



Strathmore  
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES  
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING  
END OF SEMESTER EXAMINATION

EMT 2211: Engineering Mathematics II

Instructions

Time: 13:30-16:30

Date: 17th March, 2025

1. This examination consists of FIVE questions.
  2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.
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QUESTION ONE (30 MARKS)

- (a) Determine whether the following series converges or diverges. [3 Marks]

$$\sum_{k=1}^{\infty} k \left(\frac{1}{2}\right)^k .$$

Show a full method, justifying every step in the workings.

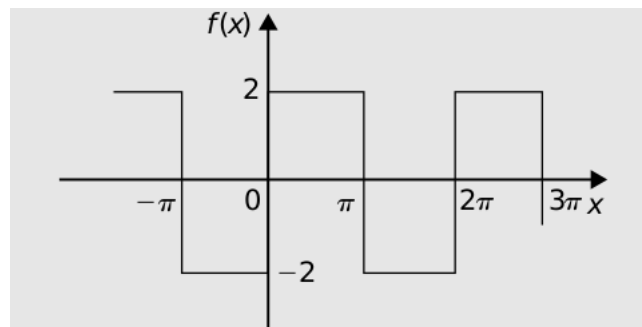
- (b) Find each of the following Laplace transforms, showing, where appropriate, the technique used.

(i)  $\mathcal{L}\{e^{2t} \cos t\}$  [2 Marks]

(ii)  $\mathcal{L}\{(t+2)(t+3)\}$  [2 Marks]

- (c) A drilling machine is to have six speeds ranging from 50 rev/min to 750 rev/min. If the speeds form a geometric progression determine their values, each correct to the nearest whole number. [4 Marks]

- (d) Obtain the Fourier series for the square wave shown below. [5 Marks]



(e) Evaluate  $\int_0^{0.4} x \ln(1+x) dx$  using Maclaurin's theorem, correct to 3 decimal places. [3Marks]

(f) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

[4 Marks]

(g) A direct current (d.c) circuit comprises three closed loops. Applying Kirchhoff's laws to the closed loops gives the following equations for current flow in milliamperes:

$$2I_1 + 3I_2 - 4I_3 = 26$$

$$I_1 - 5I_2 - 3I_3 = -87$$

$$-7I_1 + 2I_2 + 6I_3 = 12$$

Use determinants to solve for  $I_1$ ,  $I_2$  and  $I_3$

[4 Marks]

(h) Find the inverse Laplace transform, showing, where appropriate, the technique used.

[3 Marks]

$$\mathcal{L}^{-1} \left\{ \frac{s+2}{s^2+4s+13} \right\}$$

## QUESTION TWO (15 MARKS)

(a) In determining a Fourier series to represent  $f(x) = x$  in the range  $-\pi$  to  $\pi$ , Fourier coefficients are given by:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos nx dx$$

$$\text{and } b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx dx$$

where  $n$  is a positive integer. Show by using integration by parts that  $a_n = 0$  [5 Marks]

(b) (i) Determine the Fourier series for the function  $f(\theta) = \theta^2$  in the range  $-\pi < \theta < \pi$ . The function has a period of  $2\pi$ . [7 Marks]

(ii) For the Fourier series in part (i), let  $\theta = \pi$  and show that  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ . [3 Marks]

## QUESTION THREE (15 MARKS)

(a) Determine whether the following series converges or diverges.

$$(i) \sum_{n=1}^{\infty} \left[ \frac{2^n}{n!} \right] \qquad (ii) \sum_{n=0}^{\infty} \left[ \frac{4n^2 - n^3}{10 + 2n^3} \right] \qquad [6 \text{ Marks}]$$

Show a full method, justifying every step in the workings.

- (b) Use standard results to find the series expansion of  $y = e^{2x} \sin 3x$ , up and including the term in  $x^4$ . [4 Marks]

Hence, evaluate, correct to 3 decimal places,  $\int_0^{0.1} e^{2x} \sin 3x \, dx$ . [2 Marks]

- (c) A hire tool firm finds that their net return from hiring tools is decreasing by 10% per annum. If their net gain on a certain tool this year is £400, find the possible total of all future profits from this tool (assuming the tool lasts forever). [3 Marks]

### QUESTION FOUR (15 MARKS)

- (a) The velocity of a car, accelerating at uniform acceleration  $a$  between two points, is given by  $v = u + at$ , where  $u$  is its velocity when passing the first point and  $t$  is the time taken to pass between the two points. If  $v = 21\text{m/s}$  when  $t = 3.5\text{s}$  and  $v = 33\text{m/s}$  when  $t = 6.1\text{s}$ , use matrix inverse to find the values of  $u$  and  $a$ , each correct to 4 significant figures. [5 Marks]

- (b) The  $3 \times 3$  matrix  $\mathbf{A}$  is given below.

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -3 \\ 2 & 4 & 3 \\ -4 & 2 & -1 \end{pmatrix}$$

The matrix  $\mathbf{A}$  is non singular.

- (i) Evaluate  $\mathbf{A}^2 - \mathbf{A}$ . [2 Marks]

- (ii) Show clearly that  $\mathbf{A}^{-1} = \frac{1}{20}[\mathbf{A} - \mathbf{I}]$ . [2 Marks]

- (c) For the matrix  $\mathbf{A} = \begin{pmatrix} 1 & -4 & -2 \\ 0 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}$  find the (i) eigenvalues (ii) eigenvectors. [6 Marks]

### QUESTION FIVE (15 MARKS)

- (a) Find each of the following Laplace transforms or inverse Laplace transforms, showing, where appropriate, the technique used.

- (i)  $\mathcal{L} \{ 5e^{-3t} \sinh 2t \}$  [2 Marks]

- (ii)  $\mathcal{L}^{-1} \left\{ \frac{7s+13}{s(s^2+4s+13)} \right\}$  [3 Marks]

- (b) Prove that [3 Marks]

$$\mathcal{L} \{ \sinh at \} = \frac{a}{s^2 - a^2} \quad \text{provided that } s > a.$$

(c) Solve the following pair of simultaneous differential equations

[7 Marks]

$$3\frac{dx}{dt} - 5\frac{dy}{dt} + 2x = 6$$

$$2\frac{dy}{dt} - \frac{dx}{dt} - y = -1$$

given that at  $t = 0$ ,  $x = 8$  and  $y = 3$

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**END OF PAPER**

### Formula sheet

Function, $f(t)$	Laplace transforms $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$
1	$\frac{1}{s}$
$k$	$\frac{k}{s}$
$e^{at}$	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$t^n (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
Function, $e^{at} f(t)$	Laplace transforms $\mathcal{L}\{e^{at} f(t)\}$
$e^{at} t^n$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2 - \omega^2}$
$e^{at} \cosh \omega t$	$\frac{s-a}{(s-a)^2 - \omega^2}$

Fourier series for periodic functions	
Over period $2\pi$	$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ <p>where <math>a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx</math>; <math>a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx</math></p> <p>and <math>b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx</math>; <math>(n = 1, 2, 3, \dots)</math></p>
Over period $L$	$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos (\frac{2\pi nx}{L}) + b_n \sin (\frac{2\pi nx}{L})]$ <p>where <math>a_0 = \frac{1}{L} \int_{-L/2}^{L/2} f(x) dx</math>; <math>a_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \cos (\frac{2\pi nx}{L}) dx</math></p> <p>and <math>b_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \sin (\frac{2\pi nx}{L}) dx</math>; <math>(n = 1, 2, 3, \dots)</math></p>