Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

BACHELOR OF BUSINESS SCIENCE FINANCIAL ENGINEERING

END OF SEMESTER EXAMINATION

BSM 4124: COMPUTATIONAL METHODS IN FINANCE

DATE: 25th July 2024 TIME: 2 HOUR

INSTRUCTIONS

I. This examination consists of FIVE questions. Answer **Question 1** (**COMPULSORY**), choose **2 optional** questions out of **Question 2 to 5.**

II. Ensure you have clearly **indicated the question number** on EVERY QUESTION.

QUESTION ONE [30 marks]

i. Derive the *binomial option pricing formula* via the *risk-neutral* strategy, assuming that at time t, the price of an asset is $S(t) = S$, and r represents the interest rate for both lending and borrowing. (5 marks)

ii. Show that with T = 0.5, r = 0.05/250, σ = 0.01, N = 5, if we take λ = 1.25 then the probabilities in *Boyle's model* are close to 1/3 for the *trinomial lattice tree model*. Explain the implication of this result in comparison to other parameter estimation schemes. (5 marks)

iii. Finite difference discretization techniques are frequently applied in finance for numerical estimation of continuous time models that would have otherwise proven intractable. To this effect, what would you say are the key (mathematical modelling) differences between the three popular finite difference techniques used in option valuation i.e., the

Explicit, Implicit, and Crank-Nicolson schemes? Use the derivation process to differentiate, assuming the Black-Scholes-Merton PDE is given as

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

with time t, asset value S, and r representing the risk-free rate. (10 marks)

iv. Discuss the steps underlying the Monte Carlo valuation of derivatives (3 marks)

v. Describe the *antithetic variate* method of variance reduction for *path-dependent options* whose payoff at time T is given by

$$\psi(S_s, s \leq T)$$

And $(S_t, t \geq 0)$ is the lognormal diffusion

$$S_t = x \exp\left(\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right)$$

(3 marks)

vi. Describe the *importance sampling* variance reduction procedure, including its key characteristics that differentiate it with other variance reduction techniques (4 marks)

QUESTION TWO [15 marks]

i. How would you describe the truncation/discretization errors and stability of the three popular finite difference methods, i.e., the explicit, implicit, and Crank-Nicolson methods? (6 marks, 2 marks per method)

ii. **Algorithm**: Write a *generalised algorithm* to evaluate the price of a *European option (call or put)* within the *Crank-Nicolson scheme*. (9 marks)

QUESTION THREE [15 marks]

i. Show that by substituting the values of α and B from the *replication BOPM*, where α represents the units of stock and B the cash amount in a riskless bond, into $V(S,t) = \alpha S + B$, we get the same value for $V(S,t)$ as the *risk-neutral BOPM for a European call option*. (10 marks)

ii. **Algorithm**: Write a *general algorithm* to evaluate the price of an *American call option* with the *trinomial lattice tree model*, using the *Boyle model* conditions. (5 marks)

QUESTION FOUR [15 marks]

i. Why is Monte Carlo Variance reduction important in financial applications? (2 marks)

ii. Differentiate any three popular variance reduction techniques in Monte Carlo simulation (6 marks)

iii. **Algorithm**: Write a *generalised algorithm* for the *Control Variate* method to estimate $\theta = \mathbb{E}[h(X)] = \mathbb{E}(Y)$. (7 marks)

QUESTION FIVE [15 marks]

i. Derive the Euler discretization method for a generalised/arithmetic Brownian motion process (5 marks).

ii. Comment on the convergence properties of the Euler discretization scheme and how this can be improved upon (3 marks)

iii. **Algorithm**: Use the *Euler discretization and approximation scheme* to detail an algorithm for calculating the price of an *Asian call option* whose underlying stock's initial price and option's strike price equal to 100, expiration time is one month, risk-free interest rate of 10 %, and volatility of 25 %. (7 marks)

[TOTAL: 60 MARKS]

END OF EXAM