Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN INFORMATICS AND COMPUTER SCIENCE
BACHELOR OF SCIENCE IN COMPUTER NETWORKS AND CYBER SECURITY
END OF SEMESTER EXAMINATION
ICS 1205 /CNS 1205: LINEAR ALGEBRA

DATE:12 /03/2025 Time: 13:30-15:30

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE [30 MARKS]

(a) State whether the following statements are true or false.

(i) For a matrix A to have an inverse, it must be square. **[1 Mark]**

(ii) Every square matrix does not have inverse. **[1 Mark]**

(iii)If a matrix has an inverse, it is said to be non-singular. **[1 Mark]**

(b) Solve the following system by Gaussian elimination method:

$$a+3b-c=6$$
$$8a+9b+4c=21$$
$$2a+b+2c=3$$

[4 Marks]

(c) Find the matrix A such that $A\begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix}=\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$.

[3 Marks]

(d) Show that the matrix $A=\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ satisfies the equation $x^2-4x=5$. Hence find A^{-1}.

[4 Marks]

(e) Show that the square matrix $\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$ is orthogonal.

[3 Marks]

(f) Are the vectors $X_1 = (1, 3, 4, 2)$, $X_2 = (3, -5, 2, 2)$, $X_3 = (2, -1, 3, 2)$ linearly dependent? If so, express one of these as a linear combination of others.

[4 Marks]

(g) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$.

[5 Marks]

(h) Using the properties of determinants evaluate

$$\begin{vmatrix} 1 & 3 & -3 & 5 \\ 4 & 2 & 1 & 2 \\ 3 & 2 & -2 & 2 \\ 0 & 1 & 2 & -1 \end{vmatrix}$$

[4 Marks]

QUESTION TWO [20 MARKS]

(a) Given that

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \\ 3 & -1 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & -1 & 1 \\ 4 & -10 & -8 \\ -1 & 7 & 5 \end{pmatrix}$$

i. Determine AB. Hence A^{-1}. **[4 Marks]**

ii. Hence solve the following system of simultaneous equations:

$$x + 2y + 3z = -6$$
$$-2x + y + 2z = 1$$
$$3x - y - z = 1$$

[4 Marks]

(b) A company makes three types of security locks A, B and C, each of which requires cutting, assembly, and finishing. Each unit of A requires 2 hours for cutting, 1 hour for assembly, and 3 hours for finishing. Each unit of B requires 1 hour for cutting, 2 hours for assembly, and 1 hour for finishing. Each unit of C requires 1 hour for cutting, 2 hours for assembly , and 3 hours for finishing.

Available machine resources provide exactly 10 hours for cutting, 14 hours for assembly, and 18 hours for finishing each week.

i. Use the information provided to form a system of simultaneous equations. **[3 Marks]**

ii. Apply Cramer's rule to calculate the number of locks of each type produced given that there is optimum utilization of machine time at the factory in a week. **[9 Marks]**

QUESTION THREE [20 MARKS]

(a) Find the value of λ such that the following equations have unique solution.

$$\lambda x + 2y - 2z - 1 = 0$$
$$4x + 2\lambda y - z - 2 = 0$$
$$6x + 6y + \lambda z - 3 = 0$$

and, by matrix method , solve these when $\lambda = 2$.

[5 Marks]

(b) In a mass spring-damper system, the acceleration $\frac{d^2x}{dt^2}$ m/s^2, velocity $\frac{dx}{dt}$ m/s and displacement x m are related by the equations:

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 4x = 35$$
$$2\frac{d^2x}{dt^2} + \frac{dx}{dt} + 2x = 20$$
$$-3\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 24$$

By using Gauss Jordan method, determine the acceleration, velocity and displacement for the system.

[5 Marks]

(c) Reduce the matrix A to a diagonal form, where $A = \begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$.

[10 Marks]

QUESTION FOUR [20 MARKS]

(a) The eigenvalues of a 2×2 matrix A are $\lambda_1 = 3$ and $\lambda_2 = 2$ with corresponding eigenvectors $x_1 = [-2 \quad 1]^T$ and $x_2 = [-1 \quad 1]^T$. Determine the

i. The modal matrix M and spectral matrix Λ of A. **[2 Marks]**

ii. the matrix A. **[4 Marks]**

iii. A^2 **[2 Marks]**

(b) Given that $[1 \quad 0 \quad 1]^T$ is an eigenvector of the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & b \\ 0 & b & c \end{pmatrix}$

Find the:

i. value of b and c. **[3 Marks]**

ii. eigenvalues and corresponding eigenvectors. **[9 Marks]**

QUESTION FIVE [20 MARKS]

(a) Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ and hence find A^{-1}.

[6 Marks]

(b) Use Cayley Hamilton theorem to express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ in terms of A where $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$. **[8 Marks]**

(c) Given that $A = \begin{pmatrix} 2 & x \\ x & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$. AB is a singular matrix, determine the possible values of x.

[6 Marks]

END