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A Comparative Modelling of Price Dynamics Of Certified Emission Reductions using Diffusion Processes:A Case Study of The European Energy Exchange

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Submitted in partial fulfillment of the academic requirements for the Degree of Masters of Science in Mathematical Finance and Risk Analytics at Strathmore University.

Institute of Mathematical Sciences
Strathmore University
Nairobi,Kenya.

November 2021

A research thesis submitted to the Strathmore Institute of Mathematical Sciences in fulfillment of the requirements for the award of the Master's Degree in Mathematical Science in Mathematical Finance and Risk Analytics.

Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

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12th November 2021.

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Acknowledgment

I thank the Almighty God for His provision, grace and care and this far He has brought me.
My sincere gratitude goes to my supervisor Mr. Meleah Oleche who tirelessly devoted his time to guide me and whose humble criticism assisted me throughout the study.
I am greatly indebted to Strathmore University Institute of Mathematical Sciences fraternity for the support that they have given me this far.
Lastly, I acknowledge the support and encouragements I have received from my parents, siblings and classmates.

Abstract

In this study,the price dynamics of Certified Emission Reductions were forecasted by comparing and acquiring the most consistent and accurate forecast model using the diffusion processes: Geometric Brownian Motion, Vasicek Model and Mean Square Root Process.The assumption of each model with drift and diffusion component were investigated focusing on the Certified Emission Reductions prices traded within European Energy Exchange(EEX) between the years of 2012-2020.The forecasted prices were compared to the actual to evaluate the validity of the models.

Based on the research findings, Bayesian information criterion, which determined the goodness-of-fit to assess the performance of the model with respect to how well it explains the data, shows that vasicek model is the most preferred model among the three models since it has the lowest Bayesian information criterion (BIC). This study compared the accuracy of the models, Geometric Brownian Motion, Vasicek Model and Mean Square Root Process, in terms of forecasting the price dynamics of CERs, and concluded the vasicek most is the most accurate among the three models since the predictive power is high toward a specified time frame proven by the lower value of mean absolute error and Mean Absolute Percentage Error.

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Keywords: Certified Emission Reductions(CERs), Geometric Brownian Motion Process(GBMP), Mean-Reverting Square Root Process, (MRSRP),Vasicek Model.

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Abbreviations and Acronyms

AAU:Assigned Amount Units

CDM:Clean Development Mechanism

CO_2:Carbon dioxide

$(CO2e)$:Carbon-dioxide equivalent

CERs:Certified Emission Reductions

EU ETS:European Union Emissions Trading System

EEX:European Energy Exchange

ERUs:emission reduction units

GHGs:Greenhouse Gases

GBMP:Geometric Brownian Motion Process

GBMPJ:Geometric Brownian Motion Process augmented with jumps

MRSRP:Mean-Reverting Square Root Process

MRSRPJ:Mean-Reverting Square Root Process augmented with jumps

RMSE:Root Mean Square Error

MAPE:Mean Absolute Percentage Error

LDCs:Least Developed Countries

UNFCCC:United Nations Framework Convention on Climate Change

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1 Introduction

1.1 Background of the study

In this chapter, the background to the study is discussed, making reference to previous research at the global and local levels on the understanding of carbon markets and its derivatives. The extent to which these studies have investigated the problem is examined, noting in particular where gaps exist in which this study attempts to address. The problem statement is outlined in this context along with the resulting research objectives. The significance of the study is also highlighted.

1.1.1 Certified Emission Reductions

Carbon market is an environmental toll whose aim is to address the issue of global warming by reduction of greenhouse Carbon gases. This type of market involves the buying and selling of carbon offsets based on a certain set limit by the policy makers. In 1997, the United Nation, developed the Kyoto protocol, and the carbon trading in its semblance which began with the Protocol in 2005. The Kyoto protocol [3], provides a framework for trading emission rights and established explicit and mandatory limits on industrialized and those set by the governments that wish to ratify the protocol. The idea behind this agreement was because countries will find it easier and cheaper to reduce emissions than other countries. For example, through the introduction of energy-efficient technology or carbon sequestration activities such as tree planting, the protocol allows countries to trade emission rights to reduce the overall costs of meeting global targets. Countries wishing to emit more that their limit, must purchase additional rights from others that find it less costly to reduce their emissions.

To meet the set targets, the protocol has set out three potential flexible mechanisms that could permit emission rights trading: International Emission Trading mechanisms, that is defined in Article 17 , allows Annex B countries to trade emission permits known as Assigned Amount Units (AAUs);Joint Implementation (JI), defined in article 6 of the protocol allows countries to earn Emission Reduction Units(ERUs) through projects in other Annex B countries and Clean Development Mechanism (CDM), defined in article 12 of the protocol allows the generation of Certified Emission Reductions(CERs) from projects in non-annex B countries (Developing countries that are outside the cap regime).

These allowances and derivatives are traded on many exchanges worldwide, for example, the European Union Emission Trading System (EU ETS): Powernext, Nord Pool and European Energy Exchange (EEX), which is by far the largest global carbon market, the Chicago climate exchange, Australian Carbon Trading Scheme and other upcoming trading schemes. Under this agreement binding limits, expressed in AAU and measured in tones of Co_2 equivalent GHG, they are imposed on the emissions of participating countries. This study however focuses more on the CERs which are rooted in international efforts to control GHG emissions, specifically in the Kyoto Protocol's country-level, emission reduction targets for the developing countries.
There are restrictions when polluters surrender their permits for compliance. On average, CERs and ERUs altogether cannot comprise more than 10-20% of all the submitted permits, which means EUAs should be the major emission rights in the compliance account. The regulations vary from country to country, and different installations may be subjected to different rules. Since the use of CERs and ERUs is so much restricted and the supply is abundant, the CERs and ERUs are thus over-supplied in the market nowadays and their prices are significantly lower than the EUA prices.

1.1.2 Model Specifications

The movements of prices play an important role in the decision of investment and risk management in the financial markets.Diffusion models are a popular class of models which is used for these purposes, especially in the pricing of carbon derivatives Daskalakis [8]).Diffusion models are represented by the following stochastic differential equation

$$dS_t = \mu(S_t, t)dt + \sigma(S_t, t)dW_t,$$

where Wt is a standard Wiener process,$\mu(St, t)$,$\sigma(St, t)$ are the drift and diffusion term of the models The Geometric Brownian Motion is well known for incorporating the random movement of market prices whereas the Vasicek model and Mean-Reverting Square root process are two important models of short rate in the class of diffusion models. Although there are many extensions of these models in the literature, they are still popular because of their tractability and their closed form solutions for various derivatives. In this work, we will focus on these three models.

1.1.3 Price Dynamics of Carbon Prices

Most market participants especially investors strive to maximize returns or profits, with the lowest possible risk, and by designing portfolios believed to perform well in the future. One way to accomplish this is by having a good understanding of different factors that have effect on specific sectors within a market. Commodities are directly or indirectly essential for all sectors in the market, thus showing the importance of understanding various commodities connected to that sector and how it responds to price movements. However, relevant academic research outside the scope of environmental economics and policy has been sparse despite the size, growth and importance of the carbon market. The limited empirical research has concentrated almost entirely on the EU ETS markets since these are by far the largest, most liquid and most developed.

In support of previous research (see Daskalakis [8]), carbon derivatives thus EUAs based on the market information, are traded with increasing liquidity within the EU emissions trading scheme with the main objective being to address explicitly the open issue of correctly pricing carbon futures and to do an econometric analysis for more understanding of the carbon derivative on an existing market. Although, option pricing problem is one of the predominant concerns in the financial market. Since the advent of the Black-Scholes option pricing formula in Black and Scholes(1973), there has been an increasing amount of literature describing the theory and its practice. Due to drawbacks of the Black-Scholes model which cannot explain numerous empirical facts such as large and sudden movements in prices, heavy tails, volatility clustering, the incompleteness of markets, and the concentration of losses in a few large downward moves. Many option valuation models have been proposed and tested to fit those empirical facts. Jump-diffusion models with stochastic volatility could overcome these drawbacks of the Black-Scholes model. Based on those advantages, the research will examine the ability of various popular diffusion continuous-time processes suggested by Daskalakis in capturing the dynamics of the CERs market prices and the adequacy of the models capturing price behaviours.

1.2 Problem Statement

Certified Emission Reductions are rooted in international efforts to control carbon emissions.Unlike EUAs,CERs are the only commodity that enables developing countries to be part of the global market under the Kyoto protocol. One of the major benefits of carbon trading market to developing countries like Kenya include; providing a flexible and profitable mechanism in combating climate change and help them reach their sustainable development goal.Further,reducing their financial burdens in the public sectors and need of funds spurs their development.

Despite of all these benefits, it is projected that less than two per cent of projects are hosted in Africa (Reddy[14]), and less than 30 per cent of revenue streams from carbon trades go to the developing countries. Beneficiaries of a large portion of the funds being brokers, bankers, investors, and consultants in rich countries. This is because, there is lack of transparency, accountability and integrity of the system that is aimed at achieving professed goals of climate change (Reddy [14]).

Motivated by previous studies, the research investigates the appropriability of the diffusion processes under study in predicting the price dynamics of CERs.This is achieved by comparative study the diffusion processes under study.Specifically, we employ the Geometric Brownian Motion process,Mean-Reverting Square Root process and the vasicek model to identify best model in predicting of CERs prices.The research will also explores on the effectiveness of the best model to developing countries on trading of CERs within the global market. Therefore, this research aims at filling this gap by studying the price dynamics of CERs traded in a global carbon market.

1.3 Research Objective

The research objective of this study is to do a comparative modelling of price dynamics of Certified Emission Reductions prices using the diffusion processes under study.

1.3.1 Specific Objectives

Based on the above objective, the specific objectives of the study are:

1. To calibrate parameters of the diffusion processes under study.

2. To determine how well the the models are fitted to the data set.

3. To assess the fit of the three models, Geometric Brownian Motion, Vasicek and Mean-Reverting Square Root process and to compare their forecasting accuracy.

1.4 Significance of the Study

By investigating the research objectives outlined above, the research will make unique contributions to the existing literature in the following respects:

- It will establish the link between CERs and the perceptions about the global carbon market by outlining the dynamics driving the CERs in the global markets, attention been increasingly drawn to the econometric analysis of carbon allowance prices.
- The study will contribute to existing literature in the ways developing countries especially those in Africa can be part of the initiative and the markets benefits.
- The determined price behaviors of CERs will be helpful to the market participants(Private Sector, NGOs, Private Corporations, Governments) in implementing carbon management strategies, discerning trends (Buying and selling of CERs), hedge future carbon liabilities e.g carbon taxes and have a prospect of attracting competing offers (suppliers).
- Having a reliable pricing and forecasting model will allow companies, investors and traders to realize effective trading strategies, risk management, and investment decisions in the carbon market.

1.5 Scope of the study

The scope of this study it to determine the price dynamics of CERs, and the impacts they have on trading of CERs in the global market. The study will focus on CERs traded under European Energy Exchange (EEX) and will examine the approaches by conducting an in- sample and out-of-sample forecasting analysis, and by comparing the results to alternative approaches.

1.6 Organization of the thesis

This thesis study is organized in five chapters. Chapter 1 briefly introduces the study and objective of the thesis. Chapter 2 explains price dynamics of Certified Emission Reductions forecasting models in the literature. Chapter 3 gives the general review to the panel data and explains GBM, Vasicek and MRSP data analysis methodology in detail. Estimation and forecasting strategies to be used for each model are expressed in this chapter. Chapter 4 describes the data, shows empirical results and forecast results of the models and compares the results. Finally; Chapter 5 presents the summary of the study and conclusions based on the forecast results. Moreover, it gives some perspectives about future investigation.

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2 Literature Review

2.1 Introduction

This chapter outlines a review of literature on price dynamics of Carbon emission allowances of various research, which have mainly been focused on the EU ETS.
The precise area of focus will be empirical studies relating to various models which have been considered appropriate in capturing different behaviors of the commodity. As a basis on why this research preferred a comparative analysis of diffusion and jump diffusion processes in determining the dynamics of CERs, will also be discussed.

2.2 Empirical Review of the study

Carbon pricing is a cost levied by governments to encourage polluters to reduce the amount of greenhouse gas they emit and provide cleaner options. It helps reveal the true cost of unclean energy, making clean energy even a better value, Unilever [17]. More specifically, carbon trading is seen as a useful vehicle to help member state government of the Kyoto protocol (which amended the UNFCCC by assigning mandatory Carbon emission reduction targets) in achieving their targets instead of using the old command and control approach such as environmental taxes.
Despite the size, growth and importance of the carbon market, relevant academic research outside the scope of environmental economics, the policy has been sparse. The limited empirical research has concentrated almost entirely on the EU ETS markets since these are by far the largest, most liquid and most developed. According to Nel [12], the EU ETS is referred to as a Cap-and-trade system. The cap indicates a limit which lowers over time whereas the trade refers to trading, which in this case are the carbon credits that gives the holder the right to emit one tone of CO_2. The EU ETS determines the amount of CO2 credits each of their industry sectors and ultimately companies will receive. However,the cap on the total number of credits granted or auctioned to companies that participate in the EU ETS is what creates the scarcity in the market. Making the cap and trade one of the approaches to which the carbon prices are set. Another approach that few countries especially in Africa, for example South Africa is to tax the distribution based on the carbon content of fossil fuels. See Shi et.al [16].
According to World Bank, price on carbon is an aspect that every country and business is on

the verge to incorporate with the aim of a favorable climate change. However, price on carbon has helped shift the burden for the damage back to those who are responsible for it, and who can reduce it. In this way, the overall environmental goal is achieved in the most flexible and least cost way to society. The carbon price has also stimulated clean technology and market innovation thus providing low-carbon drivers of economic growth to both developed and developing countries.

Although the United Nations Climate Change Framework Convention on Climate Change (UNFCCC) is responsible for maintaining a global registry for many types of carbon units around the globe, various regulatory bodies at the national and regional level for example the World Bank group and the European Union, oversee and monitor transactions in this market. However, there are a number of major exchanges where carbon allowances and credits trade in real time.

Similarly, various market mechanisms were put into place under the Kyoto protocol including the Clean Development Mechanism (CDM), which is a project-based, whose objective is to reduce the global cost of carbon mitigation by opening up the market for those countries with legally binding emission reduction targets to gain from trade with countries that do not have legally binding targets. Aiming at helping the developing countries to be part of the global carbon market.

Despite the fact that developing countries would benefit from Clean Development Mechanism projects, only few developing countries like South Africa, and Ethiopia have developed its national policies to tackle climate change as they are considered the countries with the highest number of CDM projects. The main aim of CDM to some of these countries has contributed in diversifying the energy system and creating human and institutional capacity for carbon governance (Friberg.L [9])

In 2011, total value of the markets stood at more than 175 billion dollars, which is over 20 times higher than in 2005 (World Bank [4]), and business activities in all sectors of the economy were influenced by carbon emission trading(Calel [6]). Given the importance of the carbon emission markets, there has been a growing body of literature studying the characteristics of carbon emission allowance prices and the financial markets for carbon assets. The most important carbon emission market, the European Union emission trading scheme (EU ETS), which is a "cap-and-trade" system requiring firms to surrender a certain amount of tradeable permits corresponding to the firms GHGs emissions by a specific deadline, to avoid incurring a penalty. However, under the Kyoto protocol the EU ETS has three commitment periods: Phase I: 2005-2007; Phase II: 2008-2012; Phase III: 2012-2020), among which each portrays a different trait based on the behavior of the carbon allowance during that phase. This research will focus on the issues associated with the third phase of the EEX, which is ranked one of the main carbon markets in Europe.

As it is important to understand the underlying dynamics driving the carbon emission markets where attention has been increasingly drawn to the econometric analysis of carbon allowance prices. For example, Paolella and Taschini [13], analyzed the time series properties of carbon allowance prices and examined the fitness of a series of GARCH models. Based on their findings, they found out that the GARCH model with generalized asymmetric t distribution performs best in the in-sample fitness; however, none of the models considered can provide accurate value of risk (VaR) forecasting. Other studies have focused on determining the stochastic properties of on carbon emission allowances in carbon emission markets by modelling the relationship between carbon spot and futures prices (e.g. Chevallier [7];Rittler[15]).In other studies (e.g.Daskalakis et al [8];Benz and Trück [5])they have shown that carbon allowance prices experience price jumps, spikes, high volatility, and are very sensitive to government policies. In addition to a very important issue, the impact of this policies carbon emission markets on the characteristics of carbon allowance prices and the implications they have on special properties for hedging, arbitrage and other trading activities in carbon emission markets. Therefore, highlighting the importance of modeling the price dynamics of carbon spot and futures markets by considering the special characteristics of carbon emission markets, and analyses how these models can be applied to trading.

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2.2.1 International Carbon-Reduction Policies

According to Zhang et.al, [19], at different phases of the Kyoto Protocol,CERs portray different characteristics. The CDM set down in the first commitment period had promoted the development of carbon market, but the price of CERs started declining due to slow progress of climate policy negotiations and uncertainties, Nazifi[11]. In the second commitment of the Kyoto Protocol, most countries have not yet reached agreement on targets reductions. At the same period CDM faces some down fall due of some of the main emitters like the United States exiting the market. This means that the demand for CERs may reduce, leading to lower price of CERs in the market.

More specifically, new CERs are only allowed if they originate from the developing countries in Phase III of EU ETS,World Bank [4]. Consequently, the policy changes should rather reduce the number of available CERs, which may have a positive impact on the price of CERs if demand remains constant. Therefore, to capture the influence of carbon-reduction policies, this research will investigate the CERs price volatility change of Phase III to observe and captured by the alternative processes rather than those used by previous studies.

2.2.2 Carbon Price Determination

Price determination theory, is a principle that uses the concept of supply and demand to determine the appropriate price point for a given good or service. In a carbon market set up the main determinants of price are the market participants (that is, Investors) and governments. According to Ji et.al[10], several factors may affect the quantity of demand, such as the price of the commodity, consumer's income level,consumer's preferences and expectations of the product, the number of consumers, the weather, and political factors. However, the price of the commodity have negative effects on the demand, while factors such as the consumer's income level and the consumer's preference have positive effects on the demand of the product. Therefore, if consumers expect future prices will rise, they will also have to increase the current demand on the supply-side factors that might affect the price of the product, the cost of production, the product's technical level, the producer's expectations in the future, the price of the relevant product and the number of producers. Specifically, factors such as the price of the product, the production technology, the price of the complementary product, and the number of producers that have positive effects on the supply, while others such as the cost of production when the price is constant, the price of the substitutes and the expected future price of the product have negative effects.

2.2.3 Review of modeling approaches of the study

Diffusion models are generative models that are flexible and analytical in evaluating the distribution of a quantitative data.Each model studied plays an important role in prediction of market price movements both in the current state and in the future.Therefore this study explores different models namely: Geometric Brownian Motion, Vasicek Model and Mean Square Root Process in predicting CER prices.

Agbam et.al,[2]describes the GBM as a model that incorporates the idea of random walk in market prices through its uncertain component along the idea that prices maintain same trend over time as the certain component.However it assumes that the returns and profits incurred on market are independent and normally distributed but due to its unpredictable path it makes it difficult to outperform a market without assuming additional risk.

Vasicek Model and Mean Square Root Process on the other hand describes the evolution of interest rate based on time and market risk.However both model exhibit a mean reversion which helps predict future interest rate movements.Zeytun et.al,[18] studies the basic features of the two models and did a comparative study to see how their parameters affect the dynamics and pricing of bonds evaluated using the two models.The study showed that the Vasicek model can be given an edge over the CIR model, due to the tractability of the model and also the availability of closed form solutions even for more complex financial interest rate derivatives.However, if the interest rates are closer to zero then working with the Vasicek model can become cumbersome due to the possibility of negative interest rates which can yield some unreasonable results and prices. In recent research,Daskalakis et al, [8], Benz, and Truck, [5], research has shown that carbon allowance prices experience frequent jumps and spikes and that the volatility of carbon emission markets is high compared to other financial markets. Thus, managing the financial risk in carbon emission markets is important for market participants.

The work of Daskalakis et al, [8] was evaluated in terms of both statistical accuracy and model parsimony. By estimating parameters and corresponding t-statistics, the log-likelihood values (I) and the Bayesian Information Criterion (BIC) for both the diffusion(GBMP and MRSRP)and jump diffusion processes(GBMPJ and MRSRPJ). They concluded, that EU-ETS markets observe that the GBMPJ is better than competing models in terms of parameter significance, log-likelihood and the BIC. Second, the addition of mean-reversion appears to decrease the goodness-of-fit, especially in the case of the jump-diffusion models. For the diffusion processes, GBMP is only slightly better than MRSRP. Thirdly, the addition of jumps improves performance significantly since all jump diffusion processes outperform their diffusion counterparts.

The findings indicate that EUA spot prices have a proportional, non-mean reverting structure with jumps, i.e., they are subject to large movements that cannot be explained by standard diffusion processes. These three main conclusions are supported by all the criteria they used. However, they were consistent with the descriptive analysis findings which concluded that the existence of jumps, the non-normality of returns, and the non-stationarity of the price process. A further investigation of the two best performing models, that is, the GBMPJ and MRSRPJ, was also done showing that the addition of mean reversion almost doubles the diffusion volatility parameter. However, it was noted that both mean reversion parameters kappa and theta are not statistically significant. A comparison between the best performing diffusion and jump-diffusion process, respectively that is the GBMP and GBMPJ. On the other hand, it indicated that the addition of jumps leads to a lower expected mean and volatility.

2.3 Summary of Literature Review

Motivated by previous findings in the literature(Daskalakis et al[8]), this research shall evaluate both statistical accuracy and model parsimony of the diffusion processes (GBMP and MSRSP) for the CERs traded in the EEX in order to define the price dynamics of this type of commodity in a global market and determine the impacts this dynamics have on trading of CERs in the global market.

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3 Research Methodology

3.1 Introduction

In this section, we investigate the appropriateness of the suggested time series models for log returns of daily Certified Emission Reduction (CER) prices. The considered time period is from January 3, 2008 - December 30, 2020. Hereby, the data from period January 3, 2008 - December 30, 2012 will be used for the calibration of the models, while the period January 3, 2013 - December 29, 2020 is used for out-of-sample testing.

3.2 Data Description

The data set used in this research will be quantitative Secondary data from the European Energy Exchange (EEX) under the Intercontinental Exchange(ICE) recorded on a daily basis. ICE is an electronic exchange platform designed originally for the trading of energy related products, but which has expanded to offer trading in other commodities.
In this section we shall investigate the appropriateness of the suggested time series models for daily Carbon Emission Reductions(CERs) prices traded in EEX. The considered time period will be from January 3, 2008–December 30, 2020(Second, and the third Commitment of the Kyoto Protocol). Hereby, the data from period of January 2, 2013 - December 30, 2016 will be used for the calibration of the models,while the period of January 2, 2017 - September 9, 2020 will be used for out-of-sample testing.

3.3 Descriptive Analysis

Analyzing prices from different trading periods allows us to investigate the potential effect of different trading platforms and of local market conditions. Several descriptive statistical tests will be performed on the stock return series which include reporting on the mean, skewness, kurtosis and the test for normality and mean-reversion. The purpose of this is to check the distributional properties of CER spot prices (P) and logarithmic returns (LR).
The return is estimated as follows.

$$R_t = LnS_t - LnS_{t-1},$$

where R_t are the daily returns for period t. S_t And S_{t-1} are the share indices for days t and t-1 and Ln is the Natural logarithm

3.3.1 Tests for Normality

Normality test are used to determine if the given data set is well-modeled by a normal distribution and to compute how likely it is for the random variable underlying the data set to be normally distributed. In this research QQ plots and histogram will be used. QQ plots: test for normality and randomness of the data. Normality can be checked more carefully by plotting the normal scores on the QQ plots. Such a plot displays the quantiles of the data and compares them to the theoretical quantiles of a normal distribution. With normal distributed data, the QQ plots look approximately like a straight line and hence the values are expected to fall along or close to the line otherwise the data is not.

In addition to the above tests, Jarque bera test, which is a goodness-of-fit test of whether sample data have the skewness and kurtosis matching a normal distribution. Data that is normally distributed has a skew of zero (perfectly symmetrical around the mean), and a kurtosis of three, whereby kurtosis tells us how much data is in the tails and gives us an idea about how peaked the distribution is. Therefore,the null hypothesis for the test is that the data is normally distributed and the alternate hypothesis is that the data does not come from a normal distribution.

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3.3.2 Testing for auto-correlation-Ljung-Box test

Ljung-Box test will be to test whether series have significant auto-correlation or not. The test can also be used as a diagnostic tests which is performed on the residuals of the estimated models to establish whether the model has been correctly specified and guard against spurious inferences. Diagnostic tests checks for non-linearity in the residuals. The common principle behind these tests is that once a model has been generated non-linear structure is removed from data and the remaining structure should be due to an unknown linear data generating process (Addelaal et.al, [1]).

The hypothesis is:

Ho : data are not correlated.

H1 : data are correlated (not random).

Ljung-Box test statistic for a number of tested lags is:

$$Q_k = N(N+2) + \sum_{i+1}^{k}(\frac{\rho_i^2}{N-i}),$$

where N is the sample size, ρ^2 is the sample auto correlation at lag
Null hypothesis is rejected at $\alpha\%$ significance level if:
$Q(k) > \chi^2_{1-\alpha,k}$
where: $\chi^2 1-\alpha, k$
α is a quantile of the chi-square distribution with k degrees of freedom.

3.3.3 Test for stationarity

Augmented Dickey-Fuller(ADF)test for stationarity. ADF tests will be used to determine whether a data is stationary or, specifically whether the null hypothesis of a unit root can be rejected.
Consider the model:

$$Y_t = \alpha Yt - 1 + X_t,$$

where where, Yt is the value of the time series at time 't' and Xt is an exogenous variable (a separate explanatory variable, which is also a time series). For t=1,2$\cdots$ Xt is a stationary process, process Yt is non-stationary if the coefficient $\alpha = 1$ but it becomes stationary if the absolute $\alpha < 1$.
This means that,the presence of a unit root means the time series is non-stationary.Therefore, the number of unit roots contained in the series corresponds to the number of differencing operations required to make the series stationary. Suppose that Xt is an AR(k) process

$$X_(t) = \alpha X_{t-1} + ... + \alpha X_{t-k} + \epsilon_t,$$

Under the null hypothesis that: $\alpha = 1$; $X_t = Y_t - Y_{t-h} X_t$, Letting $\alpha = \alpha - 1$, We have

$$\begin{aligned} Y_t - Y_{t-1} &= (\alpha - 1) + Y_{t-1} + X_t, \\ &= \alpha Y_{t-1} + \alpha_1 X_{t-1} + ... + \alpha_1 + X_{t-k} + \epsilon_t, \\ &= \alpha Y_{t-1} + \alpha_1 (Y_{t-1} - Y_{t-2}) + \cdots + \alpha_1 (Y_{t-k} - Y_{t-k-1} + \epsilon_t), \end{aligned}$$

where y(t-1) = lag 1 of time series and $\alpha Y(t-1)$ = first difference of the series at time (t-1)
Test statistic is basically a t-statistic

$$t^{*} = \frac{\tilde{\alpha}}{s.e\tilde{\alpha}},$$

For the hypothesis testing:

H0:$\alpha = 0$ (the data is not stationary (unit root))

H1:$\alpha < 0$ (the data is stationary (no unit root))

Decision rule:

If $t^{*} > ADF$ critical value, we do not reject null hypothesis, i.e., unit root exists.

If $t^{*} < ADF$ critical value, we reject null hypothesis, i.e., unit root does not exist. This will call for differencing if it is found that $P-value > \alpha = 0.05$ then we conclude Ho that the data is not stationary. The lag structure in the ADF test will be selected automatically on the basis of the Bayesian Information Criterion (BIC).

3.3.4 Test for Mean Reversion

Mean reversion processes, assumes that the process has a tendency to revert to its average level over time. This average level is usually determined by physical or economical forces such as long term supply and demand. Prices might deviate from that long term mean due to sentimental or short-term disturbances on the market but eventually will revert back to its intrinsic value. To test for mean reversion, in this study we shall consider two methods: Mean-reverting Test using t-statistics and mean-reverting test using gamma component.

For the mean-reverting test using the t-statistics, considering the data under study, we accept the null hypothesis if the calculated value of the t-statistics is greater than the critical value at a given confidence level. On the other hand,the data is considered mean-reverting if the gamma is negative. For both methods:

H0: The data does not have a mean reverting time-series

H1: The data has a mean reverting time-series.

3.4 Model Description

Uncovering the continuous-time dynamics of spot prices is a necessary step for choosing the appropriate option pricing model. Under the real probability measure P, the candidate processes considered are nested in the following stochastic differential equation:

$$dS_t = \mu(S_t, t)dt + \sigma(S_t, t)dW_t,$$

where: S_t is the CER spot price at time t, Wt is a standard Wiener process,$\mu(St, t)$ is the drift, and $\sigma(St, t)$ is the diffusion coefficient. The drift are the diffusion coefficients are assumed to be general functions of time and the CER spot price.

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

The Geometric Brownian Motion(GBMP)has been very popular for modeling the evolution of stocks, commodity prices and indices and basic for most option pricing models.
The GBM Model's drift term exhibits drift parameter μ,volatility σ functions on the random variable $S_t.dW_t$ is the change in the Brownian motion over a short period of time.
This model, says the percentage change in the random variable S_t is normally distributed with instantaneous mean μ and instantaneous variance σ^2
By applying Ito on $f(t, dt) = LnS(t)$. The solution of the above model will be

$$S(t) = S(0)exp((\mu - \frac{1}{2}\sigma^2)t + \sigma W(t)),$$

which illustrates that stock price has independent growth factors that are log-normally distributed with expected value and variance given by:

$$E(S_t) = S_0 e^{\mu t},$$

$$Var(S_t) = S_0^2 e^2 \mu t(e^{\sigma t} - 1),$$

3.4.1 Vasicek Model

$$dS_t = \kappa(\Theta - S_t)dt + \sigma dW_t,$$

The Vasicek Model's drift term exhibits mean reversion, meaning that interest rates will converge to the mean reversion level Θ with speed κ over time.When the short rate wanders above the long term rate it is pulled down and likewise when it drifts below the long term rate it is pushed up. In the case of carbon emission, the parameter κ is the rate at which the carbon emission is pulled back to the long term mean,Θ. In that, the bigger the long term mean, the more the carbon emission.
To solve the SDE, we use Ito formula, by first using an integrating factor $e^{\kappa t}$. The solution of the Vasicek SDE will be:

$$S_t = S_o e^{-\kappa t} + b[1 - e^{-\kappa t}] + \int_0^t \sigma e^{-\kappa(t-s)} dw_s,$$

Based on the above solution the Short-term, and Long-term Distribution of the Vasicek model will be as follows; For the Short-term Distribution:

$$E(S_t) = S_o e^{-\kappa t} + b[1 - e^{-\kappa t}],$$

$$Var(S_t) = \frac{\sigma^2}{2\kappa}[1 - e^{-2\kappa t}],$$

Hence,

$$S_t \sim \mathcal{N}(S_o e^{-\kappa t} + b[1 - e^{-\kappa t}], \frac{\sigma^2}{2\kappa}[1 - e^{-2\kappa t}]),$$

For the long-term distribution,since the $t \geq \infty$

$$E(S_t) = \Theta,$$

$$Var(S_t) = \frac{\sigma^2}{2\kappa},$$

Hence,

$$S_t \sim \mathcal{N}(\Theta, \frac{\sigma^2}{2\kappa}),$$

. However, this model has been adapted in credit markets, and useful in modeling stochastic investments. Just like other stochastic models, the model has its advantages and disadvantages. Some of its advantages: Used to capture mean reversion of the interest rates movements, may be viewed as a short rate model, and used in the valuation of the interest rate derivatives. In particular, the model is analytically tractable and has an explicit solution since the distribution of interest rates is normal. On the hand, some of its disadvantages: It has no term structure of volatility, in that, volatility is assumed constant, it is a one factor model, therefore assuming that the short rates are correlated and can only cater for parallel shifts in the yield curve, and some observed short rate terms structures are difficult making it impossible to fit using this model. Due to Vasicek model shortcomings, it leads to the development of other short term models i.e Cox, Ingersoll and Ross(CIR) 1985, and Hull-White 1990.

3.4.2 Mean-Reverting Square Root Process

$$dS_t = \kappa(\Theta - S_t)dt + \sigma\sqrt(S_t)dW_t,$$

where κ , μ and σ are positive constants and W(t) is a scalar Brownian motion. We assume that the initial condition S(0) is independent of the Brownian motion and has bounded second moment. Therefore,For MRSRP , κ is the speed of mean reversion, Θ is the unconditional long-run mean, and σ the volatility of the asset price. We also assume that $S(0) \geq 0$ with probability 1. It is known that a unique strong solution exists for (1), and that non-negativity of the initial data is preserved. However, the above process was widely used in mathematical finance as an alternative to geometric Brownian motion. Most notably by Cox, Ingersoll and Ross as an interest rate model as it forms the stochastic volatility component of Heston's asset price model. Since the SDE nonlinear and non-Lipschitzian and cannot appeal to standard convergence theory for numerical simulations to deduce that the numerically computed paths are accurate for small step sizes.

A numerical method applied to SDE above may break down due to negative values being supplied to the square root function we replace the SDE by the equivalent, but computationally safer, problem:

$$dS(t) = \kappa(\Theta - S(t))dt + \sigma\sqrt{|S(t)|}dW_t,$$

. Given that;

$$E[s_t|s_0] = s_0e^{-\kappa t} + \Theta(1 - e^{-\kappa t}),$$

,so the long term is Θ.

$$Var[s_t|s_0] = s_0\frac{\sigma^2}{\kappa}(e^{-\kappa t} - e^{-2\kappa t}) + \frac{\Theta\sigma^2}{2\kappa}(1 - e^{-\kappa t})^2,$$

. Therefore,given a step size $\Delta t > 0$, the Euler–Maruyama (EM) method applied to the above SDE sets $s_0 = S(0)$ and computes approximations $s_n \approx S(t_n)$, where $t_n = n\Delta t$, according to:

$$s_{n+1} = s_n(1 - \kappa\Delta t) + \kappa\Delta t\mu + \sigma\sqrt{|s_n|}\Delta W_n,$$

where

$$\Delta W_n = W(t_{n+1}) - W(t_n),$$

provided the ϵ_t is n.i.i.d(0,1)

3.5 Model Estimation and Simulation

In this research the parameters of the models shall be calibrated using the MLE. Using the empirical data, the MLE will perform parameter estimation for the specified model. In this case our models specifications will include: drift(μ), volatility(σ), intensity of the compound Poisson process(λ), long term mean(κ), mean reverting size (Θ). Through the summary, we will also be able to visualize other statistics such as trends and mean reversion and ultimately we will simulate a plot from randomly generated model sample paths.
For model evaluation, we shall examine the log-likelihood values, the information criteria such as the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) for each one of the models under study. The AIC or BIC for a model is usually written in the form $[-2logL + kp]$, where L is the likelihood function, p is the number of parameters in the model, and k is 2 for AIC and Ln(n) for BIC. In this case, AIC estimates a constant plus the relative distance between the unknown true likelihood function of the data and the fitted likelihood function of the model, so that a lower AIC means a model is considered fit. On the other hand, BIC estimates a function of the probability of a model being true, under a certain Bayesian setup, so that a lower BIC means that a model is considered to be more likely to be the true model.Then use, the estimated parameters to simulate the out-of-sample data.Based on the results, conduct out-of-sample forecasting analysis and by comparing the results to alternative approaches to determine the best model for forecasting the prices. Evaluate forecast accuracy using RMSE and MAPE.
The mean absolute percentage error (MAPE), is the average of the absolute percentage errors

of forecasts which is defined as:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} |\frac{(A_i - F_i)}{A_i}| * 100,$$

where n is the number of observations, A_i,is the observed values and F_i,is the predicted values. Therefore, the smaller the MAPE the better the forecast accuracy of the model.

The Root Mean Square error, measures measure the error of a model in predicting quantitative data, without considering their direction which is defined as:

$$RMSE = \sqrt{\sum_{t=1}^{n} \frac{|(F_i - A_i)|^2}{n}},$$

Therefore, the smaller the RMSE the better the forecast accuracy of the model.

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4 Data Analysis and Results

In this chapter, we calibrate the models considered in the previous chapter to the data set used in this research from the European Energy Exchange (EEX) this research. We simulate them, and show how the models can be compared in terms of accuracy by comparing the measures of fit obtained for the prediction of each model. We break the data into two subsets: January 2, 2013 to December 31, 2016 and January 3, 2017 to September 09, 2020 and perform econometric analysis on the first set and the calibrations and simulations separately on second set. In this chapter,we also make use of a user-friendly software package: R for testing, calibration and simulation.

4.1 Data Description

This research uses the certified reduction emissions daily data as a proxy to estimate the parameters of mentioned diffusion and jump-diffusion process. The sample is a data set of 2620 daily observations covering the period from January 2013 to September 2020. The data is provided European Energy Exchange.

Studying the graphical representation of the CER price shown in Figure 4.1.1 for the period covered, we notice a persistent increase in price since 2015 to 2016 followed by a persistent decrease until 2017. The highest price of the CERs for the period covered was observed in 2016.

The daily returns (S_t) are calculated as the continuously compounded returns which are the first differences of log prices of EEX CER prices all of successive days under study:

$$LP = ln(\frac{S_t}{S_{t-1}}),$$

. where S_t and S_{t-1} are respectively the closing market prices of CERs at the current day and previous day.

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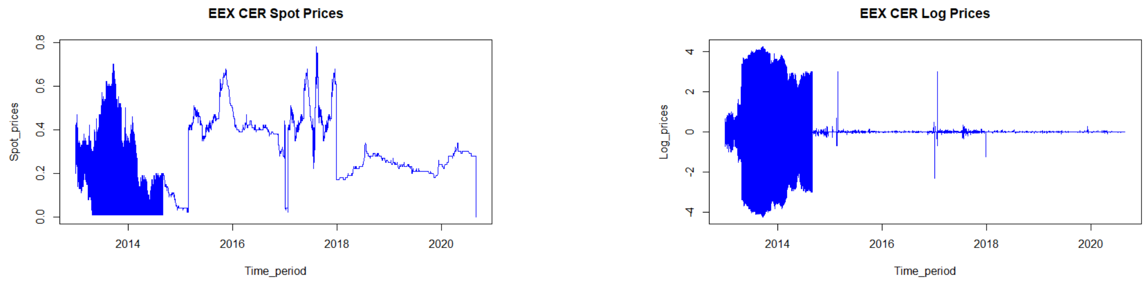

(a) EEX CER Spot Prices (b) EEX CER Log Prices

Figure 1: Corresponding trends for spot prices and logarithmic prices Jan-2012 to Sep 2020 traded in EEX

From Figure 1 the closing prices are very irregular with varied degree of fluctuations. The time plots clearly shows a drop in prices from a high value in 2016 to a low value in 2017, the high level were due to most countries especially developing countries who got ratified into the agreement by investing fully in the carbon market. But in 2017 some of the great emitters like USA and Canada pulled out of the agreement thus affecting the demand of the carbon offsets. Therefore, the series such as these cannot be used for further statistical inferences because of their implications (Gujarati, 2004), thus the need to transform them to log returns. The plots of daily returns of EEX CERs is presented in Figure 1(b). The plot clearly shows that the log returns are stationary and exhibit no trend.

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4.2 Descriptive analysis of the data

Table 1 below shows summary statistics for daily closing price of a CERs and the daily logarithmic prices of CERs at the end of the day t. The results for the log returns indicate high volatility and the risky nature of the market since the standard deviation of the market returns is high in comparison with the mean. Both price series have positive skewness implying that the distribution has a long right tail. On the other hand, the kurtosis both price series is high (above one) implying it is leptokurtic.

Table 1: Descriptive statistics of EUA spot prices (P) and logarithmic returns (LR)

Descriptive Analysis	Prices	Returns
Mean	0.2942770	-0.00009588
Standard Error	0.0035047	0.03055346
Minimum	0.0100000	-4.24849500
Maximum	0.7800000	4.24849500
Standard Deviation	0.1794280	1.56420600
Sample Variance	0.0321944	2.44674100
Kurtosis	2.2595490	5.22035700
Skewness	0.0310865	0.00849107
Range	0.7700000	8.49699000
Coef Variance	0.61	-16313.37
Count	2620	2620
Confidence Level(95.0%)	0.0068724	0.05991137

4.2.1 Normality Checks

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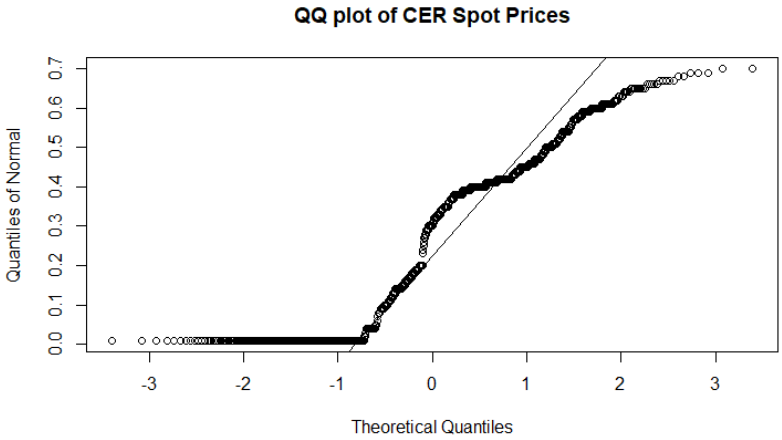
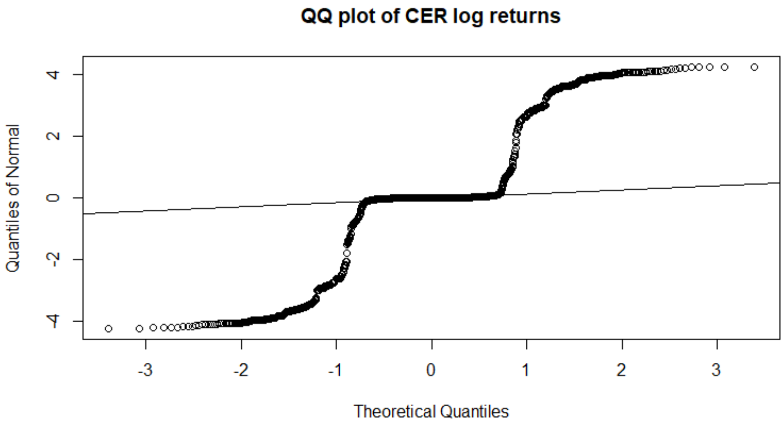
Statistical analysis should satisfy the assumption that the population follows a normal distribution with a common variance and additive error structure. When the relevant theoretical assumptions are approximately satisfied, the usual procedures can be applied in order to make inferences about unknown parameters of interest. Before starting the confirmatory analysis, we have created histogram and normal QQ plot of the daily CER EEX prices in order to have an idea about normality of data.



(a) Normal QQ Plot for daily EEX CER spot Prices



(b) Normal QQ Plot for daily EEX CER log returns

Figure 2: QQ Plot for daily EEX CER spot prices (P) and logarithmic returns (LR)

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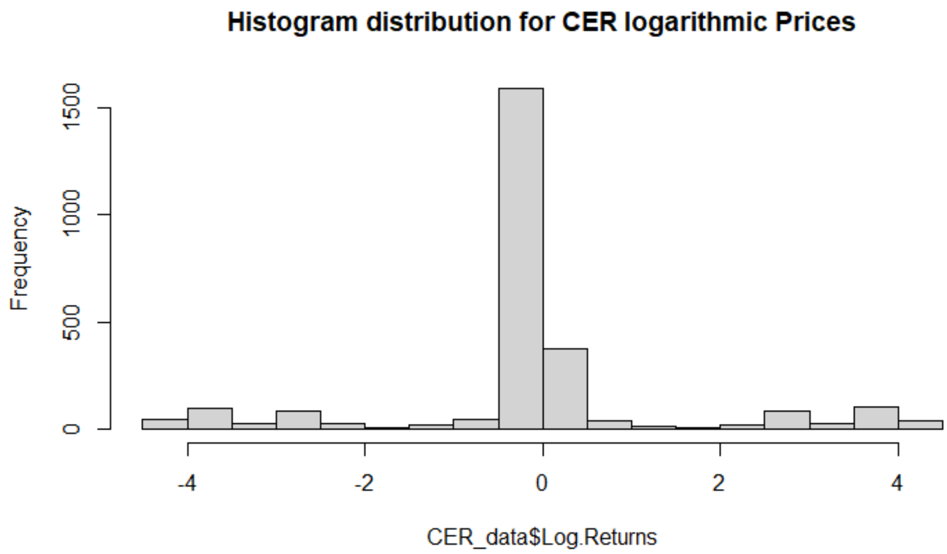

Figure 3: Histogram Distribution for EEX CER logarithmic returns (LR)

Notice the points fall along a line in the middle of the graph, but curve off in the extremities. Since our Q-Q plot exhibit this behavior usually the data under study have more extreme values hence both the histogram and Q-Q plot show that daily CER Prices traded in EEX are distributed as normal distribution.

Histogram distribution: It was observed that the histogram distribution for the real prices is right skewed, indicating that most of the prices are greater than the mode. whereas the histogram distribution for the log returns is uniform a clear indication of a normal distribution.

For more clarification on normality of the sample data, Jarque-Bera Test is used to test for normality in the series which are shown in the **Table 2** below. The p-values are less than 0.05, thus we fail to reject the null hypothesis of normality in the data so it can be concluded that the closing prices stock price is normally distributed and feasible to do stock price forecast on the data.

Table 2: Jarque-Bera for normality test for EEX CER prices.

	Jarque-Bera Test for Normality for RP and LP		
	X-value	df	P-value
Real-Prices	60.298	2	8.06E-14
Log Returns	538.43	2	< 2.2e-16

4.2.2 Unit Root Check

A stationary check for both closing prices and returns using Augmented Dickey Fuller (ADF) test shows that under the alternative hypothesis, unit root is not detected in both price series.

Table 3: Unit Root test for EEX CER prices.

ADF Test for auto-correlation for RP and LP		
Values	Prices	Returns
Augmented Dickey Fuller value	-2.0063	-11.352
Lag-order	11	11
P-value	0.5756	0.01
Hypothesis	HO:Stationary	HO:Stationary

4.2.3 Auto-correlation Check

To be able to see whether the chosen sample data is highly related with our response chosen models, we use Ljung-Box test given in **Table 4** to test whether series have significant auto-correlation or not.

Table 4: Auto-correlation test for EEX CER prices.

Ljung-Box Test for daily CER prices	
Values	Results
n	2620
df	2619
alpha	0.05
crit	2739.170821
BP	0
LB	2429.565409
P-Value	-0.996253294
sig	yes

From the above table, it is evident that the sample data under study have a significant auto-correlation since the p-value ≤ 0.05 thus we reject the null hypothesis.

4.2.4 Mean-Reversion Check

A market price follows a mean-reverting process if it has a tendency to return to its trend path over time,which means that market participants and investors may be able to forecast future returns better using information on past returns to determine the level of reversion to the long-term trend path.

Mean-reverting Test using t-statistics under ADF

A mean-reversion check for the closing prices understudy using Augmented Dickey Fuller (ADF) test shows that under the alternative hypothesis, mean-reversion detected in the price series.

Table 5: Mean-Reversion test for EEX CER prices.

Augmented Dickey-Fuller Test	
data: CER_data$CER.Spot.Price	
Dickey-Fuller(t-value)	-2.7024
Lag order	13
p-value	0.2809
df	2620
Upper Limit	1.645435
Lower Limit	-1.645435

From the above table, it is evident that the sample data under study is mean-reverting since the t-value $<$ the critical values thus we reject the null hypothesis.

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Mean-reverting Test using gamma

An alternative to the mean-reversion test described above, is to investigate if the data under study is a mean reverting series.
If the CERs spot prices is a mean reverting series, and has a negative gamma(γ), then this shows that the spot prices are mean-reverted to the mean. To calculate the above test and assess whether the spot prices are mean-reverting, we used R.We found that, the spot prices are mean-reverting since the calculated gamma is negative (-0.4844466).

4.3 Parameter Estimation

In this subsection, parameters of the model are estimated using the maximum likelihood estimation method. The parameters of the fitted models are shown in the table 5 below:
In this study Geometric Brownian Motion has been used to find the expected CER price, which expresses the change in market price using a constant drift μ and volatility σ. Therefore,in validating a GBM model, it was important to verify that the assumptions for the GBM are satisfied for the dataset under study, including normality and independence of ratios. Using actual data for CER prices, we found that the log returns did have a normal distribution. Therefore, a GBM would be an accurate representation of the price. Estimating the drift and volatility parameters for a GBM provides insights on future behavior and provide a basis for informed decision-making.
The Vasicek model predicts where interest rates will end up at the end of a given period of time, given current market volatility, the long-run mean interest rate value, and a given market risk factor. In this case,drift term exhibits mean reversion, meaning that interest rates will converge to the mean reversion level Θ with speed κ over time.
Based on the observation, the parameter κ is the rate at which the CER prices are pulled back to the long term mean,Θ.In that, the bigger the long term mean, the higher the price of CER at a defined period(t).

Table 6: ML Estimation results of continuous-time models in-sample data

Parameters	Diffusion Models		
	GBM	MRSRP	Vasicek
μ	561.931		
σ	33.52405	3.28128	3.28128
κ		166.9299	166.9299
α	0.5	0.5	0.5
Θ		0.2939934	0.2939934
ξ	-2539.805	-1929.45	133.9488
BIC	5093.768	3895.24	-253.7392
AIC	5083.609	3868.9	-263.8976

From Table 6 above the models were fitted to the data, diagnosed and from the diagnosis and goodness of fit statistics, the vasicek model was found to be the best choice. Since in order to check if they are appropriate for modeling price dynamics of CERs. We select the model with the lowest AIC and BIC. In this case,vasicek model with BIC:-253.7392 and AIC:-263.8976.

4.4 Model Forecasting

The main objective of this study is limited to the task of modeling. However, since forecasting is the prime object of modeling, forecasts of the series based on the models chosen have been generated.

In our analysis, for each model under study the predicted prices are plotted against the actual prices between the period of 2017 and 2020 which is the out-of sample data. Figure 4, 5, and 6 is a clear representation of this objective. The performance of these estimated models are determined on the basis of some accuracy measures. In our study, we compute Mean Absolute Error(RMSE) and the Mean Absolute Percentage Error(MAPE). The results are displayed in Table 6.

4.4.1 Graphical representation of the predicted CER prices vs Out-of-sample dataset

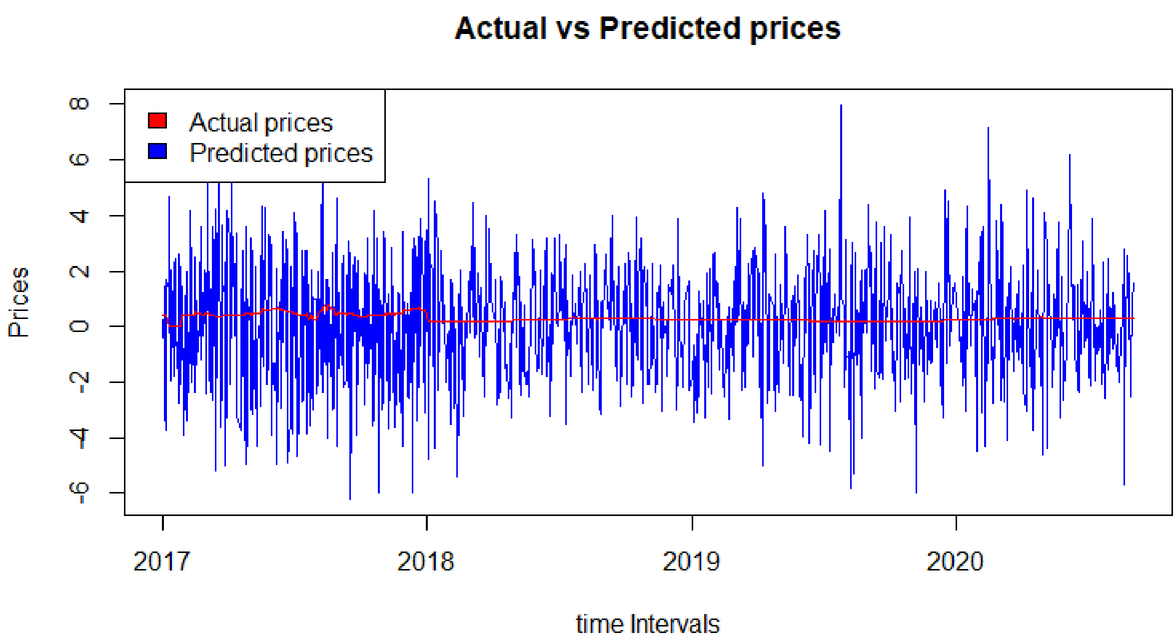

Figure 4: Geometric Brownian Motion forecasting

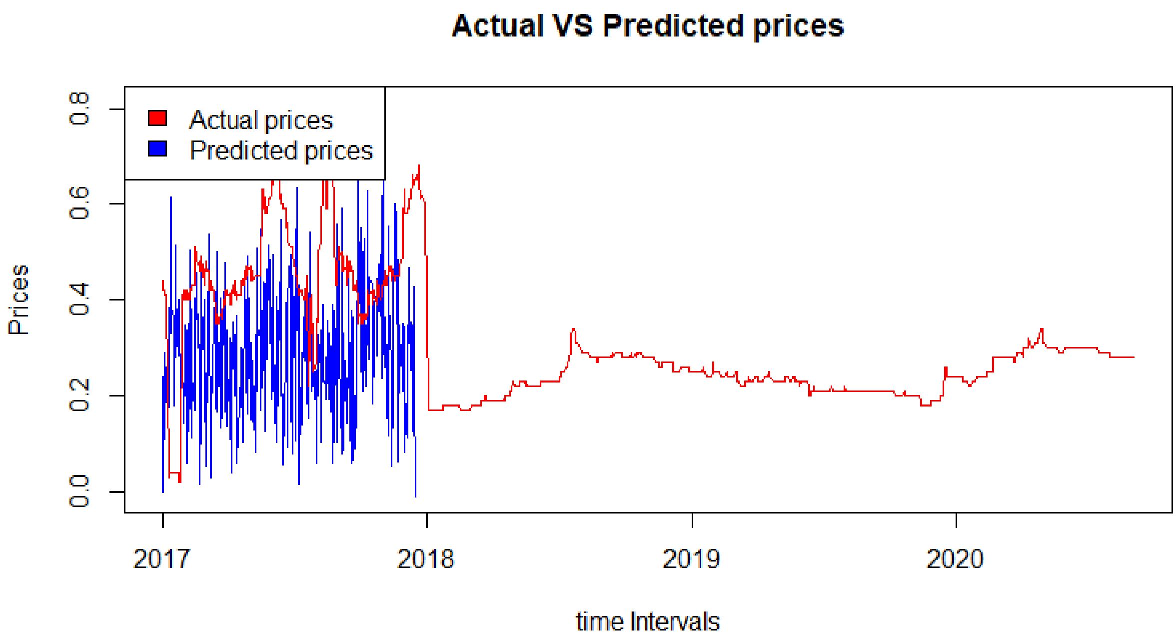

Figure 5: Mean-Reverting Square Root process forecasting

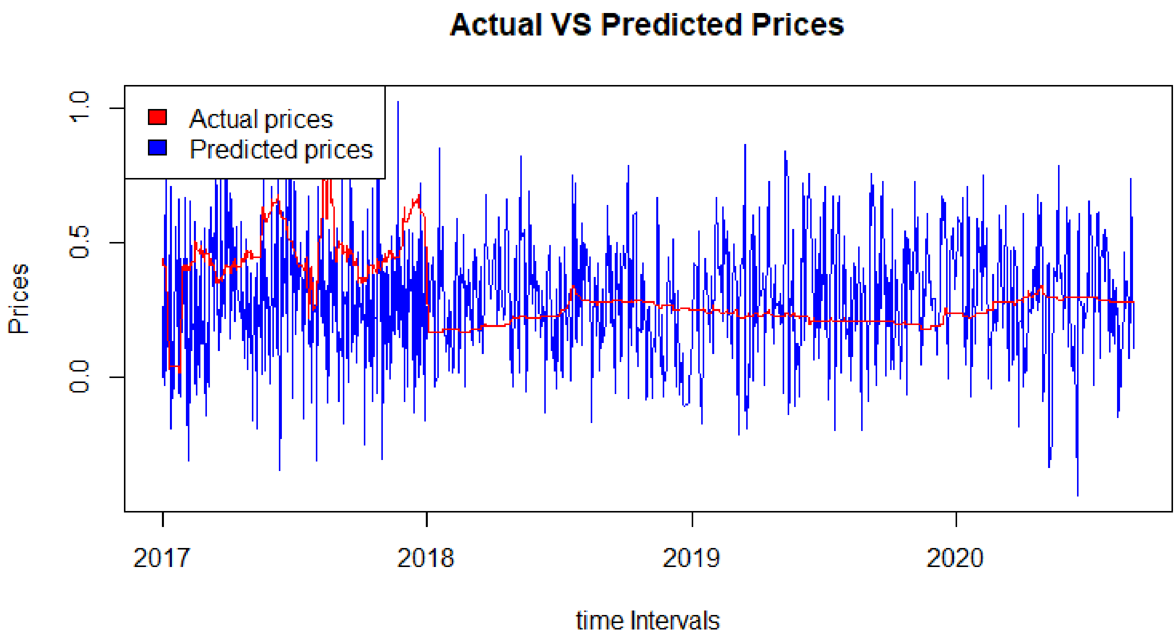

Figure 6: Vasicek Model forecasting

4.4.2 Forecasting Model Accuracy

Table 6 below shows the forecasts. From R output the Mean Absolute Error and MAPE were as follows:

Table 7: RMSE and MAPE Forecasting Analysis

Parameters	GBM	MRSRP	Vasicek
RMSE	0.3937	NAN	0.2985
MAPE	635.9094	NAN	81.75714

In this study,the Mean Absolute Error (RMSE) and Mean Absolute Percentage Error (MAPE) are computed to measure the performance of model on the dataset under study.
Mean Absolute Percentage Error (MAPE) being a statistical metric, measures how accurate a forecast system is; this is measured as the average absolute percent error of actual values minus predicted values divided by actual values whereas the mean absolute error (RMSE) is a quantity used to measure how close forecasts are to the eventual outcomes. Therefore, lower values indicate that the forecasted values are good predictions and the higher they increase in percentage indicates that forecaster is doing worse predictions.
As shown in table 6, the Vasicek model with MAPE accuracy metric, compared to other models, has became the best performing one as it has smallest error rate percentage. This proves that the Vasicek model is the best model to fit our data set with performance rate.
In MRSRP,the square root element $\sqrt{|(S_t)|}$ does not allow for negative rates and the model assumes mean reversion toward a long-term normal interest rate level.Therefore,under MRSRP, both measures are 'NAN' because, of the aspect of the square root element does not allow for negative rates and the model assumes hence eliminating almost 70% of the predicted values.

5 Summary, Conclusion and Recommendations

5.1 Summary of the study

This study, studies the price dynamics of CERs traded under the EEX market. The main objective being getting appropriate model for predictability of CER prices. To achieve this objective, the analysis involved descriptive analysis, model identification, order, parameter estimation, and forecasting. Main focus being the CERs daily returns series over the period January 2012 to September, 2020 forming a sample of 2621.

To specify the descriptive properties of CER daily returns over the period under study, descriptive analysis are calculated and reported using R in Table 1. According to results in table 1, the mean of the CER daily returns is -0.0009588 with a standard deviation of 0.0305535. The results indicate high volatility and the risky nature of the market since the standard deviation of the market returns is high in comparison with the mean. The results also indicate a positive skewness of 0.0084 implying that the distribution has a long right tail. A high value of kurtosis of 5.22 is also observed implying a leptokurtic distribution or heavy tailed since the value is above three. According to the JB test, the log returns are normally distributed which we fail to reject normality at 0.05 level.

From Figure 2, the closed prices and returns are stationary which is in support of normality test, the histogram distribution of the log returns is uniform with a mean of zero. The quantile-quantile (Q-Q) plot shows that the CER log return series are normal supporting the Jarque-Bera test indicated in Table 1. There is little evidence of significant correlation in the return series according to the Ljung-Box in Table 4. In addition to investigations about the data stationarity, the level of series are also defined. Augmented Dickey-Fuller (ADF) statistics clearly reject the hypothesis of a Unit Root at the 0.05 level of significance for the CER log return series. Table 3 summarizes the ADF test results.

The first and second specific objective was achieved by performing parameter estimation for each model using the Maximum Likelihood method and evaluated the models using the Information criteria and the fitted the likelihood function of the models under study.

From the results outlines in Table 5, it evident that the Vasicek model is the best fitted model with the lowest BIC of -23.7392, lowest AIC of -263.8976 and the the highest likelihood function of 133.9488, thus compared to other models in the study. This is because,in order to check the models if they are appropriate for modeling price dynamics of CERs. We select the model with

the lowest AIC and BIC.
To assess the fit of each model in the study and compare their accuracy in terms of forecasting the CERs price dynamics,the Root Mean Square Error(RMSE) and the Mean Absolute Percentage Error(MAPE) were computed.Based on the results in Table 6,the square root element $\sqrt{|(S_t)|}$ in MSRSP does not allow for negative rates resulting in 'NAN'.Therefore, it is evident that the, Vasicek model is the best performing model in terms of forecasting the price dynamics of CERs since it has the lowest RMSE and MAPE accuracy metric.
From the empirical results, it has been found that USE returns are normal, positively skewed and stationary. Overall, vasicek model outperformed the other diffusion models(Geometric Brownian Motion and Mean-Reverting Square Root process)in modeling price dynamics of EEX CER prices.

5.2 Conclusion of the study

The first step was to analyse the data whereby the descriptive analysis indicated that Certified Emission Reductions spot prices are likely to be characterized by mean reversion and stationarity. In line with these results, an information criteria amongst continuous time processes showed that spot prices are better approximated by vasicek model evidence with a lower BIC value of -253.7392. An empirical evaluation, Figure 1,using actual market price data demonstrated that the existence of extreme discontinuous variations(movements) in carbon prices meaning that much caution is needed when dealing with carbon emission derivatives. This will help in achieving the primary goal of the market, i.e the emission reductions will be achieved at the least possible cost.
The findings of this study is valuable in designing of other emission trading schemes especially in the developing countries (which have already ratified the Kyoto protocol). This study also has several important implications such as risk management practices for both operating and upcoming emitting firms and the growing community of carbon investors. As with every market, the ability of carbon market stakeholders to speculate and hedge their positions is of paramount importance for sustaining a healthy volume and for ensuring market efficiency.

5.3 Recommendations

These findings are strongly recommended to financial managers and modelers dealing with carbon market. It is of value to investors who are interested in investing in carbon market and wish to avoid large, erratic swings in portfolio returns. They should structure their investments to produce a leptokurtic distribution. To the management of carbon market, the Investment relation officers should bear technical skills to understand and interpret issues that determine the value of the firm in order to disseminate information for their stocks and increase investor participation.

Further, in emerging markets, diversification and return benefits provided have attracted significant investors' interest which have led to significant portfolio equity inflows into these financial systems, and as a result, motivated the study of various aspects of price dynamics of commodities in these markets. For that reason, an imperative and contemporary filament of empirical researches should focus on the calculation of risk an investor or participant is bound to face in these markets. Future research should examine the performance of multivariate time series models when using daily returns.

This research is however limited because it considers CERs traded in EEX. Other researchers can study other markets listed under the ratification of the kyoto protocol putting into consideration markets in the developing countries and consider other alternative models like the mean reverting logarithmic process and the jump-diffusion processes.

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Appendix

Rcodes used for Analysis

Rcodes used for Descriptive Analysis

```
#Import data
CER_data<- read.csv("C://Users//Nyaitaha//Desktop//CER Spot Prices.csv")
#Spot_prices<-CER_data$CER.Spot.Price
#Log_Returns<-log(diff(Spot_prices))
Spot.Price<-CER_data$X.t.dt.
n <- length(Spot.Price)
Log_Returns<- log(Spot.Price[-1]/Spot.Price[-n])
#date for data
Time_Period<- as.Date(CER_data$Trading.Date,format = "%m/%d/%y")
##Packages Used
install.packages("zoo")
install.packages("xts")
install.packages("timedate")
install.packages("fitdistrplus")
#install.packages("stats")
install.packages("stats4")
install.packages("MASS")
library(zoo)
library(xts)
library(timeDate)
library(stats4)
library(MASS)
library(fitdistrplus)
#library(stats)
library(tseries)
##Descriptive Analysis
#stationarity Test
ADF_TEST<-adf.test(na.omit(Spot.Price))
ADF_TEST1<-adf.test(na.omit(Log_Returns))
#Tests
```

```
#Is the mean log return significantly different from zero=T test
t.test(Spot.Price)
t.test(Log_Returns)
#Normality Tests
#JB Test
library(AutoSEARCH)
jb.test(as.numeric(Spot.Price))## JB(PValue<0.05)= Accept Ho The data is normally distri
library(tseries)
#conduct Jarque-Bera test
jarque.bera.test(Spot.Price)
jarque.bera.test(Log_Returns)
#QQ plots
qqnorm(Spot.Price,ylab="Quantiles of Normal", main="QQ plot of CER Spot Prices");qqline(
qqnorm(Log_Returns,ylab="Quantiles of Normal", main="QQ plot of CER Prices");qqline(Log_
ggplot(Spot.Price[-1], aes(Time_period[-1],Log_Returns))+geom_line()+labs(x="Date",y="Lo
#Histogram
library(ggplot2)
Datap<-as.data.frame(Spot.Price)
ggplot(aes(x=Spot.Price, y=Time_Period, fill=Time)) +
  geom_bar(stat="identity") +theme_bw() +
  geom_text(aes(label=Date), vjust=1.6,size=3.5)
#L-Jung Box Test
#serial correlation in the log return series
Box.test(Spot.Price,lag=11,type=’Ljung’)
Box.test(Log_Returns,lag=11,type=’Ljung’)
```

Rcodes used for Model Estimation and Forecasting

```
    #Importing Data into R
CER_data<- read.csv("C://Users//Nyaitaha//Desktop//CER Spot Prices.csv")
Linear_regression<-lm(X.t.dt. ~ CER.Spot.Price,data=CER_data)
summary(Linear_regression)
plot1<-hist(CER_data$X.t.dt.,main = "Histogram distribution for CER Spot Prices")
PLOT2<-hist(CER_data$Log.Returns,main = "Histogram distribution for CER logarithmic Pric
```

```
##Original data plotting
Spot_prices11<-CER_data$X.t.dt.
n <- length(Spot_prices11)
Log_Return11<- log(Spot_prices11[-1]/Spot_prices11[-n])
Log_Returns12<-Log_Return11[1:2621]
Time_Period11<- as.Date(CER_data$Trading.Date,format = "%m/%d/%y")
Time.Period12<-Time_Period11
plot(Time.Period12,Spot_prices11,type="l",xlab="Time_period",ylab="Spot_prices",col="blu
plot(Time.Period12,Log_Returns12,type="l",xlab="Time_period",ylab="Log_prices",col="blue
# Estimating parameters of the original data
## Geometric Brownian Motion model
## dXt=\mu Xtdt+\sigma Xtdwt
#Spot prices
#Spot_prices<-CER_data$CER.Spot.Price[1:1435]
#Log_Returns<-log(diff(Spot_prices))
Spot_prices<-CER_data$X.t.dt.[1:1434]
n <- length(Spot_prices)
Log_Return<- log(Spot_prices[-1]/Spot_prices[-n])
Log_Returns<-Log_Return[1:1434]
#date for data
Time_Period<- as.Date(CER_data$Trading.Date,format = "%m/%d/%y")
Time.Period<-Time_Period[1:1434]
# Estimating parameters of the original data
summary(Spot_prices)#mean of the empirical data
Price_var<-var(Spot_prices)#variance of the empirical data
Delta<-1/252# Converting the daily data to yearly data
##Pre-estimate of GBM parameters
GBM_var<- var(diff(log(Spot_prices)))/Delta
GBM_sd<-sqrt(GBM_var)
GBM_mean <- mean(diff(log(Spot_prices)))/Delta + 0.5 *
suppressWarnings(X<-zoo(Spot_prices,order.by = Time_period1))
# Calibration/Estimating the parameters
Regression_Sum<-summary(Linear_regression)
Coeffi<-Regression_Sum$coefficients
```

```
Intercept<-Coeffi[1,1]
Gradient<-Coeffi[2,1]
Std_error<-Regression_Sum$sigma
dt<-1/252
Result<-data.frame("mu"=GBM_mean,"sigma"=GBM_sd)
#Simulations
#simulated data
library(Sim.DiffProc)
N<-1187
#N<-2020-2017#Number of simulations
M<-1#Number of paths
dt<-1/252
X<-matrix(0,N,M)
X[1,]<-CER_data$CER.Spot.Price[1435:2621]
for (i in 2:N){
  Z<-rnorm(M,0,1)
  dw<-sqrt(dt)*Z[1]
  X[i,1]=X[i-1,1]*exp(GBM_mean-(0.5*GBM_sd^2))*dt+GBM_sd*dw
}
Predicted_Prices<-as.data.frame(X)
#Packages Used
install.packages("stats4")
install.packages("MASS")
install.packages("fitdistrplus")
library(stats4)
library(MASS)
library(fitdistrplus)
##Goodness of fit
Dist_model<-fitdist(Predicted_Prices, "norm", method = "mle",start=NULL, fix.arg=NULL, k
summary(Dist_model)
plot(Dist_model)
library(ggplot2)
#date for data
Time_Period<- as.Date(CER_data$Trading.Date,format = "%m/%d/%y")
```

```
Time.Period<-Time_Period[1:1434]
Time.Period1<-Time_Period[1435:2621]
Graph<-matplot(Time.Period1,Predicted_Prices,type='l', xlab='time Intervals',
               ylab='Prices',main='Actual and Simulated Data',col="blue")
lines(Time.Period1,CER_data$CER.Spot.Price[1435:2621],type="l", col="red")
legend("topleft", c("Actual data","Simulated data"),
       fill=c("red","blue"))
##forecasting
library(tseries)
#Estimated Mean Equation
library(forecast)
library(lubridate)
library(ggplot2)
#date for data
Time_Period<- as.Date(CER_data$Trading.Date,format = "%m/%d/%y")
Time.Period<-Time_Period[1:1434]
Time.Period1<-Time_Period[1435:2621]
Graph<-matplot(Time.Period1,Predicted_Prices,type='l', xlab='time Intervals',
               ylab='Prices',main='Actual and Simulated Data',col="blue")
lines(Time.Period1,CER_data$CER.Spot.Price[1435:2621],type="l", col="red")
legend("topleft", c("Actual data","Simulated data"),
       fill=c("red","blue"))
library(forecast)
##Accuracy
library(Metrics)
library(class)
library(tidyverse)
library(modelr)
library(broom)
Observed_Prices<-as.numeric(CER_data$CER.Spot.Price[1435:2621])
Predicted_prices<-as.numeric(unlist(Predicted_Prices))
##Goodness of fit
library(fitdistrplus)
Dist_model<-fitdist(Predicted_prices, "norm", method = "mle",start=NULL, fix.arg=NULL, k
```

```
summary(Dist_model)
MAPE_GBM<-(Predicted_Prices-actual_price)/actual_price
MAPE<-abs(MAPE_GBM)
head(MAPE,n=3)
tail(MAPE,n=3)
library(ie2misc)
MAPE<-mape(Predicted_prices,Observed_Prices, na.rm = FALSE)
RMSE<-sqrt(mean(Predicted_prices-Observed_Prices, na.rm = FALSE)^2)
##########################################################################################
df<-as.data.frame(Time_period1)
##Mean Reverting Square Root Process
# Calibration/Estimating the parameters
a<--(log(Gradient)/dt)
b<-Intercept/(1-Gradient)
vola<-sqrt(((Std_error^2)*2*a)/(1-Gradient^2))
Result<-data.frame("a"=a,"b"=b,"sigma"=vola)
#Simulations
#simulated data
library(AutoSEARCH)
library(Sim.DiffProc)
N<-1187
#N<-2014-1960#Number of simulations
M<-1#Number of paths
dt<-1/252
X<-matrix(0,N,M)
X[1,]<-CER_data$CER.Spot.Price[1435:2621]
for (i in 2:N){
  Z<-rnorm(M,0,1)
  dw<-sqrt(dt)*Z[1]
  X[i,1]=X[i-1,1]+a*(b-X[i-1,1])*dt+vola*prod(sqrt(X[i-1,1]))*dw
}
Predicted_Prices2<-as.data.frame(X)
library(ggplot2)
#date for data
```

```
Time_Period<- as.Date(CER_data$Trading.Date,format = "%m/%d/%y")
Time.Period<-Time_Period[1:1434]
Time.Period1<-Time_Period[1435:2621]
Graph<-matplot(Time.Period1,Predicted_Prices2,type='l', xlab='time Intervals',
               ylab='Prices',main='Actual and Simulated Data',col="blue")
lines(Time.Period1,CER_data$CER.Spot.Price[1435:2621],type="l", col="red")
legend("topleft", c("Actual data","Simulated data"),
       fill=c("red","blue"))
#Forecasting Accuracy
library(Metrics)
library(class)
#Estimated Mean Equation
library(forecast)
##Accuracy
library(Metrics)
library(class)
library(tidyverse)
library(modelr)
library(broom)
Observed_Prices1<-as.numeric(CER_data$CER.Spot.Price[1435:2621])
Predicted_prices2<-as.numeric(unlist(Predicted_Prices2))
##Goodness of fit
library(fitdistrplus)
Dist_model<-fitdist(Predicted_prices2, "norm", method = "mle",start=NULL, fix.arg=NULL,
summary(Dist_model)
MAPE_CIR<-(Predicted_prices2-Observed_Prices1)/Observed_Prices1
MAPE1<-abs(MAPE_CIR)
head(MAPE1,n=3)
tail(MAPE1,n=3)
library(ie2misc)
MAPE<-mape(Predicted_prices2,Observed_Prices, na.rm = FALSE)
RMSE<-sqrt(mean(Predicted_prices-Observed_Prices, na.rm = FALSE)^2)
########################################################################################
Coeffi<-Regression_Sum$coefficients
```

```
Intercept<-Coeffi[1,1]
Gradient<-Coeffi[2,1]
Std_error<-Regression_Sum$sigma
dt<-1/252
Result<-data.frame("mu"=GBM_mean,"sigma"=GBM_sd)
actual_price<-CER_data$X.t.dt.[1435:2621]
##Vasicek Model
# Calibration/Estimating the parameters
a<--(log(Gradient)/dt)
b<-Intercept/(1-Gradient)
vola<-sqrt(((Std_error^2)*2*a)/(1-Gradient^2))
Result<-data.frame("a"=a,"b"=b,"sigma"=vola)
#Normalcy test
library(fitdistrplus)
library(stats)
library(stats4)
library(MASS)
library(AutoSEARCH)
#Simulations
#simulated data
library(Sim.DiffProc)
N<-1187
#N<-2020-2017#Number of simulations
M<-1#Number of paths
dt<-1/252
X<-matrix(0,N,M)
X[1,]<-CER_data$CER.Spot.Price[1435:2621]
for (i in 2:N){
  Z<-rnorm(M,0,1)
  dw<-sqrt(dt)*Z[1]
  X[i,1]=X[i-1,1]+a*(b-X[i-1,1])*dt+vola*dw
}
Predicted_Prices3<-data.frame(X)
Standard_Error1<-Predicted_Prices3-CER_data$CER.Spot.Price[1435:2621]
```

```
##forecasting
library(tseries)
#Estimated Mean Equation
library(forecast)
library(lubridate)
library(ggplot2)
#date for data
Time_Period<- as.Date(CER_data$Trading.Date,format = "%m/%d/%y")
Time.Period<-Time_Period[1:1434]
Time.Period1<-Time_Period[1435:2621]
Graph<-matplot(Time.Period1,Predicted_Prices3,type='l', xlab='time Intervals',
               ylab='Prices',main='Actual and Simulated Data',col="blue")
lines(Time.Period1,CER_data$CER.Spot.Price[1435:2621],type="l", col="red")
legend("topleft", c("Actual data","Simulated data"),
       fill=c("red","blue"))
library(forecast)
##Accuracy
library(Metrics)
library(class)
library(tidyverse)
library(modelr)
library(broom)
Observed_Prices<-as.numeric(CER_data$CER.Spot.Price[1435:2621])
Predicted_Prices3<-as.numeric(unlist(Predicted_Prices3))
##Goodness of fit
library(stats4)
library(MASS)
library(fitdistrplus)
Dist_model<-fitdist(Predicted_Prices3, "norm", method = "mle",start=NULL, fix.arg=NULL,
summary(Dist_model)
MAPE_Vasic<-(Predicted_Prices3-actual_price)/actual_price
MAPE2<-abs(MAPE_Vasic)
head(MAPE2,n=3)
tail(MAPE2,n=3)
```

```
install.packages("ie2misc")

library(ie2misc)

MAPE<-mape(Predicted_Prices3,Observed_Prices, na.rm = FALSE)

RMSE<-sqrt(mean(Predicted_prices-Observed_Prices, na.rm = FALSE)^2)
```

Similarity Checker/Plagscan Report

Curiginal

Document Information

Analyzed document	Final Thesis.pdf (D118694200)
Submitted	2021-11-15 20:10:00
Submitted by	
Submitter email	Evalin.Kariuki@strathmore.edu
Similarity	1%
Analysis address	library.strath@analysis.urkund.com

Sources included in the report

W	URL: http://aplicaciones.cimat.mx/Personal/sites/default/files/jbhayet/files/clase4.pdf Fetched: 2021-11-15 21:09:00	1
W	URL: https://bearworks.missouristate.edu/cgi/viewcontent.cgi?article=4491&context=theses Fetched: 2021-11-15 21:09:00	1
W	URL: http://www.columbia.edu/~ks20/FE-Notes/4700-07-Notes-GBM.pdf Fetched: 2021-11-15 21:09:00	3

Figure 7: Similarity Checker/Plagscan Report

Originality Report

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RHInnO Ethics - SU-IERC1125/21 - 1 of 1

Final Decision

This document certifies that the study:

\\\"A COMPARATIVE MODELLING OF PRICE DYNAMICS OF CERTIFIED EMISSION REDUCTIONS\\\"

Principal Investigator: Mrs. Kariuki, Evalin Wanjiru
Reference number: SU-IERC1125/21

Was reviewed and received the following status:
\\\"done\\\"
Additional Comments: Final decision: **approved**
Comments sent:

Reviewer #1:
'Amendment Approved '

Figure 8: Originality Report

Ethical Certificate

Strathmore UNIVERSITY

4th August 2021

Ms Kariuki, Evalin
evalin.kariuki@strathmore.edu

Dear Ms Kariuki,

<u>REF: SU-IERC1125/21 (*AMENDMENT*) PROPOSAL "A Comparative Modelling of Price Dynamics of Certified Emission Reductions"</u>

I make reference to your application for the approval of a proposed amendment submitted on 15th July 2021.

We acknowledge receipt of the following submitted documents for amendment.

a) Amendment Cover letter.
b) Amended Study Proposal dated 30th June 2021
c) Study budget
d) CV of Investigator

The committee noted the following amendment(s):

1. ***Study Design and Sampling: - Study now forecasts the price dynamics of Certified Emission Reductions by comparing and acquiring the most consistent and accurate forecast model using the diffusion approaches: Geometric Brownian Motion, Vasicek Model, and Mean Square Root Process. Focusing on the Certified Emission Reductions prices traded within European Energy Exchange(EEX) between the years of 2012-2020.***
2. ***Study Procedure: - there is a change in the methodology by the addition of one diffusion process for comparative purposes.***

The Committee concluded that the suggested amendments are justified and will not result in increased risk to the participants. The proposed changes have therefore been granted **approval** for implementation. You may continue with your study.

You are required to submit any further changes to this version of the protocol to SU-IERC for review and approval prior to implementing any additional changes.

Yours sincerely,

STRATHMORE UNIVERSITY INSTITUTIONAL ETHICS REVIEW COMMITTEE (SU-IERC)
04-Aug-2021
TEL: +254 (0)70303400
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Figure 9: Ethical Certificate

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