Strathmore University

Strathmore Institute of Mathematical Sciences

Bachelor of Business Science in Economics, Financial Engineering and Actuarial Science

BSM 2111: Statistical Inference

Repeat Class Examinations

Date: 3rd April 2024 **Time: 2 Hours**

Instructions: Answer Question ONE and ANY other TWO questions.

Question One [30 marks]

(a) (i) An efficient and a sufficient estimator. **[2 marks]**

(ii) Point estimator and an interval estimator. **[2 marks]**

(b) Let $X_1, X_2, \cdots, X_n$ be a Bernoulli random sample with parameter θ where $0 < \theta < 1$. Show that $\tau(X) = f(X_1, X_2, \cdots, X_n)$ is a sufficient statistic for θ. **[5 marks]**

(c) Every semester, students undertaking Probability & Statistics II must sit for a continuous assessment test. The amount of time allowed for the test is found to be normally distributed with mean, $\mu = 130$ minutes and standard deviation, $\sigma = 45$ minutes. Determine the probability that a student takes

(i) Less than 100 minutes. **[3 marks]**

(ii) Between 2 hours to 3 hours. **[4 marks]**

(d) Let $X_1, X_2, \cdots, X_n$ denote a random sample from a population with mean μ and variance σ^2. Show that $\hat{\mu} = \frac{1}{2}(X_1 + X_2)$ is an unbiased estimator for μ. **[3 marks]**

(e) Two investigators have studied the income of a group of persons by the method of sampling. The following table gives their findings

Investigator	Poor	Middle Class	Well-to-do	Total
A	160	30	10	200
B	140	120	40	300
Total	300	150	50	500

Using Chi-squared test, show that the sampling technique of at least one of the investigators is a suspect at 5% level of significance.

[6 marks]

(f) A software company uses the model below to predict the lifespan of new computers in their warehouse

$$f(y;\theta) = \begin{cases} \theta e^{-\theta y}, & y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Use the method of moments to estimate the unknown parameter θ. **[5 marks]**

Question Two [20 marks]

(a) Let $X_1, X_2, \cdots, X_n$ denote a random sample from a normal distribution with mean μ and variance σ^2. Two unbiased estimators of σ^2 are given as:

$$\hat{\sigma_1}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

and

$$\hat{\sigma_2}^2 = \frac{1}{2}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Determine the efficiency of $\hat{\sigma_1}^2$ relative to $\hat{\sigma_2}^2$. **[5 marks]**

(b) An investor buys a stock whose return (including both dividends and the change in stock prices) depends on whether the nation's gross domestic product is rising, constant or falling. If the gross domestic product is rising, the return is 20 percent per dollar invested; if its constant, the return is 5 percent; and if its falling, the return is - 10 (minus 10) percent. Jane believes that it is equally likely that gross domestic product will rise, remain constant, or fall. Determine:

(i) Develop the probability distribution table. **[2 marks]**

(ii) Find the expected value of return from this stock. **[4 marks]**

(iii) Calculate the standard deviation in percent of the return. **[4 marks]**

(c) The amount of time, X in minutes a bank teller spends with a customer is known to have an exponential distribution with an average amount of time of 4 minutes. The density function is defined below:

$$f(x) = \begin{cases} \frac{1}{4}e^{-\frac{1}{4}x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that a teller spends $four$ to $five$ minutes with a randomly selected customer.

[5 marks]

Question Three [20 marks]

(a) Suppose deposit amounts, X, by customers to a savings account are uniformly distributed over the interval (α, β). Given also that the density of the distribution is defined by

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & \alpha \leq x \leq \beta \\ \\ 0, & \text{Otherwise} \end{cases}$$

(i) Show that the function $f(x)$ is a probability density function. **[3 marks]**

(ii) Find the moment generating function of X. **[4 marks]**

(iii) Find the mean of X. **[3 marks]**

(iv) Obtain the variance of X. **[5 marks]**

(b) The average commission charged by full service brokerage firms on a sale of common stock is \$ 84, and standard deviation is \$ 10. Janet has taken a random sample of 75 traders by her clients and determined that they paid an average commission of \$ 81.50. At 0.10 significance level, can Janet conclude that her clients' commission are higher than the industry average? **[5 marks]**

Question Four [20 marks]

A random sample of 5 employees $(y_1, y_2, y_3, y_4, y_5)$ who have taken an insurance policy in a certain company is drawn from a normal population with an unknown mean μ and variance σ^2. Consider the following estimators to estimate μ.

$$\hat{\theta}_1 = \frac{y_1 + y_2 + y_3 + y_4 + y_5}{5}$$

$$\hat{\theta}_2 = \frac{y_1 + y_2}{2} + y_3$$

$$\hat{\theta}_3 = \frac{2y_1 + y_2 + \alpha y_3}{3},$$

where α is such that $\hat{\theta}_3$ is unbiased estimator for μ.

(a) Find the value of α. **[4 marks]**

(b) Are $\hat{\theta}_1$ and $\hat{\theta}_2$ unbiased? Show your workings. **[6 marks]**

(c) Which one among the estimators is efficient between $\hat{\theta}_1$ and $\hat{\theta}_3$, assuming that the samples are independent? **[10 marks]**

Question Five [20 marks]

(a) Let X be a discrete random variable with probability mass function given by

X	0	1	2	3
$P(X = x)$	$\frac{1-\alpha}{3}$	$\frac{\alpha}{3}$	$\frac{2(\alpha-1)}{3}$	$\frac{2\alpha}{3}$

where $0 \leq \alpha \leq 1$ is a parameter. The following 12 independent observations were obtained from such a distribution.

1, 2, 1, 0, 3, 1, 2, 1, 0, 3, 0, 1

Find the method of moments (MoM) estimator of α. **[6 marks]**

(b) Let $Y_1, Y_2, \cdots, Y_n$ be independently and identically distributed random variables with mean μ and variance σ^2 i.e. $Y_i \sim N(\mu, \sigma^2)$. Obtain the maximum likelihood estimator of σ^2. **[6 marks]**

(c) A laptop manufacturing company claims that its top quality laptops are good, on average, for at least 200 months before expiry. A consumer protection agency tested 60 such laptops and found that on average they last 265 months with a standard deviation of 10 months. Construct:

(i) 95% confidence interval for the mean. **[3 marks]**

(ii) 99% confidence interval for the mean **[3 marks]**

(iii) Which confidence interval should the agency use for better precision? Why? **[2 marks]**