolio in Equation (3.7). Figure 3.10 includes the efficient frontier, UGA, the tangency line and tangency point for a zero risk-free rate, the equal weights portfolio, all single assets risk vs return points. The tangency is on the efficient frontier, with the tangency portfolio point is with UGA & BATU. The right-hand side of Figure 3.10 shows the range of the Sharpe ratio.

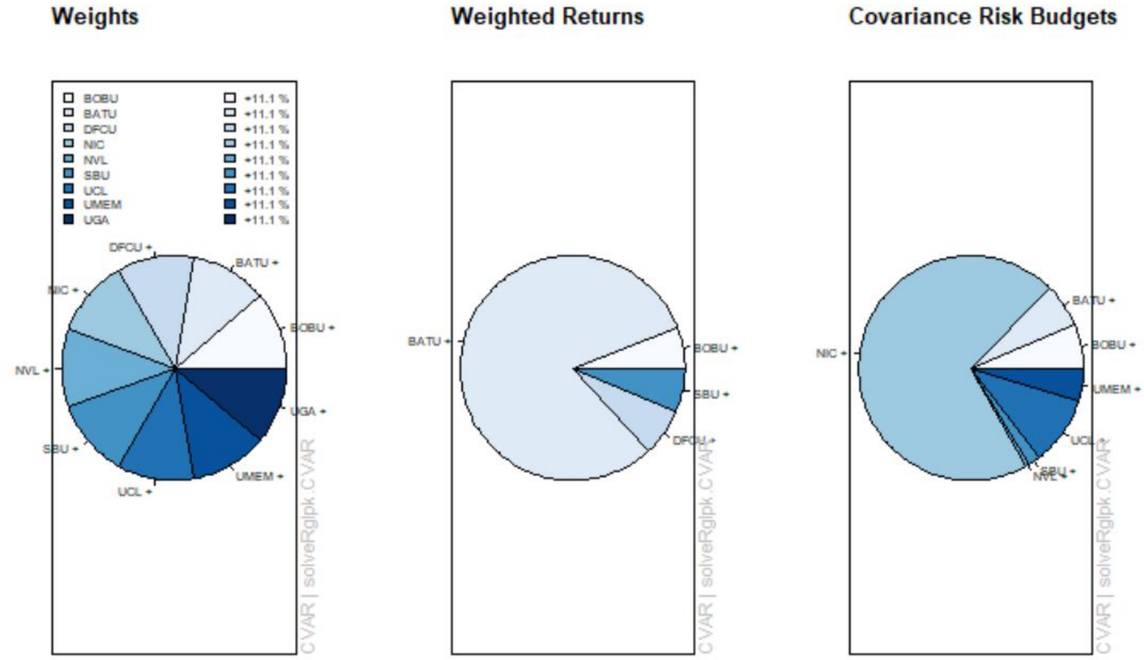


Figure 3.11: CVaR Feasible Portfolio Equal Weights Mean-CVaR Portfolio.

The middle graph on Figure 3.11 shows weights and the performance attribution. The right-hand graph shows the risk attribution expressed by the covariance risk budgets in the Mean-CVaR portfolio.

(d) **The Shrinked Mean Variance (SMV) Portfolio** The shrinked Mean-Variance (SMV) Portfolio was coded in R under specifications ofWürtz et al. [2015] framework. We performed an optimization that allocated weights for the eight stocks that we included in this portfolio. The portfolio weights, covariance risk budgets and target returns and risks for the shrinked Mean-Variance (SMV) Portfolio are shown on Figure 3.12 below,

```
Title:
 MV Portfolio Frontier
 Estimator:         shrinkEstimator
 Solver:            solveRquadprog
 Optimize:          minRisk
 Constraints:       LongOnly
 Portfolio Points:  4 of 4

Portfolio Weights:
    BOBU   BATU   DFCU    NIC    NVL    SBU    UCL    UGA
1 0.0411 0.0000 0.2009 0.0180 0.2883 0.0807 0.1548 0.2163
2 0.1116 0.1640 0.6052 0.0000 0.0184 0.1008 0.0000 0.0000
3 0.0344 0.5738 0.3918 0.0000 0.0000 0.0000 0.0000 0.0000
4 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

Covariance Risk Budgets:
    BOBU   BATU   DFCU    NIC    NVL    SBU    UCL    UGA
1 0.0565 0.0000 0.0817 0.0274 0.2441 0.1163 0.2398 0.2342
2 0.1198 0.4170 0.4069 0.0000 0.0018 0.0545 0.0000 0.0000
3 0.0037 0.9738 0.0226 0.0000 0.0000 0.0000 0.0000 0.0000
4 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

Target Returns and Risks:
     mean      mu     Cov   Sigma    CVaR     VaR
1 -0.4256 -0.4256  2.5374  2.7981  6.6736  4.1441
2  0.8805  0.8805  3.6721  3.4906  6.8779  4.2169
3  2.1867  2.1867  8.5469  8.5843 10.3910  2.3767
4  3.4928  3.4928 14.7476 14.7476 15.0875  0.1997
```

Figure 3.12: Shrinked Mean Variance (SMV) Portfolio

Figure 3.12 shows the Shrinked Mean-Variance (SMV) Portfolio Frontier we obtained for the *nine* stocks we included in this portfolio listed on USE. We considered the optimal weights on this portfolio where most assets are represented in the portfolio (check line 1 of portfolio weights on Figure 3.12). The optimal results show that NVL has the highest portfolio weight of 0.2883 and the highest covariance risk budget of 0.2441, followed by UGA with 0.2163 and 0.2342, receptively. In terms of Target Returns and Risks of the Shrinked Mean-Variance (SMV) Portfolio, CV aR risk measure has the highest values with [15.0875 highest, 6.6736 lowest] followed by variance or Sigma with [14.7476 highest, 2.7981 lowest]. Covariance risk measure also has higher values [14.7476 highest, 2.5374 lowest]. VaR has the lowest values, that[4.2169 highest, 0.1997 lowest]. The mean return from the Shrinked Mean-Variance (SMV) Portfolio is [3.4928 highest, -0.4256 lowest], which means at some time this portfolio had losses(negative returns). *Cov* and *Sigma* have almost the same values because they all measure the deviation from the portfolio's actual values. Considering the optimal results from Shrinked Mean-Variance (SMV) portfolio, VaR would be the best risk measure since it has the lowest values in this portfolio than other risk measures ($Sigma$, $CVaR$ & VaR) available.

(e) **Alternative Covariance Estimator (ACE) Portfolio** We also implemented the Alternative Covariance Estimator (ACE) model to come up with the optimal portfolio. We explored the work of Würtz et al. [2015] framework to come up with the Alternative Covariance Estimator (ACE) Portfolio by using the MASS package iR and the $function(cov.trob())$ was used to estimate a covariance matrix. This implementation was made to assume that the 11 years' historical price data used for this study come from a multivariate Student's t distribution. We believe this approach provides better results in terms of robustness to outliers, as suggested by Würtz et al. [2009]. Under model specification, we performed an optimization that resulted in MV Portfolio Frontier illustrated in Figure 3.13. Figure 3.14 shows that every portfolio on this frontier is a portfolio that results in the greatest possible expected return for that level of risk. In contrast, Figure 3.15 shows the weights, the performance attribution, and the covariance risk budgets, which measure the risk attribution (weighted returns)and covariance risk budgets.

```
Title:
 MV Portfolio Frontier
 Estimator:         covtEstimator
 Solver:            solveRquadprog
 Optimize:          minRisk
 Constraints:       LongOnly
 Portfolio Points:  4 of 4

Portfolio Weights:
    BOBU   BATU   DFCU    NIC    NVL    SBU    UCL    UGA
1 0.0049 0.0247 0.0662 0.0052 0.0793 0.0017 0.0048 0.8132
2 0.0196 0.2551 0.3314 0.0131 0.2049 0.0603 0.0000 0.1156
3 0.0000 0.5768 0.3869 0.0000 0.0000 0.0363 0.0000 0.0000
4 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

Covariance Risk Budgets:
     BOBU    BATU    DFCU     NIC     NVL     SBU     UCL     UGA
1  0.0025  0.0058  0.0031  0.0005 -0.0127  0.0006 -0.0015  1.0019
2  0.0103  0.8073  0.0910 -0.0011  0.0385  0.0188  0.0000  0.0352
3  0.0000  0.9785  0.0204  0.0000  0.0000  0.0011  0.0000  0.0000
4  0.0000  1.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000

Target Returns and Risks:
     mean      mu     Cov   Sigma    CVaR     VaR
1 -0.4256 -0.4256  5.2334  0.4938 14.4908  1.2534
2  0.8805  0.8805  4.0759  1.5192  6.6138  3.0753
3  2.1867  2.1867  8.5667  3.0423 10.2928  2.2115
4  3.4928  3.4928 14.7476  5.0265 15.0875  0.1997
```

Figure 3.13: Alternative Covariance Estimator (ACE) Portfolio Frontier

Figure 3.13 shows the ACE Portfolio we obtained. We included *eight* stocks in this portfolio listed on USE. We considered the optimal weights on this portfolio where most assets are represented in the portfolio(check line 1 of portfolio weights). The optimal results show that UGA has the highest weight of 0.8132 and the highest covariance risk budget of 1.0019, followed by NVL with 0.0793 and -0.0127, receptively. In terms of Target Returns and Risks of the ACE Portfolio portfolio, CV aR risk measure has the highest values with [15.0875 highest, 6.6138 lowest] followed by Cov with [14.7476 highest, 4.0759 lowest, Sigma with [5.0265 highest, 0.4938 lowest] and then V aR with [3.0753 highest, 0.1997 lowest]. The mean return from the ACE Portfolio is [3.4928 highest, -0.4256 lowest]. Figure 3.14 shows the Efficient frontier for the ACE Portfolio, and Figure 3.15 shows ACE portfolio weights, the weighted returns and the covariance risk budgets. Considering the optimal results from ACE Portfolio, VaR would be the best risk measure since it has the lowest values in this portfolio than other risk measures (Cov, $CVaR$ & $Sigma$) available.

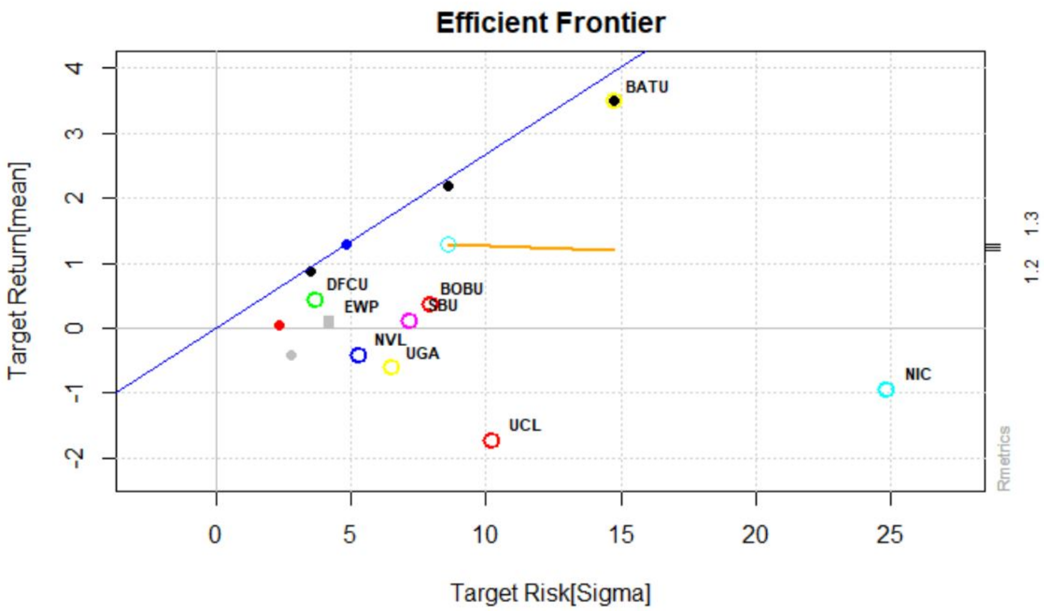


Figure 3.14: ACE Portfolio Frontier with BOBU, BATU, DFCU, NIC, NVL, SBU, UCL, & UGA stocks.

The efficient frontier of the ACE portfolio on figure 3.14 of a long-only constrained ACE portfolio did not produce the optimal efficient frontier line for the ACE portfolio. Figure 3.14 includes the efficient frontier, UGA, the tangency line and tangency point for a zero risk-free rate, the equal weights portfolio, all single assets risk vs return points. The yellow line between [5,15] shows there is the line of Sharpe ratios. The maximum of the line of Sharpe ratios coincides with the tangency portfolio point with UGA & BATU. The right-hand side of Figure 3.14 shows the range of the Sharpe ratio.

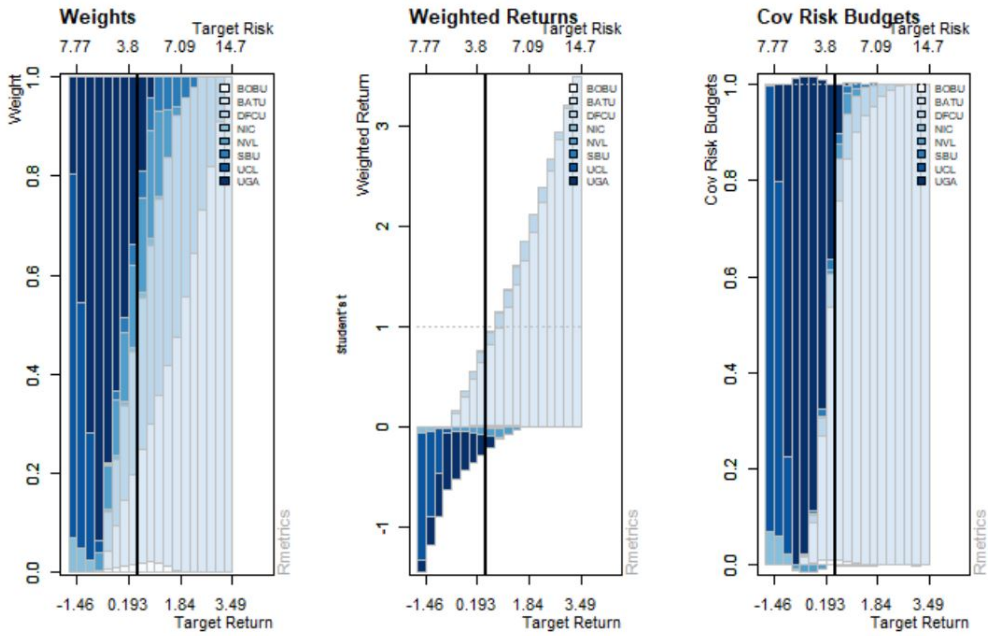


Figure 3.15: ACE Portfolio - Long Only Constraints, portfolio weights, the weighted returns and the covariance risk budgets for BOBU, BATU, DFCU, NIC, NVL, SBU, UCL & UGA stocks.

From Figure 3.15, both weights and covariance risk budgets for all stocks increases at a decreasing rate with the target return. Weighted returns increase with a target return for BOBU, BATU, DFCU, NIC, NVL and SBU stocks and are increasing at a decreasing rate with target return for UGA & UCL stocks; the weighted return is negative.

Chapter 4

Results and Discussion

4.1 Measuring Portfolio Performance, Expected return and Risk

4.1.1 Risk measure Performances

In terms of risk measure for each portfolio discussed in subsection (3.4.2), we illustrated the portfolio formations and how risk measure would be the best based on the lowest values. Therefore, we use this reference (taking the smallest value for each risk measure of each portfolio as the risk measure value), as summarized in Table 4.1.

Risk measure estimates in optimal portfolios					
Model	No. of assets	Variance (Sigma)	Covariance	VaR (C.I of 95%)	CVaR
Mean-CVaR (3.7)	9	-	3.5993	3.8959	0.5816
MAD (3.6)	8	-	1.8806	2.5374	4.5954
MV (3.1)	8	-	1.8332	0.0441	2.4103
SMV (3.1.4)	8	2.7981	2.5374	0.1997	6.6736
ACE (3.1.4)	8	0.4938	4.0759	0.1997	6.6138

Table 4.1: Different risk measures in portfolios

Table 4.1 presents the results of the investment portfolios and the risk measures. We analyzed only the $Variance$, $Covariance$, VaR and the $CVaR$. The reason for this is that both risk measures or measures of dispersion have the same unit of measurement. The portfolios generated using traditional portfolio optimization models i.e. Markowitz's based models (MAD (3.6), MV (3.1) and SMV (3.1.4)) except ACE (3.1.4) with covariance 4.0759 (higher covariance than the Mean-CVaR model) showed lower values in terms of risks. This result is consistent with Silva et al. [2017], [3]. The values of the $Covariance$, $Variance$, VaR were lower for MAD (3.6), MV (3.1) and SMV(3.1.4) models while $CVaR$ has higher values across all portfolios or models except in Mean-CVaR (3.7) model, this is because the primary objective of Mean-CVaR model is to minimize the $CVaR$ as a risk measure. When we considered all the assets(9) for the Mean-CVaR portfolio to make sure all assets are represented, there was a reduction in the risk(lower values of the risk measures), that is when we reduced assets from 9 to 7 assets on the Mean-CVaR portfolio the values for risk measures $Covariance$, VaR and the $CVaR$ increased. This result shows that a portfolio with more assets performs better in terms of risk. From Table 4.1 , in terms of which risk measure would be the best(lower values), we observe that regardless of the primary objective of some models, for example, minimizing variance for the Markowitz's based models and minimizing CVaR for the Mean-CVaR models, the results we get some violet the models' primary objectives. For example, the results in Table 4.1 show that for MV (3.1) model whose primary objective is to minimize variance or covariance, VaR has the lowest value of 0.0441 compared with the Cov value of 18332. Therefore, taking variance or covariance from the MV (3.1), which takes variance or covariance as a risk measure, would not be optimal since VaR has better values than other risk measures. This result is the same for both SMV (3.1.4)) Moreover, ACE (3.1.4) models have a lower VaR of 0.1997 compared to Cov and $Sigma's$ higher values. Only Mean-CVaR and MAD models results are consistent with these models' primary objective, which is minimizing portfolio $CVaR$ and MAD or Cov receptively for the two models. For the Mean-CVaR portfolio, $CVaR$ has the lowest value of 0.5816 compared to higher values of Cov and VaR hence $CVaR$ would the best risk measure for the Mean-CVaR portfolio. For the MAD portfolio, Cov has the lowest value of 1.8806 compared to higher values of VaR and $CVaR$; hence Cov would the best risk measure on the MAD portfolio.

4.1.2 Portfolio Performances

In section (2.1.2), we reviewed how portfolio performance is determined based on past research and concluded that portfolio performance is calculated using the reward per risk equation as shown in equation (2.1). In this subsection, we analyze the five models' portfolio performances discussed in (3.4) for the five models. We compare the performance of the traditional portfolio optimization models and the proposed risk models at USE. For the portfolios generated using Markowitz's based models, we considered variance or covariance as risk measures that is, by taking the smallest value of variance or covariance of each portfolio as the risk measure value) while for the mean-CVaR portfolio model, we considered the lowest value of conditional Value at Risk as the risk measure in this portfolio. For the portfolio expected return, we considered the highest fraction value of mean in each portfolio as the Expected Return of that particular portfolio, as shown in table 4.2. Using this criterion, we considered the lowest value of mean absolute deviation or covariance as a risk measure in the MAD portfolio for the MAD portfolio. For MV, SMV and ACE, we used the lowest values of variance or covariance as a risk measure; for this case, for the MV, we took the smallest covariance of 1.8332 as the risk measure, for SMV, we took the lowest covariance value of 2.5374 since it is the smallest among the covariance and sigma values available and the 0.4938 for ACE portfolio. For the Mean-CVaR portfolio, we considered the smallest value of $CVaR$ as the risk measure.

	Mean-CVaR(3.7)	MAD(3.6)	MV(3.1)	SMV (3.1.4)	ACE(3.1.4)
Expected Return	0.6382	0.0002	0.6382	0.8805	0.8805
Risk	0.5816	1.8806	1.8332	2.5374	0.4938
Performance	1.097318	0.000106349	0.3481344	0.3470087	1.783111

Table 4.2: Summary Statistics of optimal portfolios.

The results from table 4.2 Show the performance of different portfolios. Both SMV (3.1.4) and ACE (3.1.4) portfolios have the highest mean return of 0.8805 compared to other models. SMV (3.1.4) is the riskiest portfolio with 2.5374, while Mean-CVaR (3.7) and ACE (3.1.4) have the lowest risks of 0.5816 and 0.4938, respectively. This result is because the primary objective of Mean-CVaR (3.7) and ACE (3.1.4) is to minimize $CVaR$ and $Covariance$, respectively. With the lowest risk of Mean-CVaR (3.7) and ACE (3.1.4) portfolios, they have the highest performance ratio of 1.097318 and 1.783111 ,respectively. The portfolio performance was calculated using the reward per risk equation as shown in (2.1). MAD (3.6) model has the lowest performance among other models.

4.2 Conclusion

This study used different types of portfolio optimization models and different R packages under $library(fPortfolio)$ to form optimal portfolios that we used for analysis. For the five (Mean-CVaR (3.7), MAD (3.6), MV (3.1), SMV (3.1.4), ACE (3.1.4)) models we considered for this study, we compared their performance in terms of the return and risk associated with each portfolio or model. The results obtained with the Robust Portfolios and Covariance Estimation Models(3.1.4) (SMV and ACE) were satisfactory; that is, these two models generated optimal portfolio with a higher return of 0.8805 each followed by the Mean-CVaR and MV model that generated a higher return of 0.6382 in comparison with the other models we considered. We find out that VaR has the overall lower values across all models in terms of risk measures. We find that it is not about a specific risk measure; what matters is allocating weights to assets in a portfolio and the optimization method. Also, portfolios with more assets have less risk, i.e. lower values of Cov, $Sigma$, VaR, and $CVaR$ are observed in portfolios with $8-9$ assets compared to those with $6-7$ assets. Therefore, an investor who is risk-averse and is interested in portfolio investment at USE or any other financial market should consider including as many assets with good price-performance history as he or she can as this minimizes the risk. In terms of performance, Mean-CVaR (3.7) and ACE (3.1.4) portfolios have the highest performance ratio of 1.097318 and 1.783111, respectively, compared to other models or portfolios.

Chapter 5

Conclusions and Recommendations

5.1 Conclusions and Recommendations

5.1.1 Conclusions

This study's main intention was to establish the best risk measure for portfolio optimization for the Uganda Securities Exchange (USE) and an alternative method of selecting optimal portfolios adopted in the USE. This involved;

- Developing optimal portfolios;
- Analyzing the performance of the portfolios in terms of returns, and
- Analyzing the risk associated with each portfolio using different risk measures.

We used the USE secondary data (computed monthly returns and prices for the selected stocks from February 2010 to January 2021) from USE Local Share Index (LSI). We considered a sample of nine stocks from the USE Local Share Index. We chose nine stocks out of seventeen. Mathematically the nine stocks out of seventeen stocks listed at USE by then will be an excellent sample to represent the whole stocks listed at USE for this analysis to study the portfolio composition and risk measures.

There are concerns about model appropriateness for portfolio optimization. Hoe et al. [7] find that some traditional portfolio optimization models are appropriate for investors who have a substantial downside risk aversion but not all of them. There was a general weakness of market surveillance and speculation method of selecting optimal portfolios in the literature, especially in developing markets. This study compared traditional portfolio optimization models to determine optimal portfolios with the proposed risk models. We used variance, Covariance, Value-at-Risk(VaR) and Conditional Value-at-Risk(CVaR) as risk measures to find out which model is efficient(high return, low risk) for USE.

We analyzed different portfolio optimization models by comparing the risk measures (comparing risk measure estimates) in all portfolios. We developed portfolios, measured their performances in terms of the return and the risk associated with each portfolio. We used five portfolio optimization models, MV (3.1), MAD (3.2), SMV (3.1.4), ACE (3.1.4) and Mean-CVaR (3.7), to come up with five optimal portfolios. We compared the portfolios' performance and risk measure estimates in all the portfolios. SMV (3.1.4) and ACE (3.1.4) portfolios produced the highest mean return of 0.8805 compared to other models. SMV (**??**) was the riskiest portfolio with 2.5374, while Mean-CVaR (3.7) and ACE (3.1.4) had the lowest risks of 0.5816 and 0.8805, respectively. Mean-CVaR (3.7) and ACE (3.1.4) portfolios had the highest performance ratio of 1.097318 and 1.783111, respectively, among other models.

We then compared the expected return and risk of the best performing models. In this case, we concluded that the Mean-CVaR (3.7) and ACE (3.1.4) in terms of both the expected return and risk. We find that ACE (3.1.4) performs better than the Mean-CVaR (3.7), with a performance ratio of 1.097318 and 1.783111, respectively. In terms of risk, ACE (3.1.4)was less risky than Mean-CVaR (3.7) though the difference was not too big. After considering all the factors, we conclude that the traditional portfolio optimization models generated an optimal portfolio with a higher return. The proposed risk models develop portfolios with less risk.

We also find that it is not about a specific risk measure; what matters is allocating weights to assets in a portfolio and the optimization method. Portfolios with more assets are less risky, i.e. lower risk measures' estimates were observed in portfolios with more stocks than those with fewer assets. However, we would like to quote the "decision on which risk measure to use should depend on the regulator or practitioner's intended use". Regulators or Practitioners should focus on the strengths and weaknesses of each if they are to adopt it. It is crucial to consider all the available risk measures for a regulator or practitioner to make a good decision since using one can be subjective; as seen in our results, "different risk measures yield different results".

5.1.2 Recommendations

This study analyzed several risk measures for the Uganda Securities Exchange (USE) that researchers have proposed. Based on our findings, some yields good results; we, therefore, recommend regulators at Uganda Securities Exchange (USE) and other financial institutions in Uganda and globally adopt these risk measures as some favour regulators. For example, these risk measures allow banks, practitioners to safeguard against bank insolvency, bank failure and bankruptcy and other related challenges. They should focus on the strengths and weaknesses of each if they are to adopt it. It is crucial to consider all the available risk measures for a regulator or practitioner to make a good decision since using one can be subjective. As seen in our results, different risk measures yield different results. Further, we would like to recommend regulators at the Uganda Securities Exchange (USE) and other financial institutions in Uganda and globally adopt traditional portfolio optimization models and the proposed risk models as an alternative method of selecting optimal portfolios.

We lacked enough tools to run different optimizations, especially on the models. The different types of portfolio optimization models and different R packages under the $library(fPortfolio)$ we used in optimal portfolio formation, were not efficient computationally. To overcome the computational inefficiency, further research on portfolio optimization at USE and other financial markets while comparing different portfolio optimization models for many assets should consider using other soft wares that can perform the optimization. Other methods like Eigen decomposition-based methods can be used. The Eigen decomposition-based methods are recommended to obtain high-quality bounds on the optimal portfolios' solutions of traditional portfolio optimization problems and the proposed risk models with and without cardinality constraints.

Bibliography

[1] Kasenbacher, G., Lee, J., & Euchukanonchai, K. (2017). Mean-Variance vs. Mean-Absolute Deviation: A Performance Comparison of Portfolio Optimization Models. https://cutt.ly/AveBTVz

[2] Baganzi, R., Kim, B. G., & Shin, G. C. (2017). Portfolio Optimization Modelling with R for Enhancing Decision Making and Prediction in Case of Uganda Securities Exchange. Journal of Financial Risk Management, 6(4), 325-351. https://scirp.org/journal/PaperInformation.aspx?PaperID=80120

[3] Silva, L. P. D., Alem, D., & Carvalho, F. L. D. (2017). Portfolio optimization using mean absolute deviation (MAD) and conditional value-at-risk (CVaR). Production, 27. https://www.scielo.br/scielo.php?pid=S0103-65132017000100302&script=sci_arttext

[4] Würtz, D., Setz, T., Chalabi, Y., Chen, W., & Ellis, A. (2015). Portfolio Optimization with Rmetrics. Rmetrics Association & Finance Online Publishing.

[5] Okumu, A. N., & Onyuma, S. O. (2015). Testing Applicability of Capital Asset Pricing Model in the Kenyan Securities Market. European Journal of Business and Management, 7(26), 126-135. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.735.8006&rep=rep1&type=pdf

[6] Mayanja, F. (2011). Portfolio optimization model: the case of Uganda Securities Exchange (USE) (Doctoral dissertation, University of Dar es Salaam). http://41.86.178.5:8080/xmlui/bitstream/handle/123456789/7253/Mayanja%202011.pdf?sequence=1

[7] Hoe, L. W., Hafizah, J. S., & Zaidi, I. (2010). An empirical comparison of different risk measures in portfolio optimization. Business and Economic Horizons, 1(1), 39-45. https://www.ceeol.com/search/article-detail?id=60122

[8] Sereda, E. N., Bronshtein, E. M., Rachev, S. T., Fabozzi, F. J., Sun, W., & Stoyanov, S. V. (2010). Distortion risk measures in portfolio optimization. In Handbook of portfolio construction (pp. 649-673). Springer, Boston, MA. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.316.1053&rep=rep1&type=pdf

[9] Wuertz, D., Chalabi, Y., Chen, W., & Ellis, A. (2010). Portfolio Optimization with R/Rmetrics. Rmetrics Association & Finance Online. https://www.rmetrics.org/downloads/9783906041018-fPortfolio.pdf

[10] Würtz, D., Setz, T., Chalabi, Y., Chen, W., & Ellis, A. Por/olio Optimization with R/Rmetrics.

[11] Albuquerque, G. U. V. D. (2009). Um estudo do problema de escolha de portfólio ótimo (Doctoral dissertation, Universidade de São Paulo). https://www.teses.usp.br/teses/disponiveis/55/55134/tde-01072009-225349/publico/portf_otimo_galbuq.pdf

[12] Ali, H. M. (2006). Modern Portfolio Theory: Is There Any Opportunity for Real Estate Portfolio?. Malaysian Journal of Real Estate, 1(1), 14-26. http://eprints.utm.my/id/eprint/481/1/14-26.pdf

[13] Mansini, R., & Speranza, M. G. (2005). An exact approach for portfolio selection with transaction costs and rounds. IIE transactions, 37(10), 919-929. https://cutt.ly/kve3wcx

[14] Byrne, P., & Lee, S. (2004). Different risk measures: different portfolio compositions?. Journal of Property Investment & Finance. https://cutt.ly/4ve0Ckr

[15] Brooks, C., & Kat, H. M. (2002). The statistical properties of hedge fund index returns and their implications for investors. The Journal of Alternative Investments, 5(2), 26-44. https://core.ac.uk/download/pdf/6560812.pdf

[16] Young, M. R. (1998). A minimax portfolio selection rule with linear programming solution. Management science, 44(5), 673-683. https://cutt.ly/kve2m1S

[17] Konno, H., & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. Management science, 37(5), 519-531. https://cutt.ly/Zve9Cti

[18] Markowitz, H. (1959). Portfolio selection: Efficient diversification of investments (Vol. 16). New York: John Wiley.

[19] Markowitz, H. (1952). «Portfolio Selection»-Journal of Finance, March.

[20] Uganda Securities Exchange, "UGANDA SECURITIES EXCHANGE (USE) | African Securities Exchanges Association. https://african-exchanges.org/en/membership/members/uganda-securities-exchange-use.

[21] Kimundi Gillian Nduku, "2.1 Portfolio Theory.pdf - Portfolio Theory Kimundi Gillian Nduku Portfolio Theory ReCap The Efficient Frontier Portfolio Theory As I Still See | Course Hero. https://www.coursehero.com/file/50604204/21-Portfolio-Theorypdf

Appendix A

Appendices

A.1 Ethical certificate

Strathmore
UNIVERSITY

13th April 2021

Mr Birungi Criscent
criscent.birungi@strathmore.edu

Dear Mr Birungi,

RE: Analysis of Risk Measures in Portfolio Optimization for the Uganda Security Exchange

This is to inform you that SU-IERC has reviewed and **approved** your above **SU-master's** research proposal. Your application reference number is **SU-IERC01011/21**. The approval period is **13th April 2021 to 12th April 2022.**

This approval is subject to compliance with the following requirements:

i. Only approved documents including (informed consents, study instruments, MTA) will be used
ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by SU-IERC.
iii. Death and life-threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to SU-IERC within 48 hours of notification
iv. Any changes, anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to SU-IERC within 48 hours
v. Clearance for export of biological specimens must be obtained from relevant institutions.
vi. Submission of a request for renewal of approval at least 60 days prior to expiry of the approval period. Attach a comprehensive progress report to support the renewal.
vii. Submission of an executive summary report within 90 days upon completion of the study to SU-IERC.

Prior to commencing your study, you will be expected to obtain a research license from National Commission for Science, Technology and Innovation (NACOSTI) https://research-portal.nacosti.go.ke/ and also obtain other clearances needed

Yours sincerely,

for: Dr Virginia Gichuru,
Secretary; SU-IERC

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A.2 Originality report/Similarity Checker

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A.3 Portfolio Formation Codes

A.3.1 Mean-Variance (MV) Portfolio

```
#MV Portfolio
library(fPortfolio)
library(robustbase)
library(corpcor)
USE <- as.timeSeries(uGANDAN_dATA01)
USE.ret <-  returns(USE)
Data<-100 * USE.ret[, 1:8]
lppSpec <- portfolioSpec()
setNFrontierPoints(lppSpec) <- 5
longFrontier <- portfolioFrontier(Data, lppSpec)
print(longFrontier)

#MV Portfolio Frontier
setNFrontierPoints(lppSpec) <- 25
longFrontier <- portfolioFrontier(Data, lppSpec)
tailoredFrontierPlot(object = longFrontier, risk = "Cov")

#MV Portfolio - Long Only Constraints, portfolio weights, the
#weighted returns and the covariance risk budgets
par(mfcol = c(1, 3), mar = c(3.5, 4, 4, 3) + 0.1)
```

```
weightsPlot(longFrontier)
text <- ""
mtext(text, side = 3, line = 3, font = 2, cex = 0.9)
weightedReturnsPlot(longFrontier)
covRiskBudgetsPlot(longFrontier)
```

A.3.2 Mean-Absolute Deviation (MAD) Portfolio

```
#MAD Efficient Portfolio
library(fPortfolio)
library(robustbase)
library(corpcor)
USE <- as.timeSeries(uGANDAN_dATA01)
Data<-100 * USE.ret[, 1:8]
#MAD as risk measure
MADSpec<-portfolioSpec()
setTargetReturn(MADSpec) <- (0.2/length(USE))
setType(MADSpec) <- "MAD"
setNFrontierPoints(MADSpec)<-100
setRiskFreeRate(MADSpec)<-0
MADTgtPortfolio <- efficientPortfolio(data = Data,
 spec = MADSpec, constraints = "LongOnly")
print(MADTgtPortfolio)
```

A.3.3 Mean-CVaR Portfolio

```
#Mean-CVaR Portfolio
library(fPortfolio)
library(robustbase)
library(corpcor)
USE <- as.timeSeries(uGANDAN_dATA01)
USE.ret <- returns(USE)
Data<-100 * USE.ret[, 1:9]
longSpec <- portfolioSpec()
setType(longSpec) <- "CVaR"
setAlpha(longSpec) <- 0.05
setNFrontierPoints(longSpec) <- 5
setSolver(longSpec) <- "solveRglpk.CVAR"
longFrontier <- portfolioFrontier(data = Data,
 spec = longSpec,
constraints = "LongOnly")
print(longFrontier)

#Mean-CVaR Efficient Frontier
```

```
setNFrontierPoints(longSpec) <- 20
longFrontier <- portfolioFrontier(data = Data,
spec = longSpec,
constraints = "LongOnly")
tailoredFrontierPlot(object = longFrontier, risk = "CVaR")

#CVaR Feasible Portfolio Equal Weights Man-CVaR Portfolio,
par(mfcol = c(1, 3), mar = c(3.5, 4, 4, 3) + 0.1)
weightsPie(ewPortfolio, radius = 1.0)
text <- ""
mtext(text, side = 3, line = 1.5, font = 2, cex = 0.7, adj = 0)
weightedReturnsPie(ewPortfolio, radius = 1, legend = FALSE)
covRiskBudgetsPie(ewPortfolio, radius = 1, legend = FALSE)
```

A.3.4 The Shrinked Mean Variance (SMV) Portfolio

```
#Shrinked Mean Variance (SMV) Portfolio
library(fPortfolio)
library(robustbase)
library(corpcor)
USE <- as.timeSeries(uGANDAN_dATA0)
USE.ret <-  returns(USE)
Data<-100 * USE.ret[, 1:8]
shrinkSpec <- portfolioSpec()
setEstimator(shrinkSpec) <- "shrinkEstimator"
setNFrontierPoints(shrinkSpec) <- 5
shrinkFrontier <- portfolioFrontier(
data = Data, spec = shrinkSpec)
print(shrinkFrontier)
```

A.3.5 Alternative Covariance Estimator (ACE) Portfolio

```
# Alternative Covariance Estimator (ACE) Portfolio
library(fPortfolio)
library(robustbase)
library(corpcor)
USE <- as.timeSeries(uGANDAN_dATA0)
USE.ret <-  returns(USE)
Data<-100 * USE.ret[, 1:8]
covtEstimator <- function (x, spec = NULL, ...) {
x.mat = as.matrix(x)
list(mu = colMeans(x.mat), Sigma = MASS::cov.trob(x.mat)$cov) }
covtSpec <- portfolioSpec()
setEstimator(covtSpec) <- "covtEstimator"
```

```
setNFrontierPoints(covtSpec) <- 5
covtFrontier <- portfolioFrontier(
data = Data, spec = covtSpec)
print(covtFrontier)

#ACE Portfolio Frontier
setNFrontierPoints(covtSpec) <- 20
covtFrontier <- portfolioFrontier(
data = Data, spec = covtSpec)
tailoredFrontierPlot(
shrinkFrontier,
mText = "Student's t MV Portfolio",
risk = "Sigma")

#ACE Portfolio - Long Only Constraints, portfolio weights, the
#weighted returns and the covariance risk budgets
weightsPlot(covtFrontier)
text <- "Student's t"
mtext(text, side = 4, line = 3, font = 2, cex = 0.5)
weightedReturnsPlot(covtFrontier)
covRiskBudgetsPlot(covtFrontier)
```

[1]

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