A COMPARATIVE ANALYSIS OF THE MEAN-VARIANCE OPTIMIZATION THEORY AND THE BLACK-LITTERMAN MODEL; INTEGRATING THE MONTE CARLO SIMULATION.

A Case study of the Nairobi Securities exchange

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this university or any other university. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

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This Research Project has been submitted for examination with my approval as the Supervisor.

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ABSTRACT.

This comparative study analyzed portfolio optimization using the Black-Litterman model and Harry Markowitz's mean-variance optimization model. Furthermore, the study employed GARCH (1,1) simulated returns to predict future returns. The study used the Sharpe ratio to measure the performance of the portfolios. The comparative study was conducted on the Nairobi Securities Exchange using nine selected stocks from nine sectors of the economy. The study aims to determine which of the two optimization models yields better results for an investor's wealth. The study found that the Black Litterman model portfolios performed better than the mean-variance optimization model portfolios. Furthermore, the study revealed that portfolios constructed using the historical dataset, yielded better returns with a higher Sharpe ratio and the portfolios constructed using the simulated returns.

ABBREVIATIONS.

GARCH	Generalized Autoregressive Conditional Heteroskedasticity
NSE	Nairobi Securities Exchange
MVO	Mean-Variance Optimization
B-L	Black-Litterman model
ARCH	Autoregressive Conditional Heteroskedasticity.

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CHAPTER 1: INTRODUCTION

1.1 BACKGROUND OF THE STUDY.

With the emergence of various financial instruments in the Kenyan financial markets, many investors have joined the market with different investment goals. Mbithi (2024), explained that investors in the market strive to maximize their expected return with the minimum possible risk. One of the ways an investor can minimize risk is through diversification. Mbithi (2014), explained that investment managers use diversification as one of the main concepts to eliminate as much risk as possible from their portfolios. Diversification is a risk management strategy that creates a mix of various investments within a portfolio. Odhiambo (2014), described portfolio optimization as the process of choosing the proportions of various assets to be held in a portfolio, in such a way that the portfolio performs better than any other portfolio according to some criterion.

Khan and Tanwani (2024), explained that portfolio optimization is a crucial aspect of investment management that is focused on selecting the best portfolio of assets to achieve specific investment goals. Furthermore, portfolio optimization can be described as the process of selecting an optimal portfolio out of a set of considered portfolios according to some objective. Khan and Tanwani (2024), described portfolio optimization as a fundamental concept in modern finance, that aims at constructing a portfolio that maximizes return for a given level of risk or minimizes risk for a given level of return. Markowitz (1952) stated that there is a rule which implies that the investor should both diversify and maximize their expected return. Furthermore, the rule states that the investor does (or should) diversify his funds among all those securities that give maximum expected returns. The Mean-Variance Optimization model by Markowitz (1952), proposed that every rational investor would want a portfolio where the return is maximized and their risk is minimized (for any given level of risk, one would opt for a portfolio with the most return).

In Kenya, with the emergence of the Nairobi Securities Exchange (NSE), many individuals have become interested in investing their funds in different assets. As a result, many fund managers are tasked with ensuring that their clients' portfolios are optimized.

With the emergence of many investment firms in Kenya such as Britam Insurance, CIC Asset Management Limited, and Old Mutual investment fund, there has been a need for researchers to study the dynamics of the stocks listed on the exchange and how investor funds can be allocated to the different assets to ensure the existence of an optimal portfolio.

Mbithi (2014), conducted a study on determining the optimal portfolio size on the Nairobi Securities Exchange. The study involved the use of Markowitz's 1952 mean-variance to determine the optimal portfolio size in the Nairobi Securities Exchange. The study went further to explain that determining the optimal portfolio size would help investors in making decisions in the selection of assets to invest in based on the tradeoff between risk and return. The results of the study showed that the portfolio risk was reduced with an increase in the size of the portfolio thus supporting diversification and modern portfolio theory.

Ogutu (2014), conducted a study on the effect of portfolio optimization on the returns of the listed company at the Nairobi securities exchange. The study also implemented Markowitz's (1952), mean-variance optimization model to analyze the risks and returns of the portfolios. The study explained that portfolio optimization would protect the value of investments made by the investors. The study concluded that the optimal portfolio had the highest return, Sharpe ratio, and Jensen alpha with the lowest standard deviation.

Furthermore, Masese (2017), conducted a study on portfolio optimization in the Kenyan stock market by conducting a comparison between mean-variance optimization and threshold accepting.

However, over the years improvements have been made to Markowitz's mean-variance theory. There was the introduction of the Black-Litterman model. The black-litterman model was advanced by Fischer Black and Robert Litterman in the year 1992 as they found that quantitative asset allocation models did not play the important role they should have in global portfolio management.

Mishra, Pisipati, and Vyas (2011), in their study of an equilibrium approach for tactical asset allocation, applied the Black-Litterman approach to the Indian BSE. The study concluded that the Black-Litterman model produced the best optimal portfolio with the maximum return and minimum risk in comparison to the classical mean-variance and implied return-based approach.

Furthermore, Vladimirov, Stoilov, and Stoilova (2017), in their study of the new formal description of expert views of the Black-Litterman asset allocation model explained that the concept of the model is to integrate the historical data about returns with the subjective views about the future behavior of the returns. However, despite the modifications made to the mean-variance optimization model and the Black-Litterman model both employ the use of historical price data.

Instead of using historical data of stock prices to create optimal portfolios for investors, researchers attempt to simulate future prices of assets using various simulations.

Cong and Oosterlee (2016) proposed a simulation-based approach for solving the constrained dynamic mean-variance portfolio management problem in their study of multi-period mean-variance portfolio optimization based on Monte Carlo simulation. The study yielded more satisfactory allocations under the constraints of control variables for dynamic portfolio management problems.

Ding (2024) stated that the Monte Carlo algorithm has been widely used for stochastic simulation of complex systems in the financial field. The author goes further to explain that the algorithm can simulate the consideration of changes in different influencing factors, provide a reliable reference for asset allocation and finally achieve portfolio optimization.

With the new developments in the mean-variance optimization and black-litterman models, the study aims to study how the Kenyan market will respond to these new developments.

1.2 PROBLEM STATEMENT.

According to Ding (2024), portfolio optimization is of utmost importance as adopting the right portfolio to maximize the benefits is important. A good investment portfolio can always help investors spread their risk across stocks, bonds, and other types of securities and financial instruments (Li, 2023).

Portfolio managers face significant challenges in accurately forecasting asset prices and determining optimal asset allocation strategies to maximize returns while minimizing risk. Traditionally, many of them use mean-variance analysis, which only focuses on two metrics, mean return and variance, to optimize their portfolios' returns.

However, this model has been found to have some shortcomings, and due to these shortcomings, various initiatives have been proposed to alter the model further. The Black-Litterman approach was proposed by researchers Fischer Black and Robert Litterman in 1992, who incorporated the global Capital Asset Pricing Model. The incorporated CAPM can be used as a reference point in Markowitz's mean-variance optimization model to make it a more behaved model and prevent very illogical findings (Tesfamichael, 2023).

Despite the advancements, both of these strategies rely on estimating the changes in the prices of the stocks based on historical data and this might fail to accurately represent the prices of assets. As a result, there is a growing interest in employing stochastic modeling techniques within the practices of portfolio management. Stochastic optimization by the Monte Carlo simulation offers a framework for capturing the uncertainty and randomness of asset price movements thus allowing for more robust and accurate forecasts. By integrating them into the investment decision process, portfolio managers aim to enhance their ability to predict the prices of various assets thus optimizing the portfolio allocation decisions. Over the years, multiple researchers have implemented this simulation in different countries.

Pedersen (2014) conducted a study on portfolio optimization and the Monte Carlo simulation in the United States market. The study used the Monte Carlo simulation of a simple equity growth model with resampling historical financial data to estimate the probability distributions of the future equity, earnings, and payouts of companies. The study found that the portfolios constructed using the Monte Carlo simulated stock returns were found to be optimally optimized as the " inferior performing assets were avoided in favor of better-performing assets, and the portfolio's assets were weighted to maximize long-term average growth."

Li (2023) conducted a study on the United States stock market on how the Monte Carlo simulation is used for constructing an Efficient Frontier and optimizing the portfolios of investors in the market. Ding (2024), studies Portfolio optimization based on Markowitz Investment theory and the Monte Carlo simulation while focusing on China's market. The study aimed at finding the efficient frontier to achieve the optimization of the portfolio, and at the same time, the corresponding visualization is carried out to prove that the modeling algorithm has usability and reliability on the given data set.

In Kenya, various studies have been done on portfolio optimization. Mbithi (2014), sought to study the optimal portfolio size on the Nairobi Securities exchange. The study aimed to establish the number of securities required for maximum risk diversification. The author employed the use of Harry Markowitz's Mean-Variance portfolio theory. Furthermore, Odhiambo (2016), studied the effect of portfolio optimization on the returns of listed companies at the Nairobi securities exchange. The study employed Harry Markowitz's mean-variance optimization model to analyze the risk and returns of various portfolios.

Tesfamichael (2023) advanced a comparative study of Harry Markowitz's mean-variance optimization model and the Black-Litterman optimization model.

Although studies have been done in the Kenyan market to study portfolio optimization, many of them have employed the use of historical data; including the expected returns and volatility used, which were calculated from historical performances of the individual stock returns in the NSE making the model deterministic.

However, Munechika (2005), explained that the decision-making process in portfolio analysis majorly involves the use of ex-ante returns (future returns) and the uncertainty of these returns has to be quantified in the optimization process. Thus, these returns need to be described probabilistically.

This research aims to compare the performance of Harry Markowitz's mean-variance optimization model and the Black-litterman model while incorporating the Monte Carlo Simulation of asset returns.

1.3 Research Objective.

Main Objective.

The study aims to compare the performance of the Mean-Variance Optimization and the Black-Litterman while incorporating the Monte Carlo simulation in the optimization of portfolios using the stocks listed on the Nairobi securities to see which model results in more optimized portfolios.

Specific Objectives.

1. To construct optimized portfolios using the Mean-Variance optimization model and Monte Carlo simulated asset returns with stocks listed on the Nairobi Securities Exchange (NSE).
2. To construct optimized portfolios using the Black-Litterman model and Monte Carlo simulated asset returns with stocks listed on the Nairobi Securities Exchange (NSE).
3. To determine which of the two models results in more optimized portfolios using key performance metrics such as the expected returns, risk, and Sharpe ratio.

CHAPTER 2: LITERATURE REVIEW.

INTODUCTION.

This chapter will explore theories such as Harry Markowitz's mean-variance optimization theory, the Black-Litterman model, the Monte Carlo simulation, and the Sharpe Ratio. Furthermore, this chapter will go further to explore portfolio optimization in the Kenyan stock market and identify the gaps in the existing literature.

2.1 THEORETICAL ANALYSIS.

2.1.1 Mean-variance Optimization theory.

The mean-variance analysis was proposed by Harry Markowitz in 1952 in an attempt to explain investor behavior in constructing their optimal portfolios under uncertainty. The theory is known as the mean-variance analysis as it considers only two parameters: the expected returns and variance of return when constructing the optimal portfolio (Munechika,2005).

According to Markowitz (1952), there is a rule that implies that an investor should both diversify and maximize their expected returns. The rule states that the investor should diversify his funds among all those securities that give a maximum expected return. However, Markowitz (1952) states this rule cannot be accepted because the returns from securities are too intercorrelated thus diversification cannot eliminate all variance. This means that the portfolio with the maximum expected return is not necessarily the one with a minimum variance.

The E-V rule however, stated that given a set of portfolio combinations, "the investor would (or should) want to select one of the portfolios which give rise to the (E, V) combinations that are efficient". The portfolios that have minimum variance for a given return or maximum return for a given level of variance.

Markowitz (1991) wrote that uncertainty cannot be dismissed so easily in the analysis of optimizing investor behavior. He suggests that diversification is a common and reasonable investment practice to reduce uncertainty. Variance and standard deviation are considered a measure of the risk of the portfolio. From the discussion of the principles of rational behavior under uncertainty, Markowitz stated that "one should choose a strategy which maximizes expected utility for a many-period game".

The theory assumes the existence of a portfolio that gives both maximum expected return and minimum variance and recommends this portfolio to an investor.

Mbithi (2014), explained that in constructing efficient portfolios, investors are assumed to be risk averse, and when presented with two efficient portfolios with the same expected return they will prefer the less risky portfolio.

Markowitz's theory can be expressed as:

$$Minimize: \delta_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \delta_{ij}$$

Subject to

$$\sum_{i=1}^{n} w_i E(R_i) = E(R_p)$$

$$\sum_{i=1}^{n} w_i = 1$$

The equations above can be interpreted as minimizing the portfolio variance(risk) for a given level of portfolio return.

Weaknesses of Markowitz's Mean-Variance Optimization Theory.

Mishra, Pisipati, and Vyas (2011) explain that Markowitz's mean-variance optimization results in non-intuitive and extreme portfolio allocations which are highly sensitive to variations in the inputs.

2.1.2 Black-Litterman Theory

The Black-Litterman theory was advanced by Fisher Black and Robert Litterman in the year 1992 as they found that quantitative asset allocation models have not played the important role that they should in global portfolio management. The authors explain that an integral part of the problem was that such models were difficult to use and tended to result in portfolios that were badly behaved.

Black and Litterman (1992), explained that the Black-Litterman asset allocation model uses the Bayesian approach to infer the assets' expected returns. The approach treats the expected returns as random variables themselves.

He and Litterman (2005), in their research of the intuition behind Black-Litterman model portfolios, explained that the unconstrained optimal portfolio in the model is the scaled market equilibrium portfolio plus some weighted sum of portfolios representing the investors' views. The researchers went further to explain that the model provides the flexibility of combining the market equilibrium with additional market views of the investor. Furthermore, in the model, the CAPM equilibrium distribution is the prior and the investors' views are additional information. The Bayesian approach can be used in the model to refer to the probability distribution of expected returns using the CAPM prior and the additional investor's views.
Vladimirov, Stoilov, and Stoilova (2007), explain that the model combines the results of the modern portfolio theory and the Capital Asset Pricing model.

2.1.3 Sharpe Theory.

The Sharpe ratio theory was proposed by William F. Sharpe in the year 1968 in an attempt to construct a market equilibrium theory of asset prices under the conditions of risk. The theory assumes that individuals view the outcome of every investment in probabilistic terms while assessing the desirability of a particular investment. Furthermore, investors are assumed to prefer a higher expected future wealth to a lower value. They are further assumed to be risk averse; given an investment with the same expected returns, they choose an investment offering a lower value of standard deviation.
According to Sharpe (1968), the model of investor behavior considers the investor as choosing from a set of investment opportunities the one that maximizes his utility.
Over the years, the Sharpe ratio has been employed in various attempts to optimize portfolios. Despite the success of the Sharpe Ratio in optimizing portfolios, there have been various criticisms of the ratio. Deng, Dulaney, McCann, and Wang (2013), studied robust portfolio optimization with Value-at-Risk adjusted portfolios. The authors state that the traditional Sharpe ratio estimates use a limited series of historical returns which might be subject to estimation errors. Furthermore, the authors explain that portfolio optimization based on traditional Sharpe ratios ignores this uncertainty and as a result is not robust.
The Sharpe ratio can also be used as a measure of the portfolio's performance. Zakamouline and Koekebakker (2019), in their study of portfolio performance evaluation with generalized Sharpe ratios, explained that the Sharpe ratio is a commonly used measure of performance. They further

explain that the Sharpe ratio is a meaningful measure of portfolio performance when the risk can be adequately measured by standard deviation.

This study employs the Sharpe ratio to measure the difference in portfolio performance between portfolios modeled with the Monte Carlo simulation and those without.

The Sharpe ratio in its simplest form can be expressed as:

$$Sharpe\ ratio = \frac{E[R_p] - R_f}{\delta_p}$$

2.1.4 Monte Carlo Method.

The Monte Carlo method was invented by John von Neumann and Stanislaw Ulam in 1947 during the Second World War. The method was advanced in an attempt to improve decision-making under certain conditions. The theory was named after a well-known casino town, called Monaco. Metropolis and Ulam (1949), then went further in discussing the Monte Carlo Method. The approach, it is stated, does not concentrate on the individual particles but rather studies the properties of sets of particles. The authors stated that the Monte Carlo process is a combination of stochastic and deterministic flows. That is, "it consists of repeated applications of matrices-like in Markov chains-and completely specified transformations". Metropolis and Ulam (1949) stated that the mathematical description of the method is the study of a flow that consists of a mixture of deterministic and stochastic processes which requires its laws of large numbers and asymptotic theorems.

Furthermore, the authors stated that the method allows one to obtain the values of certain given operators on functions that obey a differential equation, without exact knowledge of the functions which are the solution to the equations. The authors stated that the asymptotic theorems provide the analogs of the laws of large numbers, such as the Bernoulli and Cantelli-Borel. Since its introduction, the Monte Carlo simulation has been used to assess the impact of risk in many real-life situations such as stock prices, sales forecasting, project management, and even pricing.

Kroese, Brereton, Taimre, and Botev (2014), conducted a study that delved into why the Monte Carlo simulation is important.

Our study will employ the Monte Carlo simulation in the generation of random returns that we will use in the study of optimal portfolios.

2.2 EMPIRICAL ANALYSIS.

Portfolio optimization can be defined as the procedure of measuring and controlling portfolio risk and expected return (Speidell, Miller, and Ullman 1989). Mbithi (2014), sought to study the optimal portfolio size on the Nairobi Securities exchange. The study aimed to establish the number of securities required for maximum risk diversification. The study employed the MVO model and secondary data from the NSE between January 2009 to December 2013. The study concluded that the portfolio risk decreased as the number of securities in the portfolio increased. Odhiambo (2014), studied the effect of portfolio optimization on the returns of listed companies at the Nairobi securities exchange. The study employed Harry Markowitz's mean-variance optimization model to analyze the risk and returns of various portfolios and used data from companies listed at the Nairobi securities exchange from the period July 2008 to June 2013. The study found a "significant difference between return, risk and risk-adjusted returns of optimal portfolio and the GMVP and naive (1/N) portfolio thus portfolio optimization leads to better absolute, and risk-adjusted returns".

Masese (2017), sought to study a comparison between the mean-variance optimization and threshold accepting (a general optimization model). The study compared the two models by measuring their performance using the Sharpe ratio, Sortino ratio, and information ratio using twenty-nine stocks listed in the Nairobi Securities exchange from the year 1998 to 2016. The study concluded that the threshold-accepting model performed better than the MVO (mean-variance optimization) model. The author recognized that the study was historical and thus suggested conducting studies of a forward-looking design.

Satchell and Scowcroft (2007), in their study of a demystification of the black-litterman model, explained that the model has the potential to incorporate diverse approaches that are based on the Bayesian methodology that updates currently held opinions with data to form new opinions. Furthermore, Mishra, Pisipati, and Vyas (2011) in their study of an equilibrium approach for tactical asset allocation explain that the Black-Litterman approach enabled them to incorporate investment views(subjective) and assign confidence levels to the views at the modeler's discretion.

Vladimirov, Stoilov, and Stoilova (2017), in the study of formal description of expert views of the Black-litterman Asset allocation model explain that the current view of the model integrates historical asset returns data with subjective views of the investors about the future behavior of these returns.

However, despite the advancements made in both Markowitz's mean-variance optimization model and the black-litterman model, both models employ the use of historical returns. By incorporating the Monte Carlo simulation into the modeling of portfolio performance, portfolio managers can get more accurate predictions of the prices of assets in their portfolio thus improving their performance.

The Monte Carlo simulation has been implemented in various studies of portfolio optimization over the years in different countries.

Munechika (2005), conducted a study on stochastic mean-variance optimization in portfolio analysis. The study aimed to introduce a method of stochastic optimization by applying the Monte Carlo simulation while analyzing the future performance of asset returns to determine an efficient set of portfolios. The study found that stochastic optimization provides the forecasts of the objectives probabilistically.

Li (2023) studied the portfolio optimization of five stocks in different industries listed on the New York Stock Exchange by Monte Carlo simulation. The Monte Carlo simulation was used to construct the Efficient Frontier and find the weights of the Maximum Sharpe Ratio portfolio and Minimum Variance portfolio. Furthermore, Ding (2024), studied portfolio optimization based on Markowitz investment theory and the Monte Carlo simulation using five stocks listed on the Chinese market over the pandemic period. The study aimed to show that portfolio optimization, especially during the highly volatile period, would protect investor funds. The study concluded that the Monte Carlo algorithm based on Markowitz's investment theory successfully demonstrated the optimized portfolio for the five stocks. Furthermore, the author explains that the method obtains the optimal high-risk-high-return portfolio with the highest Sharpe ratio.

Research Gap.

Over the years researchers in Kenya have attempted to understand how portfolio optimization in the markets works. How the portfolio managers and sometimes individual investors allocate their funds to various assets to ensure an optimal portfolio; one with minimum risk and maximum return. Mbithi (2014), Odhiambo (2014), and Masese (2017) all conducted studies on portfolio optimization in the Kenyan market using Harry Markowitz's mean-variance optimization theory. The majority of these studies employ the use of the Sharpe ratio as a measure of portfolio performance.

These researchers, despite their extensive efforts in understanding portfolio optimization, acknowledge the disadvantages of the mean-variance optimization theory and suggest various models for further investigation. Tesfamichael (2023) explains that the researchers advocate for the incorporation of the Black-Litterman model.

However, despite the extensive efforts aimed at understanding the optimization of portfolios, there is a very noticeable gap in the application of advanced computational techniques such as the Monte Carlo simulation. Most of the studies have focused on the use of historical data in their analysis without incorporating stochastic components in their analysis.

Given the very dynamic nature of the markets, there is a need to explore where integrating the Monte Carlo algorithm can improve the accuracy of the optimization models. Due to the limited evidence within the Kenya context on the effectiveness of this method, the study intends to address this gap by investigating whether the use of Monte Carlo simulations in the comparison of Markowitz's mean-variance optimization model and the Black-Litterman model will result in more optimized portfolios thus resulting in better risk-adjusted returns for the investors.

CHAPTER 3: METHODOLOGY.

3.1 INTRODUCTION

This chapter described the various methodologies that were adopted in this study; this included the analysis and assessment of the impact of the Monte Carlo simulation in the comparison of Harry Markowitz's mean-variance optimization model and the Black-Litterman model in the Kenyan stock market.

3.2 Research Design.

The study adopted a quantitative and comparative research design. Quantitative analysis is an approach whereby objective theories are tested by examining the relationship between various variables. The variables in question are then measured so that the numbered data can be analyzed using statistical procedures. The study was quantitative as it involved the analysis of various stock data listed on the Nairobi Securities Exchange for us to construct optimal portfolios. The research was also comparative as it made a comparison between two optimization models; to see which one resulted in a more optimal portfolio. The models employed were Markowitz's mean-variance optimization model and the black-litterman model.

3.3 POPULATION AND SAMPLING.

3.3.1 Population.

The study focused on how the Monte Carlo simulation can improve the mean-variance optimization theory and the black litterman theory, and whether implementing the Monte Carlo simulation would result in more optimized portfolios in the Nairobi Securities exchange. The study focused on the various stocks listed on the Nairobi Stock Exchange between the years 2009-2019. The timeline of the study was chosen arbitrarily although it captured some major events that affected the prices of stocks in the market. These included the 2013 and 2017 general elections. Kabiru (2015), explains that the performance of a stock market is of interest to various parties including investors, capital markets, the stock exchange, and the government among others.

3.3.2 Sample

The Nairobi Stock Exchange has sixty-five securities under thirteen sectors that are listed on the exchange. However, in this study, we used nine performing securities from nine different sectors to construct optimized portfolios. By choosing securities in the different sectors, the portfolios are diversified in a way that ensures the behavior of the securities will not be positively correlated over time. The securities selected from each sector represent the best-performing companies in that sector.

The securities that were selected for the study include:

Table 1: Table exhibiting the selected stocks to be used in the study.

Security	Sector
Sasini	Agricultural Sector
Safaricom	Telecommunication
East Africa Breweries Limited	Manufacturing and allied sector
Britam Holdings	Insurance sector
KenGen	Energy and Petroleum sector
Equity Bank	Banking
Centum Investments	Investment services sector
Standard Group	Commercial and services sector
Unga Group	Food sector

3.4 DATA ANALYSIS

3.4.1 Model Development.

Estimating parameters.

The historical price data of the selected stocks from the Nairobi Stock Exchange was used to calculate the daily log returns of the different stocks.

The log returns were calculated using the formula:

$$r_t = ln\left(\frac{P_{t+1}}{P_t}\right)$$

Where:

P_{t+1} Is the price at the end of period t+1

P_t Is the price at the beginning of the period t

The data set was then divided into the training and the testing dataset. Using the training dataset the mean return and the covariance matrix of the stocks were computed.

To compute the mean return for each asset, the periodic returns were summed up and divided by the number of periods. This can be expressed as:

$$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

Computing the covariance matrix:

$$Covariance\ Matrix = \begin{bmatrix} \delta_1^2 & \cdots & \delta_{1n} \\ \vdots & \ddots & \vdots \\ \delta_{1n} & \cdots & \delta_n^2 \end{bmatrix}$$

The covariance matrix helped us understand how the returns move together (correlation between two or more assets). Tesfamichael (2023) explained that the covariance matrix offers a measure of how much two or more stocks move together.

3.5.2 Simulating Asset Returns.

3.5.1.2 The Monte Carlo Simulation.

The Monte Carlo simulation is said to be a powerful technique for analyzing models that involve probabilistic assumptions. The simulation's assumptions capture the uncertainty of the ex-ante returns using a probability distribution and the forecasts of the objective will also have some probability distributions of the possible returns for the model.

The simulation works by using historical data to generate random numbers that follow the distribution of the stock returns.

GARCH MODEL.

GARCH models, also known as the Generalized Autoregressive Conditional Heteroskedasticity models are widely used in the financial sector to model and forecast the volatility of asset returns.

The GARCH (1,1) model is generally preferred for simulating asset returns because of the ability to capture heteroskedasticity (time-varying volatility) in contrast to other models such as ARMA that assume homoskedasticity(constant volatility over time).

Furthermore, the GARCH (1, 1) model captures the effect of random shocks in the market because of the autoregressive and moving average components, unlike the other models that are less suited to the dynamic nature of financial return volatility.

The very basic idea behind the GARCH models is that they allow for the volatility (standard deviation) of asset returns to change over time; a common feature that is observed in the financial markets.

The GARCH model has two main components: the mean equation and the variance equation. The mean equation represents the equations for the returns themselves, often modeled as a constant or as a function of the past returns. For simplicity purposes, we assumed that the returns follow a simple AR (1) process. This can be expressed as:

$$r_t = \mu + \phi r_{t-1} + \varepsilon_t$$

Where:

r_t Is the return at the time t

μ Is the mean return

ε_t Is the residual also known as the error term.

$\emptyset$ Is the autoregressive parameter.

The error term is assumed to follow a normal distribution with a mean of zero and a time-varying variance.

The GARCH (1,1) model for the variance is given by:

$$\sigma_t^2 = w + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Where:

W is a constant

α Is the coefficient for the lagged squared residual (ARCH term)

β Is the coefficient for the lagged variance (GARCH term)

To estimate asset returns using a GARCH model, we first needed to estimate the unknown parameters above and this was done using the historical data.

Once the parameters were estimated, we went about simulating the asset returns as follows:

We set the initial values for r_0 and σ_0^2 based on the historical data and generated a series of random variables z_t from a standard normal distribution.

For each time step t, we calculated the return. r_t and updated the variance σ_t^2Using the estimated GARCH model parameters.

$$r_t = \mu + \emptyset r_{t-1} + \sigma_t z_t$$

$$\sigma_t^2 = w + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Where $\varepsilon_t = r_t - \mu - \emptyset r_{t-1}$

The process is continued for the desired number of time steps (which in the study was equal to the test data time steps) to generate a time series of simulated asset returns. The risk and return of these simulated portfolios were determined and compared to the results of the testing dataset.

3.5.3 Optimization of Portfolios.

3.5.3.1 Harry Markowitz Mean-Variance portfolio optimization.

The optimization problem in this study involved minimizing the portfolio variance for a given portfolio return known as the target return. The target return in the study was taken as the average of the mean returns. In its simple version, the Markowitz mean-variance optimization model is written as follows:

$$Min: \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Subject to:

$$E[R_P] = \sum_{i=1}^{n} w_i \, E[R_i] = T$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0 \;\; i = 1, \ldots, n$$

Where R_i Refers to the return to the individual return of an asset i.

w_i Refers to the weight of individual asset i.

w_j Refers to the weight of the individual asset j.

σ_{ij} Refers to the covariance between the assets i and j.

And T is our target return.

The equations above were interpreted as minimizing the variance(risk), of a portfolio while ensuring that it meets a specific return (target return) and adheres to the constraints of full investment and non-negativity weights.

The non-negativity constraint implied that no short selling was allowed implying that all the asset weights must be non-negative. The budget constraint ensured that the total weight of all the assets in the portfolio sum up to one.

The above model was used in conjunction with both the training dataset and the simulated datasets to generate optimal portfolio weights.

The “optimize portfolio” function in R that employs quadratic programming was used to solve the optimization problem where the objective function was defined and the constraints on the asset’s weights (sum up to one and non-negativity assumptions) were set. Finally, the optimal weights that minimize the risk of the portfolio, were obtained.

These optimal portfolio weights were applied to both the testing dataset and the simulated data set and then used to calculate the portfolio risk and return for the two datasets.

3.5.3. Black Litterman Model

Subsequently, we employ the use of the Black-Litterman model to create other portfolios to compare the performance of the two models.

The first step involves estimating implied equilibrium returns based on market capitalization weights. These returns assume that the market portfolio reflects the consensus of all the investors and is in equilibrium.

The formula to compute the implied equilibrium returns is given by:

$$\pi = \lambda \sum w_{mkt}$$

Where:

π represents the implied equilibrium returns

λ Represents the risk aversion coefficient.

$\sum$ represents the covariance of asset returns

w_m Represents the vector of the market capitalization weights.

The risk aversion coefficient is calculated as:

$$\lambda = \frac{E(r_m) - r_f}{\sigma_m^2}$$

Where:

$E(r_m)$ represents the expected return of the market

r_f Represents the risk-free rate; the 91-day government treasury bill rate from the Central Bank of Kenya ($R_f = 0.09525$).

σ_m^2 represents the variance of the market return

Then we needed to specify investor views (subjective views) on how certain assets would perform.

The core of the model above combined the market equilibrium returns with an investor's subjective views using a Bayesian framework. The process produced a new set of expected returns which adjusts the equilibrium returns based on how much weight you want to place on your views.

The formula for calculating the expected Black-Litterman returns is given as follows:

$$\mu_{BL} = ((\tau\sum)^{-1} + P^T\Omega^{-1}P)^{-1}((\tau\sum)^{-1}\pi + P^T\Omega^{-1}Q)$$

Where:

μ_{BL} Represents the Black Litterman expected returns

$\sum$ represents the covariance matrix of asset returns

τ is a scalar representing the uncertainty in the market equilibrium returns

P is a matrix representing the investors' views on assets

π represents the implied equilibrium returns

Q represents a vector's expected returns on return differentials.

The final part of this optimization process involved using Markowitz's mean-variance optimization to find the optimal weights to balance the expected return and risk.

The mean-variance optimization problem was defined as:

$$max_w \left(w^T \mu_{BL} - \frac{\lambda}{2} w^T \sum w \right)$$

Where:

W is the vector of portfolio weights for each asset.

μ_{BL} represents the Black-Litterman expected returns

$\sum$ represents the covariance matrix of asset returns

λ Represents the risk aversion coefficient.

3.5.4 Testing the Performance of the Portfolios.

Once the risk and return of the portfolios were obtained, the performance of the portfolios was tested using the Sharpe ratio which is given by:

$$Sharpe\ ratio = \frac{E[R_p] - R_f}{\delta_p}$$

Where:

$E[R_p]$ Is the expected return of the portfolio.

R_f Is the risk-free rate.

δ_p Is the volatility of the portfolio.

CHAPTER 04: DATA ANALYSIS, RESULTS AND DISCUSSION.

4.1 Introduction.

The chapter explores the data collected and the performance of two optimization models, the Mean-Variance optimization model (MVO) and the Black-Litterman model, using various stocks listed on the Nairobi Securities Exchange (NSE). Furthermore, we delve into the empirical findings and interpretations of the portfolio construction and optimization methods.

4.2 Descriptive Analysis

4.2.1 Variable Selection and Transformation.

The stock data employed was the daily closing price from the Nairobi Securities Exchange. The log returns are obtained from the stock price data and used in the portfolio optimization models. The study employs the use of log returns as they are easier to aggregate when compared to simple returns. A total of nine performing stocks from the period 2009 to 2019 from the different sectors were employed in the study.

The primary objective of this analysis is to discern trends, patterns, and variations in stock prices. Furthermore, this part of the analysis offers important insights into the performance of individual companies in the Kenyan stock market.

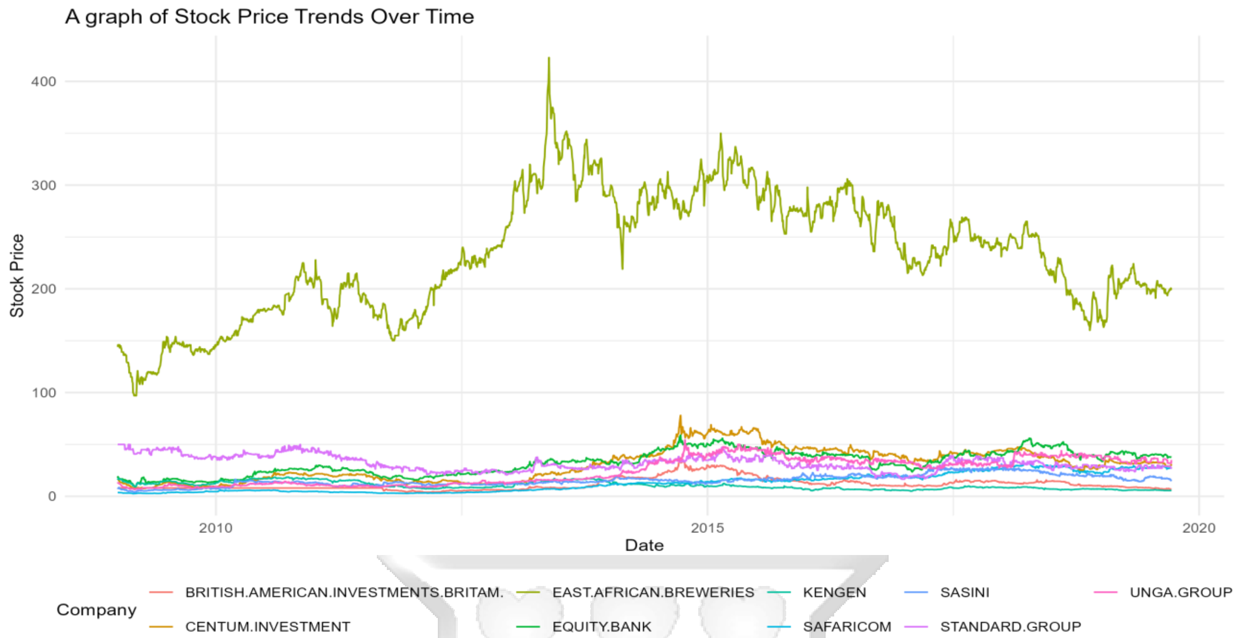

Figure 1: A Graph of Stock Price Trends during the period 2009 to 2019

The line graph above visually captures the dynamic movements in the daily stock prices for the nine chosen companies over the period. Each line in the graph represents a specific company providing a comprehensive view of the individual price trends, potential patterns, and even fluctuations over the observed time frame. The study observed that the EABL generally has higher prices than the other stocks, while Safaricom shows very noticeable trends with both increasing and decreasing trends.

Furthermore, the study examined the statistical measures of the data. Some of the key characteristics we aimed to analyze in the data set include the mean, the median, the variance, the skewness, kurtosis, and the maximum and minimum range. The summary of the price dataset is listed in the table below.

Table 2: Descriptive statistics of the stock prices over the time horizon 2009-2019

Stock	mean	median	std	variance	max	min	range	skewness	kurtosis
SAFARICOM	12.98	12.45	0.1683	79.27	32.75	2.550	30.20	0.4662	1.851
EABL	232.2	232.0	1.103	3403	423.0	97.00	326.0	-0.0139	2.261
EQUITY.BANK	32.35	33.25	0.2143	128.5	59.00	9.350	49.65	-0.0277	1.921
BRITAM.	11.55	9.520	0.1111	34.54	37.00	3.800	33.20	1.516	5.073
CENTUM.INVESTMENT	30.40	31.50	0.2938	241.4	78.00	7.020	70.98	0.3953	2.225
KENGEN	10.39	9.300	0.0678	12.88	18.50	4.950	13.55	0.5831	2.108
SASINI	15.51	14.55	0.1022	29.19	31.25	4.000	27.25	0.3064	2.730
STANDARD.GROUP	32.03	30.50	0.1363	51.96	50.00	16.50	33.50	0.3118	2.426
UNGA.GROUP	24.29	26.00	0.2407	162.1	54.50	6.500	48.00	0.1132	1.434

From the above table, it is observed that East Africa Breweries Limited had the highest mean and median indicating a consistently high price over the other stocks. KenGen had the lowest mean and median which reflected its lower pricing range. Furthermore, the mean and the median were close for many of the stocks suggesting symmetry in their price distributions except for BRITAM where the mean was slightly higher than the median.

It is also observed that EABL had the highest standard deviation and range which indicated that it was the most volatile stock in the study while. In contrast, KenGen had the lowest standard deviation which indicated that it was the least volatile stock.

Some of the key takeaways from the study were that EABL was the most volatile and expensive stock, which in turn made it very suitable for risk-tolerant investors who seek high returns. KenGen and Sasini prove to be more stable with their smaller variances, hence might seem appealing to risk-averse investors.

To capture relative changes in stock prices, daily log returns were computed for each of the nine stock prices. Log returns are preferred over simple price returns because they are more accurate representations of continually compounded returns. This offers symmetrical perspectives of both the negative and positive movement in the prices.

The descriptive statistics for the log returns were computed to understand key insights into the characteristics of their performance. This summary includes essential characteristics such as their mean, median, standard error, variance, standard deviation, maximum and minimum values, range, and kurtosis. The table below shows a more condensed presentation of these measures.

Table 3: Descriptive statistics of the log returns over the time horizon 2009-2019

Stock	mean	median	std	variance	max	min	range	skewness	kurtosis
SAFARICOM	0.0007	0	0.0003	0.0002	0.1043	-0.0809	0.1851	0.1637	6.7190
EABL	0.0001	0	0.0003	0.0002	0.0935	-0.0862	0.1797	-0.0081	8.7280
EQUITY.BANK	0.0003	0	0.0003	0.0003	0.1521	-0.1046	0.2567	-0.0157	11.2000
BRITAM.	0.0000	0	0.0004	0.0004	0.2311	-0.1001	0.3312	0.6946	14.8660
CENTUM.INVESTMENT	0.0002	0	0.0004	0.0004	0.0997	-0.1035	0.2031	0.1053	7.0120
KENGEN	-0.0004	0	0.0004	0.0004	0.1813	-0.1481	0.3294	0.4547	12.4200
SASINI	0.0003	0	0.0005	0.0008	0.1252	-0.1734	0.2985	-0.0931	5.5030
STANDARD.GROUP	-0.0002	0	0.0006	0.0011	0.0953	-0.1054	0.2007	-0.0555	4.9160
UNGA.GROUP	0.0003	0	0.0005	0.0008	0.2485	-0.1054	0.3538	0.1605	6.9570

From the analysis, it is observed that Safaricom had the highest mean returns while Britam had the lowest returns with three stocks exhibiting negative returns. These stocks are Britam, Standard Group, and KenGen. In terms of their volatility, the Standard group had the highest value while the EABL had the lowest value for the volatility. All the returns are skewed however they prevent different skewness. Some of the stock returns are positively skewed while others are negatively skewed. Moreover, all the stocks have positive kurtosis.

4.2.2. Test for Stationarity.

The next step of the analysis involved computing the stationarity tests. The stationarity test eliminates the possibility of factitious regression. The ADF test was employed in the analysis. We conducted these tests on all the stock returns. The results are displayed in the table below. From the results below, the null hypothesis is rejected therefore, all our variables are stationary.

Table 4: Table showing the results of the Stationarity Test

Stock	Test Statistic	P Value	Interpretation
SAFARICOM	-14.8518	0.01	Stationary (Reject H0)
EAST.AFRICAN.BREWERIES	-14.9484	0.01	Stationary (Reject H0)
EQUITY.BANK	-13.6711	0.01	Stationary (Reject H0)
BRITISH.AMERICAN.INVESTMENTS	-14.0053	0.01	Stationary (Reject H0)
CENTUM.INVESTMENT	-13.3769	0.01	Stationary (Reject H0)
KENGEN	-14.4698	0.01	Stationary (Reject H0)
SASINI	-15.3942	0.01	Stationary (Reject H0)
STANDARD.GROUP	-16.1327	0.01	Stationary (Reject H0)
UNGA.GROUP	-15.3827	0.01	Stationary (Reject H0)

The subsequent phase of the analysis involved computing the covariance and correlation matrices for the log returns for the stocks. The correlation and covariance are crucial metrics as they quantify the relationship between the stocks which directly impacts risk management and portfolio diversification. The covariance measures how two assets move relative to each other. The covariance matrix in Appendix 2 offers an insight into the variability of different pairs of stocks. The correlation matrix as illustrated in Appendix 3 on the other hand explains insights into the direction and strength of the relationship between the stocks.

4.2.3 GARCH simulation of asset returns.

The next phase of the analysis involved the simulation of asset returns using the GARCH model. The initial step of this analysis involved initializing the model parameters. In this scenario, they included the number of days to be simulated and the number of assets to be simulated. The number of days to be simulated was equal to the number of days in the testing dataset and the number of assets to be simulated was the same as the number of stocks for the test data set.

4.2.3.1 Test for ARCH effects

Before the GARCH model was fitted onto the training dataset, it was tested for the presence of ARCH effects to justify the use of the GARCH model. The first step was to use the Lagrange Multiplier (LM) test, which is commonly referred to as the ARCH test.

The hypotheses to be tested are:

$$H_0: No\ ARCH\ effects\ are\ present (homoskedastictity)$$

$$H_1: ARCH\ effects\ are\ present (heteroskedasticity)$$

The results of this test are displayed in the table below.

Table 5: Table showing the results of the ARCH test.

Stock	Test Statistic	P Value	Conclusion
SAFARICOM	71.39	0	Significant ARCH effects detected
EAST.AFRICAN.BREWERIES	164.1	0	Significant ARCH effects detected
EQUITY.BANK	201.5	0	Significant ARCH effects detected
BRITISH.AMERICAN.INVESTMENTS	103.8	0	Significant ARCH effects detected
CENTUM.INVESTMENT	193.3	0	Significant ARCH effects detected
KENGEN	95.57	0	Significant ARCH effects detected
SASINI	62.66	0	Significant ARCH effects detected
STANDARD.GROUP	26.73	0	Significant ARCH effects detected
UNGA.GROUP	27.84	0	Significant ARCH effects detected

Since the P values are less than 0.05, the null hypothesis is rejected, and the conclusion is that ARCH effects are present thus justifying the use of the GARCH model.

4.2.3.2 Fitting the GARCH model.

The GARCH (1,1) model was fitted onto the training data set and then the fitted GARCH model was used to simulate returns for the assets. The results are shown below.

Table 6: Table showing the Parameter Estimates for each stock.

	mu	omega	alpha1	beta1
SAFARICOM	0.0009	0.0001	0.1905	0.3301
EAST.AFRICAN.BREWERIES	0E+00	9.46E-05	0.2445	0.2566
EQUITY.BANK	0.0004	9.08E-05	0.3167	0.4320
BRITAM	-9E-05	5.65E-07	0.0376	0.9614
CENTUM.INVESTMENT	0.0001	8.93E-05	0.2454	0.5577
KENGEN	-0.0009	8.41E-05	0.1976	0.5709
SASINI	0.0002	0.0002	0.1619	0.6193
STANDARD.GROUP	0.0004	1.32E-06	0.0029	0.9961
UNGA.GROUP	0.0002	0.0002	0.1038	0.6868

Where GARCH (1,1) was used to simulate asset returns, the null hypothesis was related to the adequacy of the GARCH model in capturing the volatility dynamics of the asset returns. The adequacy of the model is tested using residual characteristics.

The test undertaken is known as the Box test for residual and squared residuals. The hypotheses to be tested are:

$$H_0: no\ autocorrelation\ in\ residuals\ or\ squared\ residuals$$

$$H_1: autocorrelation\ in\ residuals\ or\ squared\ residuals$$

The results obtained from the analysis are:

Table 7: Results of the Ljung-Box Test.

Test	Chi-square	df	P Value
Standardized Residuals	28.49	10	0.0015
Squared Standardized Residuals	2.572	10	0.9898

For the Ljung-Box test, the p values were greater than 0.05 hence we failed to reject the null hypothesis suggesting that there was no significant autocorrelation. This indicated that the GARCH model had adequately captured the autocorrelation and thus the model is adequate. Since the P-value of these tests was greater than 0.05 it suggested that the GARCH (1,1) model was adequate in modeling asset returns since there was no evidence of remaining autocorrelation. By using the GARCH to model the returns, the simulated return series reflected the statistical properties of the historical data such as the return and volatility.

4.2.3.3 Testing whether the GARCH(1,1) model is Parsimonious

To test whether the above model is parsimonious, we compared the fitted GARCH model against a null model (with no GARCH) effect. The likelihood ratio test was employed in this study. The results are displayed below:

Table 8: Table showcasing the Likelihood Ratio Results.

Stock	LR_Statistic	P value
SAFARICOM	108.3	1.65E-22
EAST.AFRICAN.BREWERIES	166.1	7.19E-35
EQUITY.BANK	355.7	1.02E-75
BRITISH.AMERICAN.INVESTMENTS	961.5	7.83E-207
CENTUM.INVESTMENT	269.1	5.04E-57
KENGEN	248.2	1.55E-52
SASINI	132.0	1.43E-27
STANDARD.GROUP	9.745	4.49E-02
UNGA.GROUP	79.11	2.69E-16

From the above results, the small P values that are less than 0.05 indicate that the GARCH model provides a significantly better fit for the stocks' returns.

4.3 Optimal Portfolio Selection.

The mean-variance optimization model and the Black-Litterman model were used to construct optimal portfolios using the selected stocks.

4.3.1 Mean-Variance Optimization Model.

This model, developed by Harry Markowitz in the 1950s, highlights the importance of expected returns (mean) and portfolio risk (standard deviation) in portfolio selection. The model attempts to design portfolios that offer investors the maximum expected return for a given level of risk or the lowest risk for a given level of return.

The first step of the analysis involves obtaining the log returns, which are calculated as the average(mean) return for each stock across all periods. The summary of the mean log returns is as follows:

Table 9: Summary of the Mean Returns

Stock	Mean Returns
SAFARICOM	0.0007
EABL	0.0002
EQUITY.BANK	0.0000
BRITAM.	-0.0003
CENTUM.INVESTMENT	0.0001
KENGEN	-0.0007
SASINI	0.0004
STANDARD.GROUP	0.0005
UNGA.GROUP	0.0003

In our results, we observe that Safaricom, EABL, Centum Investments, Sasini, Standard group, and the Unga group all exhibited positive returns. A positive return shows that the stock gained value over the specified period. The observation of a positive return meant that there was a capital appreciation for the stocks.

However, three of the stocks, Equity Bank, Britam, and KenGen exhibited negative returns which indicated that the value of the security had decreased over time. Some of the reasons that can cause this include economic obstacles or issues specific to a company.

The next step in the study involved obtaining the risk of the assets which involved calculating the covariance matrix. The covariance matrix is displayed below.

Table 10: The Covariance Matrix of the Returns.

	SAFARICOM	EABL	EQUITY.BANK	BRITAM.	CENTUM	KENGEN	SASINI	STANDARD.GROUP	UNGA.GROUP
SAFARICOM	0.00025	3.03E-05	4.70E-05	2.41E-05	3.84E-05	3.01E-05	1.19E-05	2.36E-06	3.87E-06
EABL	0.00003	1.85E-04	4.20E-05	1.44E-05	2.14E-05	2.99E-05	9.84E-06	4.01E-06	4.05E-06
EQUITY.BANK	0.00005	4.20E-05	3.34E-04	5.59E-05	6.80E-05	5.34E-05	1.50E-05	1.97E-05	7.06E-06
BRITAM.	0.00002	1.44E-05	5.59E-05	4.32E-04	7.50E-05	3.31E-05	1.05E-05	1.85E-05	1.47E-05
CENTUM.INVESTMENT	0.00004	2.14E-05	6.80E-05	7.50E-05	4.19E-04	4.19E-05	2.09E-05	2.99E-05	3.58E-05
KENGEN	0.00003	2.99E-05	5.34E-05	3.31E-05	4.19E-05	3.53E-04	3.39E-05	1.25E-05	-2.46E-06
SASINI	0.00001	9.84E-06	1.50E-05	1.05E-05	2.09E-05	3.39E-05	7.46E-04	2.65E-06	-6.33E-06
STANDARD.GROUP	0.00000	4.01E-06	1.97E-05	1.85E-05	2.99E-05	1.25E-05	2.65E-06	1.12E-03	3.06E-05
UNGA.GROUP	0.00000	4.05E-06	7.06E-06	1.47E-05	3.58E-05	-2.46E-06	-6.33E-06	3.06E-05	8.40E-04

The diagonal elements represent the variances of each asset returns while the off-diagonal elements show the covariances between the different stocks. From our analysis, we observe that the stocks that have higher values of their variance the more variability.
The covariance matrix is used in conjunction with the expected return to calculate a portfolio's volatility or risk. Furthermore, it is used to construct efficient frontiers, thus helping investors obtain a balance between risk and return for a given set of stocks.

The study followed the model described using the Markowitz equation to find the optimal portfolio weights under this particular model using an optimization function. The following stocks were allocated different optimal weights.

Table 11: Optimal Weights under the MVO model

ASSET	WEIGHT
Safaricom	16.20%
East African Breweries(EABL)	26.49%
Equity Bank	8.550%
British American Investments(BRITAM)	10.15%
Centum Investments	7.290%
KenGen	12.68%
Sasini	7.000%
Standard Group	4.700%
Unga Group	6.910%

The table above shows that the EABL (East African Breweries) stock and the Safaricom stock received the largest allocations, respectively. EABL is in the manufacturing sector, while Safaricom is in the financial services sector.

Finally, the above optimal weights were applied to the testing data set and the simulated data set, and the following risks and returns were obtained:

Table 12: Summary of the MVO return and risk

	Portfolio Return	Portfolio Risk
Testing Data	0.0260%	0.8300%
Simulated Returns	0.6200%	0.6200%

4.3.2 Black-Litterman Model

The Black-Litterman model builds on Markowitz's modern portfolio theory but considers investor views. Furthermore, the model considers the efficiency of the portfolio frontier and believes that the tangency portfolio exists for a reason.

The next step involves calculating implied returns, which are derived from the covariance matrix and the market weights using the formulas listed in equation 3.2. We then define the subjective views using a views matrix, as illustrated in Appendix 4. The views matrix specifies how each view relates to the assets. Moreover, omega, a diagonal matrix quantifying the uncertainty of each view, is defined. The lower the values, the greater the confidence in the subjective views.

The next step in the analysis includes computing the black-litterman adjusted returns using the formula listed in equation 3.3. This process involves the combination of the implied returns and the subjective views.

In the analysis, we use the training data set to obtain the optimal allocation to each stock under this particular model. The objective of the optimization function involves minimizing portfolio variance for a given target return. In this scenario, the target return is set as the mean of the Black-Litterman adjusted returns. The weights computed using the mean-variance optimization function allocated to each data set are displayed below.

Table 13: Optimal Weights under the B-L model

Stock	Weight
SAFARICOM	13.70%
EAST.AFRICAN.BREWERIES	13.69%
EQUITY.BANK	10.24%
BRITISH.AMERICAN.INVESTMENTS.	10.49%
CENTUM.INVESTMENT	9.719%
KENGEN	10.77%
SASINI	10.07%
STANDARD.GROUP	12.98%
UNGA.GROUP	8.335%

We find that using the Black-Litterman model, Safaricom received the majority share, followed closely by East Africa Breweries Limited. This is different from what we noticed using the Markowitz mean-variance optimization model.

The weights of the stocks obtained through the Black-Litterman model were applied to the test data and the simulated data to see which and if the data set yielded better results. Given the optimal weights allocated, we apply them to both the testing data set and the simulated data set to obtain the following returns and risks:

Table 14: Summary of the B-L return and risk

	Portfolio Return	Portfolio Risk
Test data	0.0002	0.0088
Simulated data	-0.0003	0.0074

4.4 Comparison of the Performance of the Two Models.

Once we obtained the risk and returns of the two models, the Sharpe ratio was calculated for the four portfolios. These two ratios help us to understand the performance of the various portfolios. The following results are obtained for the Mean-Variance Optimization Portfolios:

Table 15: Mean-Variance Optimization Sharpe Ratio

	Sharpe Ratio
Test Sharpe Ratio	-11.82
Simulated Sharpe Ratio	-15.76

From the table above, we observe that the Sharpe Ratio on both the testing data set and the simulated data set is negative. When the Sharpe ratio is negative, it indicates that the portfolio earns a return that is less than the risk-free rate of return given the risk taken.

The negative Sharpe ratio signals to an investor that the returns generated by the portfolio of stocks fail to exceed the risk-free rate of return and do not compensate for the risk undertaken by the investor.

For the Black-Litterman model, the Sharpe ratios for the two portfolios obtained were:

Table 16: Black-Litterman Sharpe Ratio.

	Sharpe Ratio
Test Data	-11.16
Simulated Data	-13.27

Even with the Black-Litterman model, we observed that the Sharpe ratio was still negative. This implied that the portfolio earns a less-than-ideal return given the risk that the investor is taking. As explained above, the negative Sharpe ratio implies that the investor earns a less-than-expected return.

CHAPTER 05: SUMMARY AND CONCLUSIONS

5.1 Introduction

The chapter below summarizes the study's key findings, conclusions, recommendations, and potential suggestions for further research.

5.2 Summary of Major Findings.

The findings from the study on the comparative analysis of portfolio optimization in the Nairobi Securities Exchange (NSE), which used both the mean-variance optimization model and the Black-Litterman model, divulge key insights into the methodologies' performance.

The study began by exploring the behavior of the daily stock prices of nine companies listed on the Nairobi Securities Exchange from 2009 to 2019. The descriptive analysis that included the exploration of log returns and various statistical measures provides an in-depth understanding of the performance characteristics of the individual stocks.

The subsequent part explored the use of the Black-litterman and MVO for the best possible portfolio selection. The mean-variance optimization model developed by Harry Markowitz emphasizes the combination of assets that achieves an optimal tradeoff between risk and returns. The B-L model developed by Fischer Black and Robert Litterman builds upon the modern portfolio theory by incorporating investor views into the optimization process. However, the research findings show difficulties in applying the Black-Litterman model, even with its theoretical foundations. The study does, however, recognize certain limitations, including sensitivity to the choice of the risk-free rate and the assumption that historical returns will be expected in the future. The study attempted to overcome this limitation of historical returns by employing the use of GARCH-simulated asset returns and compared the performance of the portfolios to see if they yielded better-performing portfolio returns.

5.3 Discussion of Variables.

5.3.1 Mean-Variance Optimization Model.

Harry Markowitz's mean-variance optimization model (MVO) formed the foundation of traditional portfolio theory. The MVO is employed in this study to help construct an optimal portfolio from a select group of stocks listed on the Nairobi Securities Exchange. The analysis began with an exploration of the daily stock prices and their log returns, which provided a clear foundation for the subsequent optimization process. This model, which focuses on the tradeoff between risk and return, calculates the efficient frontier and obtains the optimal weights for each stock.

The results of the study, however, reveal a negative Sharpe ratio of both the actual and simulated return series. This indicates that the returns of both these data sets fail to surpass the risk-free rate. Furthermore, it also indicates that the portfolios do not compensate adequately for the risk undertaken. This negative Sharpe ratio also raises concerns about the effectiveness of the mean-variance optimization model in this market context.

5.3.2 Black-Litterman model.

This model builds heavily on the MPT but includes investors' perspectives in the optimization process. Like Markowitz' model, the portfolio's return using both the actual and simulated data series is extremely lower than the risk-free rate. This raises concern about the relevance of the model in the Kenyan market. Another explanation can be that the model's assumptions are inconsistent with the behavior of the Kenyan stock market.

While the model seeks to improve the mean-variance model by incorporating investors' subjective perspectives, its performance in the Kenyan stock market creates a need for additional inspection and modification.

5.4 Conclusion.

5.4.1 Mean-Variance Optimization Model

The use of the MVO on both the actual and the simulated data series reveals portfolios with negative Sharpe ratios. This indicates that the expected returns of the portfolios do not exceed the risk-free rate while exposing investors to inherent risks. This raises important questions regarding the model's viability for the Kenyan marketplace.

5.4.2 Black-Litterman Model.

The low returns generated using both datasets raise some concerns about the model's assumptions and its ability to appropriately reflect investors' expectations. Moreover, the findings of this study raise significant questions about the models' relevance in the Kenyan stock market.

The model's detailed computations, such as the derivation of lambda and ensuing variances in the asset allocation may indicate the assumptions included in the B-L do not perfectly align with the NSE's market dynamics.

5.4.3 GARCH Simulated Returns.

One of the key weaknesses often referred to in both models is the use of historical returns as proxies for future expectations. The research employed the use of GARCH-simulated returns to see if better results would be obtained.

However, the simulated returns also resulted in a negative Sharpe ratio. This indicated that the portfolios earned lower returns compared to the risk-free rate.

Furthermore, it was also observed that the test data Sharpe ratio is higher than the simulated data Sharpe ratio for both models. This indicated that the portfolio strategy performed better on the test data set than the simulated data. This observation might indicate that the simulated data failed to capture the intricacies of real market dynamics such as underestimating or overestimating the volatility of returns, correlations, or even the return distributions.

BIBLIOGRAPHY.

Black, F., & Litterman, R. (1992a). Global Portfolio Optimization. *Financial Analysts Journal*, *48*(5), 28–43.

Black, F., & Litterman, R. (1992b). Global Portfolio Optimization. *Financial Analysts Journal*, *48*(5), 28–43. https://doi.org/10.2469/faj.v48.n5.28

Brooks, C., & Burke, S. P. (2003). Information criteria for GARCH model selection. *The European Journal of Finance*, *9*(6), 557–580. https://doi.org/10.1080/1351847021000029188

Cong, F., & Oosterlee, C. W. (2016). Multi-period mean-variance portfolio optimization based on Monte-Carlo simulation. *Journal of Economic Dynamics and Control*, *64*, 23–38. https://doi.org/10.1016/j.jedc.2016.01.001

Deng, G., Dulaney, T., McCann, C. J., & Wang, O. (2012). Robust Portfolio Optimization with Value-at-Risk Adjusted Sharpe Ratio. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.2146219

Ding, S. (2024). Portfolio Optimization Based on Markowitz Investment Theory and Monte Carlo Simulation. *SHS Web of Conferences*, *188*, 01009. https://doi.org/10.1051/shsconf/202418801009

Evans, J. L., & Archer, S. H. (n.d.). *Diversification and the Reduction of Dispersion: An Empirical Analysis*.

He, G., & Litterman, R. (2002). The Intuition Behind Black-Litterman Model Portfolios. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.334304

Khan, A. A. R., & Tanwani, L. K. (n.d.). *Portfolio Optimization: Theory, Methods, and Applications*.

Li, A. (2023). Portfolio Optimization by Monte Carlo Simulation. *Advances in Economics, Management, and Political Sciences*, *50*(1), 133–138. https://doi.org/10.54254/2754-1169/50/20230568

Markowitz, H. (2024). *Portfolio Selection*.

Markowitz, H. M. (1991). Foundations of Portfolio Theory. *The Journal of Finance*, *46*(2), 469–477. https://doi.org/10.2307/2328831

Mbithi, J. A. (n.d.). *Determining the optimal portfolio size on the Nairobi securities exchange*.

Mishra, A. K., Pisipati, S., & Vyas, I. (n.d.). *An equilibrium approach for tactical asset allocation: Assessing Black-Litterman model to Indian stock market*.

Modeling Stock Returns Volatility in Nigeria Using GARCH Models. (n.d.).

Odhiambo, O. J. (n.d.). *The effect of portfolio optimization on the returns of listed companies at the Nairobi securities exchange*.

Pedersen, M. E. H. (n.d.). Portfolio Optimization & Monte Carlo Simulation. *Monte Carlo Simulation*.

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*. *The Journal of Finance*, *19*(3), 425–442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

Vladimirov, M., Stoilov, T., & Stoilova, K. (2017). New Formal Description of Expert Views of Black-Litterman Asset Allocation Model. *Cybernetics and Information Technologies*, *17*(4), 87–98. https://doi.org/10.1515/cait-2017-0043

Zakamouline, V., & Koekebakker, S. (2009). Portfolio performance evaluation with generalized Sharpe ratios: Beyond the mean and variance. *Journal of Banking & Finance*, *33*(7), 1242–1254. https://doi.org/10.1016/j.jbankfin.2009.01.005