Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BSC ACTUARIAL SCIENCE

END OF SEMESTER EXAMS

BSA 3110: ACTUARIAL MODELING II

DATE: 11th August 2020 **TIME: 2 HRS 30 MINS**

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION 1 (30 MARKS)

a. A certain course in a certain community college has until recently been having a high withdrawal rate due to its complexity. The college has approached you as an actuarial science student and provided you with the data containing the following information.
   - The date which a student enrolled for the course
   - The year they withdrew from the course, if the student withdrew due to the complexity of the course
   - The date of withdrawal, if the student finished the four year course and left the college
   - The date of leaving, if a student left the college for any other reason.

   In each of the above cases: discuss the censoring mechanism for each of the above and explain how it/they came about. (8 marks)

b. Actuarial science student constructed a table assuming the Makeham's law. Given the following values from their tables: $\mu_{30} = 0.025330, \mu_{40} = 0.123224 \; and \; \mu_{70} = 0.231123$, calculate the probability that and individual currently aged 60 will die within three years of attaining age 70. (7 marks)

c. The training exercises in the military's camp is very stringent and very taxing to the body and mind. As such many recruits drop out of the training. The following life table obtained from the military website on new recruits is shown below (time x is measured in years).

x	l_x
0	10000
1	7200
2	5100
3	3600
4	2400
5	1500
6	1000
7	600
8	250
9	0

100 recruits join the camp in March this year, just after COVID-19 was declared a national disaster, calculate the expected number of complete years that a recruit will complete in the training. (5 marks).

d. In the recent revenue allocation debate, it was noted that a certain percentage of the allocation was based on the mortality of each county. In a certain county, the force of mortality was $\mu_x = 0.02312 + 0.05(x - 50)$ for $x \geq 45$. For an individual living in that county, calculate the probability that an individual aged 55 dies before his or her 65th birthday. (5 marks)

e. In a population of 10,000 individuals are currently aged 44 years, the force of mortality at different ages is given below.

	μ_x
$40 < x < 45$	0.0023
$45 < x < 55$	0.014
$55 < x < 65$	0.035

For individuals currently aged 44, find the expected number of these individuals who will die within 2 years of attaining age 55 (5 marks).

QUESTION TWO (20 Marks)

The following is information was recorded. Values with asterisk indicate censoring.

20*, 19, 19, 10*, 9*, 8*, 16*, 11, 7, 11, 4, 19*, 16, 3, 13

a. Give the value of $n, m, k, t_j, d_j, c_j \; and \; n_j$ for the above data. (3 marks)
b. Using the survival function, calculate the Kaplan-Meier estimate of $\hat{F}(t)$ (6 marks)
c. Estimate $var[\tilde{F}(13)]$ using Greenwood's formula and hence or otherwise its 90% confidence interval. (6 marks)
d. Estimate $var\left[\widetilde{\Lambda}(13)\right]$ and compare your results with that obtained in c above. (5 marks)

QUESTION THREE (20 Marks)

a. The following is data on individuals who received a certain treatment for COVID-19. Two covariates were of interest: $Z_{1i} = 0$ for female and 1 for male and $Z_{2i} = Age$ of the patient measured in years. The data below is on their observed lifetimes with age in brackets.

Male	Females
4 (24)	4 (33)
7 (69)	6 (47)
11(87)	11(90)
11(5)*	11(16)

Suppose the parameters for gender was 0.4243 and that for Age was 0.2321, find the contribution to the partial likelihood by the deaths at time 11. (10 marks)

b. Though unproven, it is believed that the effects of COVID-19 are more severe to older individuals than young individuals. An Actuarial modeling class has opines that the crude estimated can be graduated using the formula:

$$\mu_x = \beta_0 + \beta_1 x$$

i. Comment on their choice of mode (3 marks)
ii. Name 2 methods that can be used to estimate $\beta_0 \; and \; \beta_1$(2 marks)

c. It has been established that 15 out of 1000 individuals who observed all the guidelines on COVID-19 test positive (this occurrences do not depend on each other). Suppose 20000 individuals who adhere to the guidelines are tested. What is the probability that 60 test positive (5 marks).

QUESTION FOUR (20 Marks)

a. The table below is data on 5 ages from a certain research in 2019. Assuming $\log(e^{\mu_x} - 1)$ satisfies a polynomial of order one, obtain the graduated rates q_x^0. Determine whether your graduated results are smooth and whether the actual deaths are consistent with those of the model. (10 marks).

x	l_x	d_x	μ_x
36	98291	74	0.000 72
37	98217	81	0.000 79
38	98136	88	0.000 86
39	98048	96	0.000 94
40	97952	105	0.001 02
41	97847	114	0.001 12

b. Under the binomial model, calculate the probability that there will be more than 100 deaths at age 39 (5 marks)

c. The following is data on the number of individuals in a certain pension scheme in the year 2019.

Time	**Population**
1 January 2019	34,453
21 June 2019	39,656
3 September 2019	29,469
13 December 2019	37,256

Using census approximation, calculate the central exposed to risk for the year 2019. Assume that $P_{x,t}$ is linear between census dates. (5 marks)

QUESTION FIVE (20 Marks)

a. A certain company recently did a study to establish the possible reasons why their employees get depressed. The following are results from the survey with week indicating the week an employee got depression, PSV: whether one uses public service vehicle or not to work, age is age of the employee and gender. The following are results from a Cox-PH model fitted in R. (7 marks)

```
coxph(formula = Surv(week, Depression) ~ PSV + age + Gender,
data = Stress)

  n= 432, number of events= 114

                    coef     se(coef)   z     Pr(>|z|)
PSV(yes)         -0.57942   0.19138 -1.983  0.04742 *
age               0.5744    0.02200 -2.611  0.00903 **
Gender(Female)   -0.14980   0.21222 -0.706  0.48029
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

      Likelihood ratio test= 33.27  on 7 df,   p=2e-05
```

b. A mortality investigation covers the period 1 January 2016 to 31 December 2017. In this investigation, the rate interval is $[x, x+1]$. The table below is information on three individuals who took part in the investigation.

Life	Date of Birth	Date of Joining	Date of Exit	Reason for Exit
A	25.04.83	01.01.15	30.10.16	Death
B	01.07.83	12.09.16	----------	----------
C	04.09.82	22.07.17	04.12.17	Withdrawal

i. Assuming that the day of entry counts in the exposed to risk but the day of exit does not, determine the range of dates for which lives, A, B, C contribute to E_x^c.
Show your workings (9 marks)

ii. Suppose now the rate interval is $[x-1, x]$, give the range of dates for which lives A, B and C contribute to E_{34}^c (4 marks).