Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

Bachelor of Business Science Actuarial Science, Finance & Financial Economics

March 2018 Special Examination

BSA 3108:- Theory of Finance

Instructions: Read Carefully

1) This examination consists of FIVE questions.

2) Answer Question ONE (COMPULSORY) and any other TWO questions.

3) Question One carries 30 marks.

4) All other question carry 20 marks each.

Question 1: Compulsory

1. (a) Explain the following concepts as used in derivative pricing:

i. Replicating Portfolio *(3 marks)*

ii. Self Financing Portfolio *(3 marks)*

iii. No- Arbitrage Principle *(3 marks)*

(b) An investor faces the following investment opportunity set:

$$F(x_0, x_1) = (10,000 - x_(0))(1.08 + 0.00006x_(0)) - x_1$$

$$0 \leq x_0 \leq 10,000$$

With a utility function given by $U(x_0, x_1) = x_0^2 x_1$. x_t denotes investment in period t respectively. Assume a capital market exists and that all investors can borrow and lend at 8%.

i. Find the investors consumption opportunity set. *(6 marks)*

ii. Find the investors optimal consumption bundle. *(8 marks)*

(c) Suppose there are three states and three securities have the following payoff vectors in state S; $X_S = [x_1, x_2, x_3]$ where $X_1 = [6, 6, 2]$, $X_2 = [3, 0, 0]$, and $X_3 = [0, 3, 1]$.

Assume you can buy or sell fractions of a share. Prove that this market is complete or otherwise. *(3 marks)*

(d) TRUE or FALSE? Please explain your answers carefully and fully. No marks will be rewarded for a true/false-only answer.

i. The Capital Asset Pricing Model (CAPM) implies that all risky assets must have a positive risk premium. *(2 marks)*

ii. The value of an American call option is always equal to the value of a European call option. *(2 marks)*

Total for Question 1: 30 marks

Question 2

2. (a) Derive using suitable workings the pricing relation under **the Arbitrage Pricing Theory**. What are the implications of this results for modelling asset prices? *(12 marks)*

(b) Suppose you own a farm that, if run efficiently, can produce corn according to the following transformation formula

$$W_1 = 400 \times I_0^{\frac{1}{2}}$$

where I_0 is the number of bushels of corn planted at date 0, and W_1 is the number of bushels turned over to you at date 1, net after all payments to labor and other hired inputs. Your utility function of consumption at date 0 and consumption at date 1 is:

$$U(C_0, C_1) = min(C_0, C_1)$$

i. If 50,000 bushels of corn are planted, what will be the net output of corn at date 1? *(2 marks)*

ii. If you set a target for output of 100,000 bushels, what is the minimum number of bushels that must be planted *(2 marks)*

iii. Suppose capital markets do not exist, and you can neither borrow, lend, nor store any corn at the beginning of date 0. If you have 22,500 bushels of corn at date 0, what will your production plan be? Your consumption plan? What will be the average rate of return on investment of corn? *(4 marks)*

(12 marks)

Total for Question 2: 20 marks

Question 3

3. (a) Three models namely Mean Variance Optimization, Capital Asset Pricing Model and the Arbitrage Pricing Model have underpinned asset pricing models and the theory of finance. Briefly discuss why if any of the three models works, then it implies the applicability or otherwise of the other two in an economy. *(10 marks)*

(b) John and Mary are two investors with the same utility function i.e $U(\cdot) \Rightarrow U_{John}(\cdot) = U_{Mary}(\cdot)$ but different levels of intial wealth. Will John and Mary be characterised by identical risk aversion? **Justify** using suitable workings.

(5 marks)

(c) Two assets have the following distribution of returns:

Return (1) (%)	Probability (1)	Return (2)	Probability (2)
-1	8.33	0	50
11	91.67	20	50

i. Calculate the following for each asset. Where necessary assume a benchmark return of 0%

α) Variance *(1 mark)*

β) Semi-variance *(1 mark)*

γ) Shortfall probability *(1 mark)*

ii. Which asset would an investor with a quadratic utility function choose. Explain. *(2 marks)*

Total for Question 3: 20 marks

Question 4

4. (a) Differentiate the Grossman-Stiglitz paradox from the Cornell-Roll conclusions. Carefully highlight the implications of each model for Market Efficiency. *(10 marks)*

(b) Consider a mean-variance portfolio model with two securities X and Y where the expected return and variance of return of return for Y are *four times* the corresponding values for X. Suppose the two assets returns are negatively correlated with a correlation ρ.

i. Determine the values of ρ which allow the possibility of constructing a zero-risk portfolio, by calculating the variance of the return on a portfolio with weights w_X and W_Y invested in the two assets. *(4 marks)*

ii. Calculate the portfolio weights that lead to the most efficient zero-risk portfolio. *(3 marks)*

iii. Calculate the expected return on the portfolio in part (ii) in terms of the expected return on X. *(3 marks)*

Total for Question 4: 20 marks

Question 5

5. (a) In their famous "Financial Modelers Manifesto" of 2009 Emanuel Derman and Paul Wilmott made the following comments about financial modeling:

"Models are at bottom tools for approximate thinking; they serve to transform your intuition about the future into a price for a security today. Our experience in the financial arena has taught us to be very humble in applying mathematics to markets, and to be extremely wary of ambitious theories, which are in the end trying to model human behavior. We like simplicity, but we like to remember that it is our models that are simple, not the world. We do need models and mathematics you cannot think about finance and economics without them but one must never forget that models are not the world. Whenever we make a model of something involving human beings, we are trying to force the ugly stepsister's foot into Cinderella's pretty glass slipper. It doesn't fit without cutting off some essential parts. And in cutting off parts for the sake of beauty and precision, models inevitably mask the true risk rather than exposing it. The most important question about any financial model is how wrong it is likely to be, and how useful it is despite its assumptions. You must start with models and then overlay them with common sense and experience"

Using the **Expected Utility Hypothesis** as an example critically discuss the foregoing comments. Be careful to define all the assumptions and notations used.

(8 marks)

(b) Consider a two-period binomial model for a non-dividend paying stock whose current price is $S = 100$. Over each six-month period, the stock price can either move up by a factor $u = 1.2$ or down by a factor $d = 0.8$. The continuously compounded risk-free rate is r = 5% per six-month period.

i. Is there arbitrage in the market? *(2 marks)*

ii. Calculate the price of a standard European call option written on the stock S with strike price K =100 and maturity one year. *(8 marks)*

iii. Calculate the arbitrage fee price a put option on the stock. *(2 marks)*

Total for Question 5: 20 marks