



Strathmore Institute of Mathematical Sciences
BSc. Actuarial Science, Financial Engineering and Financial Economics
BSM 3220 - Optimization Methods in Finance

Date: Monday 4th December, 2023

Time: 2 Hours

Instruction

1. Answer **QUESTION ONE** and any other **TWO QUESTIONS**

QUESTION ONE [30 Marks]

- a) In relation to linear programming problems,
- i. When do we say that the problem is unbounded? [1 Mark]
 - ii. When does the problem have a unique optimum solution? [1 Mark]
 - iii. Define an alternate optimum solution? [1 Mark]
- b) Catherine eats a mix of Cereal *A* and Cereal *B* for breakfast. The amount of calories, sodium and protein per ounce for each is shown in the table below. Catherine's breakfast should provide at least 480 calories but less than or equal to 700 milligrams of sodium. She would like to maximize the amount of protein in his breakfast mix. [3 Marks]

	Cereal A	Cereal B
Calories (per ounce)	100	140
Sodium (mg per ounce)	150	190
Protein (g per ounce)	9	10

- c) Given the standard maximization problem $Max C^T X$ such that $AX \leq b$ with $X \geq 0$. ($A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^{m \times 1}$). Give the dual of the problem, defining the variable(s) you use. [3 Marks]
- d) As a numerical example and further to (b) above, give the dual of the following linear programming problem. [3 Marks]

$$\begin{aligned}
 &Max \quad x_1 + x_2 \\
 &s.t \quad x_1 + 2x_2 \leq 4 \\
 &\quad \quad 4x_1 + 2x_2 \leq 12 \\
 &\quad \quad -x_1 + x_2 \leq 1 \\
 &\quad \quad x_1, x_2 \geq 0
 \end{aligned}$$

- e) Give any two advantages of the dual over primal in linear programming problems. [2 Marks]
- f) Alpha outfitters produces two products X and Y each requiring same production capacity. The total installed production capacity is 9 tones per days. Alpha outfitters is a supplier of Beta fitters which must supply at least 2 tones of X and 3 tones of Y to Beta fitters. The daily production time for X and Y is 20 machine hour per unit and 50 machine hour per unit respectively. The daily maximum possible machine hours is 360 profit margin for X and Y is $KSh.80$ per tone and $KSh.120$ per tone respectively. Formulate as a linear programming model and use the graphical method to generate the optimal solution for determining the maximum number of units of X and Y which can be produced by Alpha outfitters Limited. [6 Marks]
- g) Give two reasons why sensitivity analysis in linear programming is important? [2 Marks]
- h) Define the following:
- i. Solution. [1 Mark]
 - ii. Feasible solution. [1 Mark]
 - iii. Feasible region. [1 Mark]
- i) Consider the unconstrained quadratic optimization problem
- $$f(x_1, x_2) = (3 - x_1)^2 + (4 - x_2)^2$$
- i. Express the function in matrix form. [1 marks]
 - ii. Check the first order necessary condition for optimality. [2 marks]
 - iii. Check the second order sufficient condition for optimality. [2 Marks]

QUESTION TWO [20 Marks]

- a) A Silicon Valley Tech firm specializes in making four types of silicon chips for personal computers. Each chip must go through four stages of processing before completion. First the basic silicon wafers are manufactured, second the wafers are laser etched with a micro circuit, next the circuit is laminated onto the chip, and finally the chip is tested and packaged for shipping. The production manager desires to maximize profits during the next month. During the next 30 days she has enough raw material to produce 4000 silicon wafers. Moreover, she has 600 hours of etching time, 900 hours of lamination time, and 700 hours of testing time. Taking into account depreciated capital investment, maintenance costs, and the cost of labour, each raw silicon wafer is worth 1 pound, each hour of etching time costs 40 Pounds, each hour of lamination time costs 60 Pounds, and each hour of inspection time costs 10 Pounds. The production manager has formulated her problem as a profit maximization. Use simplex method to help her out. [14 Marks]

b) Give an advantage of simplex method over graphical method. [1 Marks]

c) Solve by using graphical method: [5 Marks]

$$\begin{aligned} \text{Max } & 4x_1 + 3x_2 \\ \text{s.t } & 4x_1 + 3x_2 \leq 24 \\ & x_1 \leq \frac{9}{2} \\ & x_2 \geq 6 \\ & x_1, x_2 \geq 0 \end{aligned}$$

QUESTION THREE [20 Marks]

a) Imagine a furniture company that makes tables and chairs. A table requires 40 board feet of wood and a chair requires 30 board feet of wood. Wood costs 1 Pound per board foot and 40,000 board feet of wood are available. It takes 2 hours of skilled labour to make an unfinished table or an unfinished chair. Three more hours of labour will turn an unfinished table into a finished table; two more hours of skilled labour will turn an unfinished chair into a finished chair. There are 6000 hours of skilled labour available. (Assume that you do not need to pay for this labour.) The prices of output are given in the table below:

Product	Price
Unfinished Table	\$70
Finished Table	\$140
Unfinished Chair	\$60
Finished Chair	\$110

- i. What would happen if the price of unfinished chairs went up? [2 Marks]
- ii. What would happen if the price of unfinished tables went up? [2 Marks]
- iii. What if the price of finished chairs fell to 100 Pounds? [2 Marks]

- iv. How would profit change if wood supplies changed? [2 marks]
- v. How much would you be willing to pay an additional carpenter? [1 Mark]
- vi. Suppose that industrial regulations complicate the finishing process, so that it takes one extra hour per chair or table to turn an unfinished product into a finished one.
How would this change your plans? [3 Marks]
- vii. The owner of the firm comes up with a design for a beautiful hand-crafted cabinet. Each cabinet requires 250 hours of labour (this is 6 weeks of full time work) and uses 50 board feet of lumber. Suppose that the company can sell a cabinet for 200 Pounds, would it be worthwhile? [8 marks]

QUESTION FOUR [20 Marks]

- a) Suppose you are given the problem:

$$\begin{aligned}
 \text{Min } & 2x_1 + 3x_2 + 4x_3 + 5x_5 \\
 \text{s.t } & x_1 - x_2 + x_3 - x_4 \geq 10 \\
 & x_1 - 2x_2 + 3x_3 - 4x_4 \geq 6 \\
 & 3x_1 - 4x_2 + 5x_3 - 6x_4 \geq 15 \\
 & x_1, x_2, x_3, x_4 \geq 0
 \end{aligned}$$

Solve its dual using simplex method. [10 marks]

- b) A farmer has 20 hectares of growing barley and wheat. He has to decide how much of each to grow. The cost per hectare for growing barley is 30 Pounds and that for wheat is 20 Pounds. He has a budget of 480 Pounds. Barley needs one man day per hectare while wheat needs two man days per hectare. There are 36 man days available. The profit of barley is 100 Pounds per hectare while that for wheat is 120 Pounds per hectare. How many hectares of each crop should the he sow to maximize his returns. [10 Marks]

QUESTION FIVE [20 Marks]

- a) The County of Narok operates a small game reserve for the benefit of its residents and to attract tourists during the Easter . Among its many inhabitants is a pride of rare baboons . The County recreation board and especially the curator of the game park, want to be sure that the baboons' basic nutritional requirements are satisfied, but at the same time they do not want to spend more for their food than is necessary . Their necessary vitamins are provided in a supplement . Their daily diet must provide at least 500 units of protein and 960 units of mineral fiber . To keep the baboons from feeling hungry and hence unhappy they must be given at least 6 kilograms of food daily. There are also certain ceilings on the amounts of protein, mineral fiber and total food provided, but we will assume that the objective of minimizing the total cost of feeding the baboons will keep the diet within the maximum allowable levels. The foods that they will be given are prime meat and ground bone meal. A kilogram of the bone meal provides 50 units of protein and 240 units of mineral fiber and costs 1 per kilogram . A kilogram of prime meat provides 250 units of protein and 80 units of mineral fiber and costs 2 per kilogram. The combined weight of the bone meal and the meat must be at least 6 kilograms. How many kilograms of each food type should be included in the diet of the baboons? [10 Marks]
- b) Solve the following quadratic programming problem by the method of Lagrange multipliers. [10 Marks]

$$\begin{aligned} \text{Min } & a^2 + 0.5a + 3ab + 5b^2 \\ \text{s.t } & g_1 = 3a + 2b + 2 \leq 0 \\ & g_2 = 15a - 3b - 1 \leq 0 \end{aligned}$$