Strathmore UNIVERSITY

School of Computing and Engineering Sciences

Bachelor of Science in Informatics and Computer Science
Bachelor of Science in Computer Networks and Cyber Security

END SEMESTER EXAMINATION
ICS & CNS 1205 – LINEAR ALGEBRA

DATE: 11th December 2023 **TIME: 1030 – 1230 Hrs**

Answer Question ONE (COMPULSORY) and any other two questions in the answer booklet provided.

QUESTION ONE - [30 MARKS]:

a) Define the following terms:

(i) Eigen value [1 mark]

(ii) Eigen vector [1 mark]

b) Let $A = \begin{bmatrix} 6 & -3 \\ 3 & 6 \end{bmatrix}$, evaluate AA^T. [3 marks]

c) Find the eigen values and eigen vectors of $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ [6 marks]

d) Find x such that the determinant of $A = \begin{pmatrix} 4 & 1 & 0 \\ 3 & x & 2 \\ 0 & 0 & 4 \end{pmatrix}$ is 52. [5 marks]

e) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$ using row operations. [4 marks]

f) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 & 8 \\ 1 & -3 & -7 \end{bmatrix}$, find the value of $2A - 3B$. [4 marks]

g) Use the simplex method to find the maximum value of

$z = 2x_1 - x_2 + 2x_3$ (Objective function)

subject to the constraints

$$\begin{aligned} 2x_1 + x_2 \qquad &\leq 10 \\ x_1 + 2x_2 - 2x_3 &\leq 20 \\ x_2 + 2x_3 &\leq 5 \end{aligned}$$

where $x_1 \geq 0, x_2 \geq 0$ and $x_3 \geq 0$. [6 marks]

QUESTION TWO - [20 MARKS]:

a) Find the inverse of $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ using the cofactor method. [7 marks]

b) Write the vector $\boldsymbol{v} = (1, -2, 5)$ as a linear combination of the vectors $\boldsymbol{u_1} = (1, 1, 1)$, $\boldsymbol{u_2} = (1, 2, 3)$ and $\boldsymbol{u_3} = (2, -1, 1)$. [6 marks]

c) Solve the following system of equations by Gauss-Jordan elimination method [7 marks]

$$x - 3y - 2z = 6$$
$$2x - 4y - 3z = 8$$
$$-3x + 6y + 8z = -5$$

QUESTION THREE - [20 MARKS]:

Consider the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$

a) Find the eigen values of A [5 marks]

b) Determine the corresponding eigen vectors [5 marks]

c) Form matrices D and P such that $P^{-1}AP = D$ where P is the matrix whose columns are the Eigen vectors of A. [10 marks]

QUESTION FOUR - [20 MARKS]:

a) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & -4 \end{bmatrix}$, find $g(A) = A^2 + 3A - 10I$ [5 marks]

b) Let $A = \begin{bmatrix} 1 & 3 \\ 4 & -3 \end{bmatrix}$, find a non-zero column vector $\boldsymbol{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ such that $A\boldsymbol{u} = 3\boldsymbol{u}$. [5 marks]

c) A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B.

At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximise the combined sum of the units of X and the units of Y in stock at the end of the week.

(i) Formulate the problem of deciding how much of each product to make in the current week as a linear program. [4 marks]

(ii) Solve this linear program graphically. [6 marks]

QUESTION FIVE - [20 MARKS]:

a) Use Cramer's rule to evaluate the following system of linear equations: [5 marks]

$$2x + 3y + 4z = 9$$
$$5x + 6y + 7z = 15$$
$$8x + 9y + z = 2$$

b) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear mapping defined by $T(x, y, z) = (2x + 3y - 8z, x + y + z, 2x + z)$. Find the matrix representation of T using:

(i) the standard basis of $\mathbb{R}^3$. [3 marks]

(ii) the basis $\{\boldsymbol{e_1} = (1,1,1), \boldsymbol{e_2} = (1,1,0), \boldsymbol{e_3} = (1,0,0)\}$. [5 marks]

c) The advertising alternatives for a company include television, radio, and newspaper advertisements. The costs and estimates for audience coverage are given in the table below:

	Television	*Newspaper*	*Radio*
Cost per advertisement	£ 2, 000	£ 600	£ 300
Audience per advertisement	100, 000	40, 000	18, 000

The local newspaper limits the number of weekly advertisements from a single company to ten. Moreover, in order to balance the advertising among the three types of media, no more than half of the total number of advertisements should occur on the radio, and at least 10% should occur on television. The weekly advertising budget is £18,200. How many advertisements should be run in each of the three types of media to maximize the total audience? [7 marks]