



STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
MASTER OF SCIENCE IN STATISTICAL SCIENCE
END OF SEMESTER EXAMINATION
STA 8102: STATISTICAL INFERENCE

DATE: 6th December 2024

Time: 3 Hours

Instructions

1. This examination consists of **FOUR** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 Marks)

- (i) Let X_1, X_2, \dots, X_n be a random sample from a normal distribution $N(\mu, \sigma^2)$.
- a. Derive the maximum likelihood estimators (MLEs) for μ and σ^2 . (4 marks)
 - b. Given the data $X = \{8.1, 9.3, 7.8, 8.5, 9.1\}$, calculate the MLEs for μ and σ^2 . (4 marks)
- (ii) A researcher observes the following sample of failure times (in hours) for a machine: $\{2.5, 1.8, 3.1, 2.9, 2.2\}$, and assumes an exponential distribution with rate parameter λ . Compute the MLE of λ and construct a 95% confidence interval for λ . (6 marks)
- (iii) Consider testing $H_0: \mu = 100$ against $H_1: \mu \neq 100$, where $X_1, X_2, \dots, X_{25} \sim N(\mu, 16)$. The sample mean is $\bar{X} = 98$. Perform a two-sided hypothesis test at the 5% significance level and compute the p-value. (8 marks)
- (iv) The following data shows the number of defective products in random samples of 100 units taken from a factory:
- $$\{12, 9, 11, 14, 10, 8\}$$
- Assume the defect count follows a binomial distribution with p as the probability of defect. Estimate p using the method of moments. (4 marks)
- (v) The sample data X_1, X_2, \dots, X_n is drawn from a population with unknown mean and variance. Using the bootstrap method, outline how to compute an approximate 95% confidence interval for the mean based on the data $\{12, 14, 13, 16, 11\}$. (4 marks)

QUESTION TWO (15 Marks)

- (i) Let X_1, X_2, \dots, X_n be a random sample from a Poisson distribution with parameter λ . Derive the likelihood ratio test for testing $H_0: \lambda = 3$ against $H_1: \lambda \neq 3$ based on the following data: {2,4,3,5,3,4}. (8 marks)
- (ii) A dataset contains two independent samples from two different populations:

Sample 1 (n=8): 10,12,9,11,13,10,14,9

Sample 2 (n=6): 8,7,6,10,9,8

Assume normality with equal variances. Conduct a two-sample t-test at the 5% significance level and compute the p-value. (7 marks)

QUESTION THREE (15 Marks)

- (i) Suppose you have a dataset {20,22,23,21,19,25,24} which follows a normal distribution. Perform a nonparametric Wilcoxon signed-rank test to test whether the median is significantly different from 22 at the 5% significance level. (8 marks)
- (ii) The following data represents the time taken (in minutes) by two groups of employees to complete a task:

Group A: 34,36,35,33,34

Group B: 31,29,30,32,33

Use the Mann-Whitney U test to compare the medians of the two groups at the 5% level. (7 marks)

QUESTION FOUR (15 Marks)

- (i) Let X_1, X_2, \dots, X_n be a random sample from a uniform distribution $U(0, \theta)$. Derive the method of moments estimator and the maximum likelihood estimator for θ . Apply these methods to the following data: {3.2,4.5,5.1,6.2,4.9,5.3}. (8 marks)
- (ii) A random sample of $n = 50$ observations is taken from a population, and the sample variance is $S^2 = 4$. Construct a 95% confidence interval for the population variance σ^2 based on this sample. (7 marks)

END
