Strathmore
UNIVERSITY

The Impact of Longevity risk in Defined Benefit Private Pension funds

Mlambo Florence Mghoi, 083628

Submitted in partial fulfillment of the requirements for the Degree of Bachelor of Business Science Actuarial Science at Strathmore University

Strathmore Institute of Mathematical Sciences

Strathmore University
Nairobi, Kenya

November, 2017

DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

FLORENCE MGHOI MLAMBO [*Name of Candidate*]

.... [*Signature*]

27/11/2017 [*Date*]

This Research Project has been submitted for examination with my approval as the Supervisor.

DR. JOHN OLUKURU [*Name of Supervisor*]

.... [*Signature*]

27/11/2017 [*Date*]

Strathmore Institute of Mathematical Sciences
Strathmore University

Abstract

Retirement benefits should be able to last until the retiree dies. With improvements in fields like technology and medicine, there has been a reduction in mortality rates and an increase in life expectancy. Defined benefit pension plans are one of the stakeholders of longevity risk that will suffer great losses if they ignore longevity risk.

This study will use the Lee Carter model to forecast mortality rates and show the increasing trend of life expectancy and how this affects the defined benefit pension funds. The main purpose of this paper is to determine how uncertainty associated with future mortality and life expectancy outcomes would affect the liabilities of a defined benefit pension plan. Finally, this paper will measure longevity risk by comparing the actuarial present values of annuities in Israel over the years and show the trend in the actuarial present values.

Table of Contents

List of Figures

CHAPTER 1: INTRODUCTION

1.1 Background

According to Wang, Huang, Yang, & Tsai (2010), birth life expectancy has doubled over the last two centuries for example life expectancy for individuals age 65 has been increasing on an average of two months per year over the last decade. The life expectancy of people living in most of the OECD countries has increased in the last century by twenty five to thirty five years. The steady increase in the improvement of life expectancy is as a result of factors such as improved medical services.

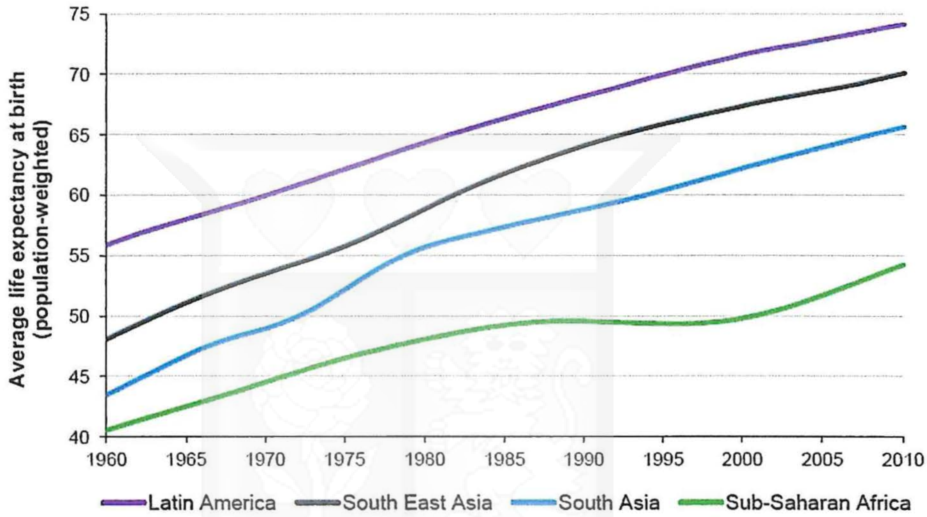
The gains in life expectancy are generally positive news although it comes with the price of longevity risk. Longevity risk is the risk that individuals will have longer lifetimes than expected. If gains in life expectancy could be forecasted and factored in in retirement planning, then the effect of longevity risk could be minimal and hence negligible but unfortunately improvement in life expectancy and mortality are uncertain. Thus, longevity risk is related with the uncertainty surrounding future mortality and life expectancy.

1.1.1 Trends in Longevity Risk

The stakeholders of longevity risk are; individuals, company pension funds, annuity providers (insurance companies), the state and the public employee pension system. The stakeholders of longevity risk respond to longevity risk in two major ways which include either ignoring longevity risk or accepting longevity risk as a legitimate business risk. Ignoring longevity risk is unwise as it could lead to serious losses to the stakeholder as longevity risk increases time horizon for pension paid or spent and therefore increases the probability of underfunding. The stakeholders that respond by accepting longevity risk as a legitimate business risk could respond by sharing longevity risk, re insurance (either by buy-out of pension liabilities or through buy-ins of bulk annuities) or managing longevity risk with index linked instruments.

Total longevity risk is made up of aggregate longevity risk (this is the systematic longevity risk) and specific longevity risk (the unsystematic longevity risk). The population basis risk forms the residual risk cannot be hedged away with standardized hedges (Coughlan, 2015).

Figure 1: Life expectancy



[1]

1.1.2 Pensions and Pension plans

According to Cannon & Tonks (2013), a pension should ensure that the consumption of an individual should not fall and this purpose is achieved by the pension when the individual is provided an income on retirement; the income should be almost similar to the labor income.

According to Forman (2012), defined contribution (DC) plans have come to dominate the pension landscape. The global popularity of Defined Contribution (DC) Pension plans is as a result of the recent increases in

---

[1] This graph has been obtained from http://www.openpop.org/?p=695 discussing how much life expectancy has increased since 1960.

longevity which has consequently increased pressure on the Defined Benefits (DB) Pension Providers (Yanga & Huang, 2009). How has the increase in longevity increased pressure on the Defined Benefits plan (DB)? Pension providers will have to pay out more cash than they had planned for due to the increase in payment period as a result of the increase in lifetime of the insured.

For the defined contribution (DC), longevity risk can be ignored by the pension provider due to the fact that the pension provider's role is contributing towards the pension not payment of the benefits; the risk falls on the retiree. According to Forman (2012), the following approaches can be taken by retirees to help manage longevity risk; systematic withdrawals, lifetime annuities, longevity insurance and guaranteed lifetime withdrawal benefits.

In Kenya, retirees who were DC plan holders are allowed to engage in income drawdown plans until the age of 70 whereby on attaining 70 years they are required to purchase life annuities("Regulation 25(6) of The Retirement Benefits (Occupational Retirement Benefits Schemes) Regulations, 2000 was amended under Legal Notice Number 77 of 2008 to provide that scheme rules may provide for the payment of retirement benefits by way of an income drawdown, as an alternative to the purchase of an annuity, for members at retirement age provided that the minimum drawdown period shall be ten years. "). This means that the government of Kenya has hedged against the longevity risk faced by retirees by imposing this law.

1.2 Problem Statement

Oeppen and Vaupel (2002) believe that life expectancy has its limits. Oeppen and Vaupel (2002) defend their argument by saying that human life has natural limits and that there is only so much that can be done in the medical field and innovation will reach a stop. Contrary to Oeppen and Vaupel (2002), Olshansky (2005) argues that life expectancy has no limits and that it will keep increasing over the years. According to Brown (2008), there is a great significance attached to the retirement period especially for defined benefit schemes as it is a large portion of the individual's average life and due to longevity improvements, the average length is growing.

The uncertainty surrounding life expectancy creates the need to incorporate longevity risk in pension contributions or benefits especially for the private defined benefit pensioners who bear longevity risk. Therefore, there is a need to study the impact of longevity risk in private defined benefit schemes.

1.3 Research Objectives

1. To forecast mortality rates
2. To measure longevity risk

1.4 Research Questions

1. What is the trend in mortality rates?
2. How can we measure longevity risk?

1.5 Motivation and justification of the research

This research will mainly benefit private defined benefit pension schemes. It will help give a better understanding of the longevity risk and its impact to available pension funds. A better understanding will consequently help the private defined benefit pension schemes to incorporate longevity risk in their pension calculations. When such schemes factor in longevity risk, it will help reduce losses as a result of longevity risk.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter discusses the theories on longevity risk in pension funds and some of the works that have been done on the same.

2.2 Theoretical Review

2.2.1 Longevity Risk

Longevity risk is present when the actual life lived exceeds the life expectancy. Theories on the increase of life expectancy are discussed below.

Survival probabilities have become more rectangular or compressed (Kannisto, 2000).The author suggested that life expectancy has limits.

Oeppen and Vaupel (2002) argued that there are no limits to life expectancy. Mortality is likely to level off thus leaving longevity uncapped and increasing in the next decades (Oeppen and Vaupel, 2002).

Whereas Oeppen and Vaupel (2002) conclude that no limits can be set to life expectancy, Olshansky (2005) is more conservative on the extent of the increases in longevity. He argued this based on historical trends and age paths. The conservative group contradicts Oeppen and Vaupel (2002).

Olshansky (2005) argues that transitions in the medical field and the decrease in mortality rates which are necessary for the increases in life expectancy suggest that the increase in life expectancy will slow down if not stop. He suggests that there are natural limits to human life.

Siegel (2005) concludes that the theory of compression of mortality (theory of rectangularization) by (Kannisto, 2000) is not conclusive.

Therefore, the extent of uncertainty surrounding the improvement of mortality rates and life expectancy is large. An approach to model future mortality rates is required.

2.2.2 Forecasting mortality rates

The basic models used in mortality rates are the Lee Carter (1992) and the Cairns, Blake and Dowd Model .They differ in the assumption of smoothness between ages. The Cairns, Blake and Dowd models have the underlying assumption that there is smoothness between ages. The underlying assumption for all models is that the quality of age, period and cohort effects is different in nature.

Lee Carter (1992) model

The Lee Carter Model (1992) is one of the models used in predicting future mortality rates and life expectancy. The model the probability that an individual who is age x in year t will die during the next year in a simple linear model. The Lee and Carter methodology focuses on replicating mortality rates from the past as well as extrapolate this model into the future.

Due to the stochastic model dynamic of changes in a future mortality, a model with a stochastic process is necessary when predicting future mortality trends and life expectancy (Lee, 2000). The Lee Carter model (1992) is often compared with the Cairns-Blake and Dowd model and both of the models have the distinctive feature of using a stochastic process to model uncertainty about the future.

Lee Carter used US mortality rates from the year 1933 to 1987. According to Lee*, 2000, the mortality rates are available at the human mortality data base

The Lee Carter model is expressed as follows:

$$\mu_{x,t} = ln(m_{x,t}) = exp(\alpha_x + \beta_x k_t + \varepsilon_{x,t})$$

(2.1)

Renshaw and Haberman (2003) Model

They proposed a multifactor model which is expressed as follows;

$$log m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)} + \beta_x^{(3)} k_t^{(3)}$$

(2.2)

Whereby $k_t^{(2)}$ and $k_t^{(3)}$ are dependent period effects.

Renshaw and Haberman (2006) Cohort Model

This model is an extension of the Lee Carter model with an additional parameter and it is expressed as follows;

$$\log m_{x,t} = \alpha_x + \beta_x^{(1)} k_{(t)} + \beta_x \gamma_{t-x}$$

(2.3)

Whereby $k_{(t)}$ is the mortality index in year t and γ_{t-x} is a random cohort effect that is a function of the years of birth t-x.

Age- Period Cohort Model

This simple age period cohort model was introduced by Curie (2006)

$$\log m_{x,t} = \beta_x^{(1)} + k_{(t)} + \gamma_{t-x}$$

(2.4)

Whereby the following constraints are imposed;

$\sum_t^n k_{(t)} = 0$ and $\sum \gamma_{t-x} = 0$

Model Selection

Given the discussed models, a lot of discussions have been invested in the Lee Carter model and it has been used to model mortality rates and consequently the quantification of longevity risks. A recommendation is given to forecasters to use the Lee Carter model for modeling mortality rates by Tuljapurkar (1998) and Tuljapurkar and Boe (1998) after they reviewed the Lee Carter model.

In their conclusion, Dowd et al (2010) agreed that the Lee Carter model (among other model) performs well most of the times. They put emphasis on the fact that their results are based on a specific set of data over limited sample duration and therefore they make no claim of how the models perform over other data sets or sample periods.

The Lee carter model provides a good fit to historical data as the age function of the lee carter model gives an allowance to model across all ages. In comparison to the other models which have been discussed, the Lee Carter model has fewer parameters and therefore in provides simplicity in fitting. Simplicity is also brought out by the singular value decomposition that is easy to put into practice. Therefore, this study will use the Lee Carter model to model future mortality rates.

2.2.3 Calculations of pensions and annuitization

Pension Replacement Ratio

Diamond (1977) conceptualizes the pension replacement ratio which is the ratio of the pension income to the labor income in the final year of employment. He examines the United States' public pension provision through the Social Security systems. Diamond (1977) conducts his analysis on the basis of redistribution of income, insurance provision whereby private markets are inefficient and the saving compulsion by individuals.

Annuitization

In the author's study, Yaari (1965) ascertained that an individual who is risk averse ought to completely annuitize his/ her savings in order to maximize utility if the alternative choice were an asset which is risk free.

Mitchell et al. (1999) defined the evasion of individuals to annuitize as a" long-standing puzzle". In their study, Mitchell et al. (1999) showed the value of choosing the route of annuitization. They used expected utility as their measure of outcome and concluded that annuities were a preferred choice when compared to income drawdown plans. They also found that risk aversion was directly related to appeal of annuities, that is, the greater the risk aversion, the more attractive the annuities were. This was attributed to the fact that annuitization had an insurance element which guarantees a consistent stream of income for life. The drawback for this study was that it did not put the value of bequests into consideration

Orszag (2000) suggested making annuitization compulsory at a particular age as a way of promoting annuitization. Mandating annuitization has the major disadvantage of annuity providers exploiting this mandate by raising prices, (Orszag, 2000).

Davidoff et al. (2005) contributed to the study done by Mitchell et al. (1999) by considering bequests and illustrated that bequest did not matter if an individual's consumption frame does not match with what is offered by the annuity and therefore efficient optimization occurs when a large portion of the individual's wealth is annuitized.

A large portion of wealth should be annuitized despite of unfair actuarial pricing and a motive for bequest (Babbel and Merril, 2006)

Brown (2009b) argued that the reason why voluntary annuitization rates remain low is because of accessibility. Brown (2009b) felt that there is a high degree of neglect toward the distribution phase of retirement of wealth by the policymakers and plan sponsors.

2.3 Empirical Framework

2.3.1 Measuring longevity risk

Renshaw and Haberman (2006) simplify the Age-Period-Cohort model in the Renshaw-Haberman model which is an extension of the Lee Carter model. They extend the Lee Carter model by adding a cohort effect to the age and period effects.

According to Cui (2008), Lee and carter (1992) marked the beginning of the literature of stochastic mortality and their model is the leading statistical mortality model (Deaton and Paxson, 2004). A linear function is used to model the natural logarithms of age specific death rates whereby the function is of a specific period index with parameters that are dependent on age. A singular value decomposition is used to fit the Lee Carter model to the matrix if United States death rates between the year 1933 and 1987 and by doing so they find there is an increase in life expectancy in 2065 to 86 years and that 46% of the population will survive to 90 years (both male and female combined).

Cairns et al (2011) propose an Age-Period-Cohort model that incorporates a stochastic spread that has mean reversion. This allows for various trends in the short run for mortality improvement rates but as for the long run, parallel improvements are allowed. Cairns et al (2011) use a Bayesian framework for fitting the model for the combination of the estimation of the unobservable state variables and the stochastic parameters leading them into a single procedure. The result of this single stage approach was consistency in estimates of the unobservable period and effects of the cohort.

2.3.2 Management of Longevity risk in pensions

Annuitization

Using survey evidence, Brown et al. (2008) showed that under annuitization is motivated by the individuals' perspective of annuities as an investment rather than a means of sustenance during retirement.

Webb (2009) suggested that due to misunderstanding risk, households tend to avoid annuities.(Webb 2009) suggested that households have great concerns with short term gains and losses and neglect the importance of long term sustenance of a smooth consumption.

Benartzi et al. (2011) added on the work of Brown et al. (2008) by showing that annuitization rates are lower when pension plans communicate the annuity as an investment frame rather than a consumption frame.

Loss of flexibility is the most common reason for not annuitization, (Gardner and Wadsworth, 2004). Gardner and Wadsworth (2004) came to this conclusion after conducting a survey in UK. Wang and Young (2009) argued that if the annuities were reversible annuities they would offer the policyholder some flexibility and hence alleviating this reason for annuitization evasion.

Brown and Scahill (2010) suggested the extension of the classes of risk for impaired annuities. The authors suggested following the results of the survey study conducted by Gardner and Wadsworth (2004) whereby it was illustrated that most of the individuals who opposed annuitization were those with lower income, health and education. Those in opposition to annuitization had characteristics associated with lower life expectancies and therefore would gain the least from grouping their longevity risk with others especially those that have higher longevity than the average population as a consequence of adverse selection.

Brunner and Pech (2005) addressed the problem of adverse selection by exploring a three period life cycle model which involved two periods of retirement. The authors suggested that annuity pay outs be paid out in two different periods and that the annuitants are grouped into two categories which are high risk and low risk. They found that the separation of the two groups could be done by diversifying contract offers and so by offering contract options that are appropriate, the problem of adverse selection is addressed.

Theoretically, annuities are an appealing way for the management of longevity risk but in the real world, few customers can purchase them at retirement (Fong et al, 2011). Fong et al (2011) evaluate the worth of money in life annuities and discuss the impact of government mandate in Singapore and its role as an annuity provider in the Insurance market. This is after Singapore's Central Provident Fund counteracted the possibility of retirees outliving their assets with a national defined contribution scheme which mandated the annuitization of the retiree's assets. With their preliminary evidence, conclude that Singapore has been able to pass cost savings from economies of scale and onto annuitants. Members of the Central Provident Fund may buy life annuities from a private Insurer (Fong et al, 2011).

Hedging against longevity risk

Coughlan (2007) expresses that longevity exposure in the world is over AC 15 trillion and this provides a lot of possibilities for marketing longevity derivatives. Blake et al (2006) included providers of pensions as one of the stakeholders of this longevity market. There are more stakeholders who are willing to sell their longevity risk rather than buy it due to the net short nature of the market.

According to Hua (2007), investors will require compensation; mortality rates used in the derivatives should be reconciled below the expected mortality rate. Coughlan (2015) discusses longevity risk associated with the provision of retirement. He proposes transferring of longevity risk via capital markets from pension plans and insurers to end investors.

Many derivatives have been developed and some proposed towards hedging longevity risk. Some of them include; longevity futures, longevity index swaps and longevity bonds among others. According to Blake et al (2006), the biggest obstacle for the emergence of a market for longevity futures is the need for a suitable underlying instrument. The USCPI future contract was listed in 1985 and delisted because of the extreme low numbers of the trades; the delisting was after two years.

Srinivasan (2004) stated that the reason for this failure was instability of pricing relationships with other instruments and inadequate publishing of the underlying index.

The use of annuities or longevity bonds as underlying instruments was suggested by Blake et al (2006). The problem is updating of prices although there is the predictability of the market because of the pricing relationship with the yield curve of the government bonds. The other problem was that the insurers will be required to reveal their prices which may not be in their best interest.

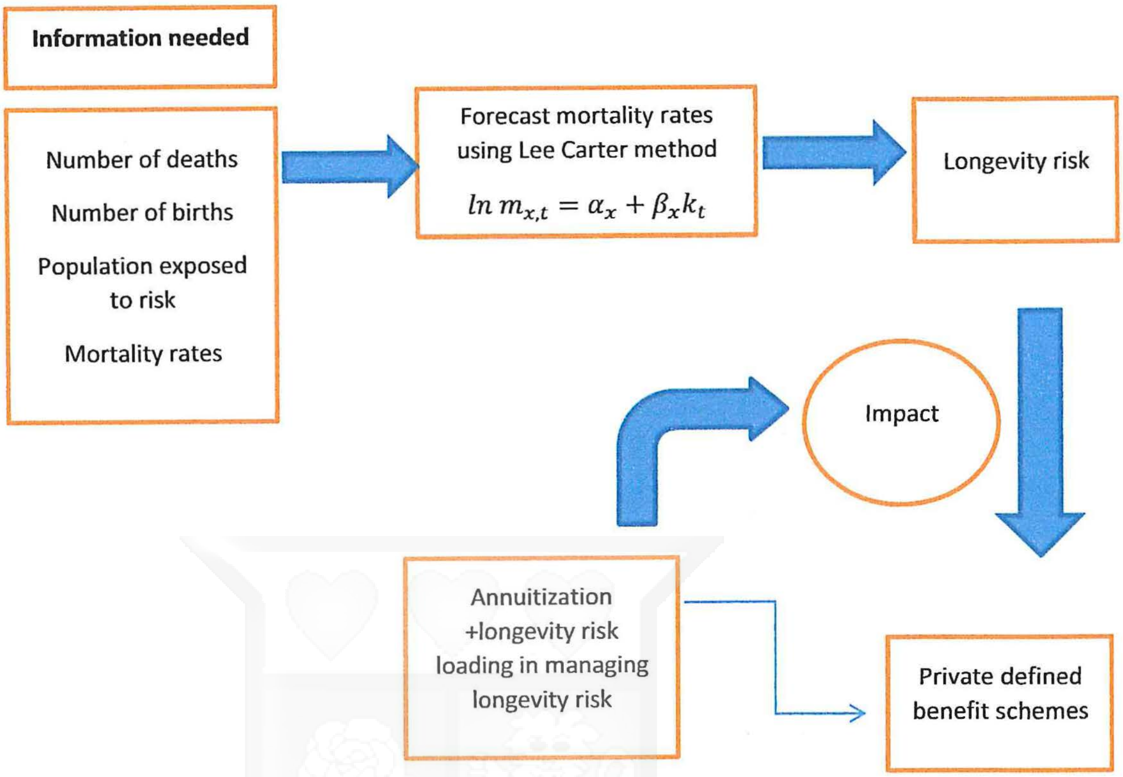
The alternative of longevity bonds is also provided. Cairns et al (2009) examines the major characteristics of longevity bonds and showed that longevity bonds can take a large variety of forms that can change widely in their sensitivities to shocks that result from longevity. They considered the problems arising from the scarcity of long government bonds. They concluded that Longevity Bonds can be used as tool for hedging longevity risks.

2.4 Knowledge Gap

As observed in the section above, a lot of research has been undertaken in annuitization and hedging as methods of managing against longevity risk but little has been done in measuring the longevity risk and formulating a formula that can incorporate the longevity risk in pension contribution or benefit pay out so that private defined benefit pension schemes do not suffer great losses.

Also, the research that has been done focused on developed countries leaving a knowledge gap for developing and under developed countries.

2.5 Conceptual Framework



CHAPTER 3: RESEARCH METHODOLOGY

3.1 Research Design

This is a descriptive study. This is a descriptive study because it is a fact-finding study that involves adequate and accurate interpretation of findings of present conditions. The present condition in this case is the increase in longevity risk. Relatively, the method is appropriate to this study since it aims to describe the impact of longevity risk in private defined benefit pension schemes.

The study will also employ a quantitative approach. The quantitative approach will focus on obtaining the numerical findings which will be used for the qualitative analysis to show the impact of longevity risk.

3.3 Population and Sampling study

The population of this study will be on Israel which is developing country with data in the Human Mortality database.

3.4 Data

Secondary data will be used in this research. This data will be derived from the findings stated in published documents and literatures related to the research.

As for the forecasting, data will be obtained from the human mortality database that is readily available in the internet.

3.5 Methodological Approach and Variables

The methodological approach that will be used is the Lee Carter method which was introduced in chapter two.

The model uses age-specific mortality and it does not include any age-specific factors therefore calibration of the model on data of male and female populations is done separately. Let m_{xt} be the central death rate of age x in year t. This rate is defined as

$$m_{(x,t)} = D_{(x,t)}E_{(x,t)}$$

(3.1)

Whereby D(x,t) is the number of deaths in year t and E(x,t) is the number of lives aged x in the middle of year t. The central death rates are fitted by the model

$$ln\, m_{x,t} = \alpha_x + \beta_x k_t + \varepsilon_{x,t}$$

(3.2)

Whereby:

Vector "α " is the average age profile

Vector "k" tracks mortality changes over time

Vector "β" determines how much each group changes when k_t changes

$\varepsilon_{x,t}\sim$ N0,σ2 ε Are white noises which reflect the variations not captured by the model

Vector "a" is estimated by averaging log rates over time whereas vector b and vector k via a singular value decomposition of the residuals.

An identification problem is present in the standard Lee Carter model and the model is invariant given the following transformations:

$\beta_x \rightarrow c\beta_x$ $k_t \rightarrow 1\,/c^*k_t$ $\forall c \in R$,cis not equal to 0

$$\alpha_x \rightarrow \alpha_x - \beta_x c \quad k_t \rightarrow k_y + c \qquad \forall c \in R.$$

Therefore, two restrictions are added (Cairns et al., 2007; Lee, 2000):

i. $\Sigma k_t = 0$

ii. $\Sigma \beta_x = 1$

(3.3)

Restriction (i) implies that for every age the estimate for a_x will be approximately equal to the average over t of the log death rates. Restriction (ii) is important in tackling the identification problem. According to Cairns et al (2007), this constraint does not have an impact on the quality for the mortality fit or the mortality forecasts.

3.6 Data Analysis

3.6.1 Parameter Estimation

The parameters of the Lee Carter model will be estimated using the lca function in R software..

The vector "α" is computed as the average over time of the logarithm of central death as shown

$$\alpha_x = \frac{1}{n}\sum_{t=1}^{n} \mu_{x,t}$$

(3.4)

3.6.2 Forecasting

This study will find a modified kt which will adjust the total number of deaths $\sum_x d_{x,t}$ to the estimated number of deaths as follows

$$\sum_x d_{x,t} = \sum_x E_{x,t} \exp(\alpha_x + \sum_i \beta_x^{(i)} k_t^{(i)}$$

(3.5)

Whereby $E_{x,t}$ is the exposure to risk and $d_{x,t}$ is the actual number of deaths at age x at time t.

Using the Lee Carter model, the prediction of mortality rates is reduced to forecasting the index k_t using a time series approach (Brockwell and Davis, 1996).

Assumptions when forecasting mortality rates;

α_x and $\beta_x^{(i)}$ remain constant over time

k_t is intrinsically viewed as a stochastic process

The following random walk with drift to model k_t was suggested by Lee and Carter (1992):

$$k_t^{\sim} = k_t^{\sim} + \theta + c\varepsilon_t$$

(3.6)

Where:

θ is a constant drift term

C is constant volatility

ε_t is a one dimensional identical independent distributed N(0,1) error

CHAPTER 4: ANALYSIS, RESULTS AND DISCUSSIONS

4.1 Sources of data

Government agencies collect and publish mortality data. The Kenyan regulatory body collects such data but it is not readily available to the public nor is it annually published like other countries. The Human Mortality Database publishes the national mortality data for many countries.

My analysis will be based on the Israel mortality data that was acquired from the Human Mortality Databases through demography package dedicated function. I chose Israel as is a third world country just as Kenya is.

The information that can be obtained by sex, age and time in the HMD includes:

- Birth counts
- Death counts;
- Population estimate;
- Population exposed to risk of death(the period & cohort :period data are indexed by the
- year of deaths; whereas cohort data are indexed by year of births); and
- Death rates (period and cohort).

4.2 Description of the software used

For the data analysis, I have used both the R software. The demography and forecast package in R software is used to fit and forecast the Lee carter model. On obtaining future life expectation results, the life contingencies package is used to project the cost of a pension annuity for specific cohorts.

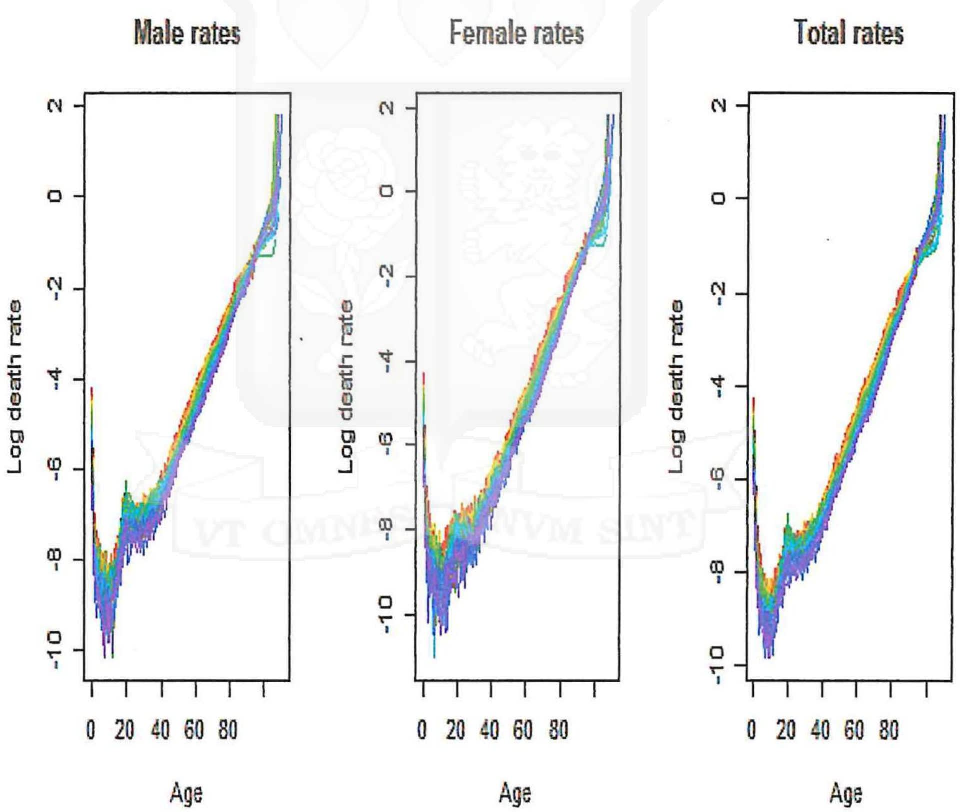
4.3 Assumptions

The following assumptions were made: The retirement age is set to 65, the pensions are paid monthly, the inflation rate of 10% and interest rate of 4 % will be used.

4.4 Analysis of data used.

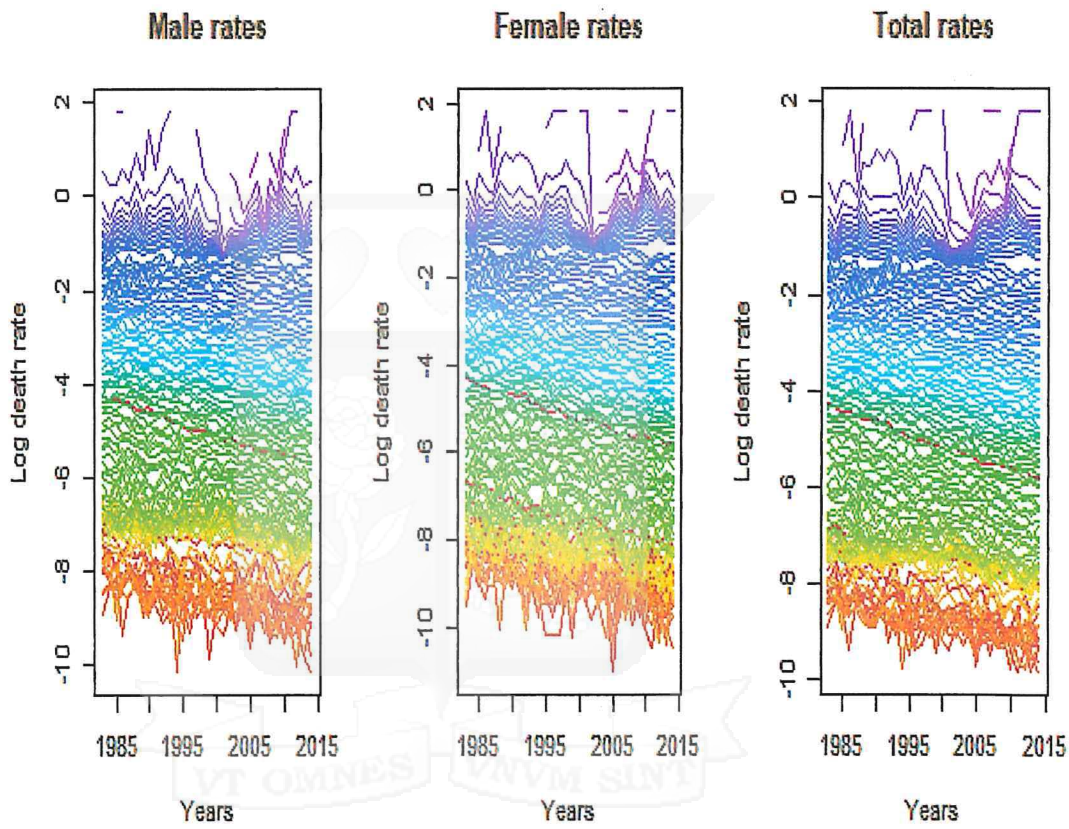
The plot method is available on demogdata.

Figure 2: Log death rates against age from 0 to 110



According to the representation of data above, mortality decreases with age. The young mortality hump between 20 and 40 could probably be attributed to factors such as abuse of drugs and road accidents. However we shall consider individuals from age 60 as this is when individuals start receiving annuities.

Figure 3; Log death rates against time from 1985 to 2015



As seen, the data confirms decreasing mortality rates over the years. This means that mortality rate is higher in 1985 compared to 2015. However, we observe that although there is a decline in mortality over time, the decrease has been uneven across the different ages at especially between age 20 and 40.

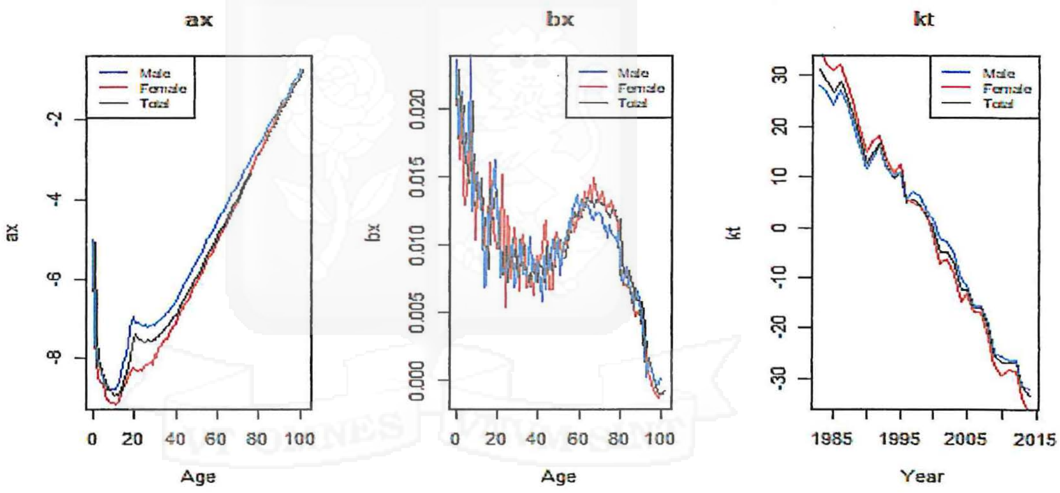
The data confirms that mortality is falling at all ages with a different behavior according to different ages.

4.5 Fitting the model

The data from Israel confirms falling mortality rates. To fit the Lee-Carter model without going through logarithms, the lca function in R software is used.

We apply Lee Carter separately between male, female and total population and we get the following results.

Figure 4; Values of Lee Carter estimated parameters



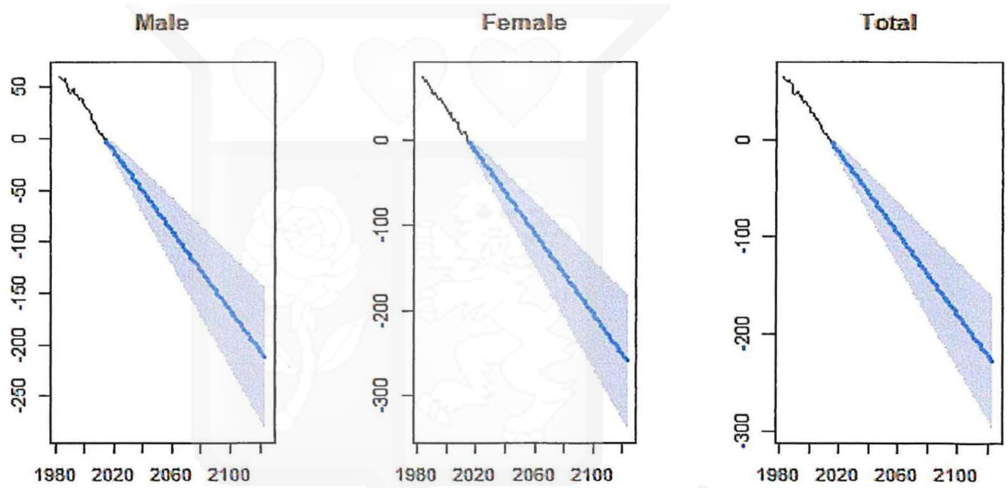
The parameters demonstrate similar behavior of have different behavior according to different data sets. As observed with the estimated parameter a_x pattern, the average mortality grows when the age increases. The young mortality hump in the age range 20 to 30 due to accidental deaths is clearly visible.

The estimated parameter b_x tracks determines how much each group changes when k_t changes. It shows a greater value for younger ages and a greatest improvement for females between the ages of 60 and 80.

The estimated parameter k_t decreases with increase in time. This is expected as $k_{t\ tracks}$ mortality changes overtime and since there is a decrement in mortality with an increment in time, k_t will follow the same pattern.

Using the forecast package, we project the future k_ts up to age 110. The projection is based on ARIMA extrapolation.

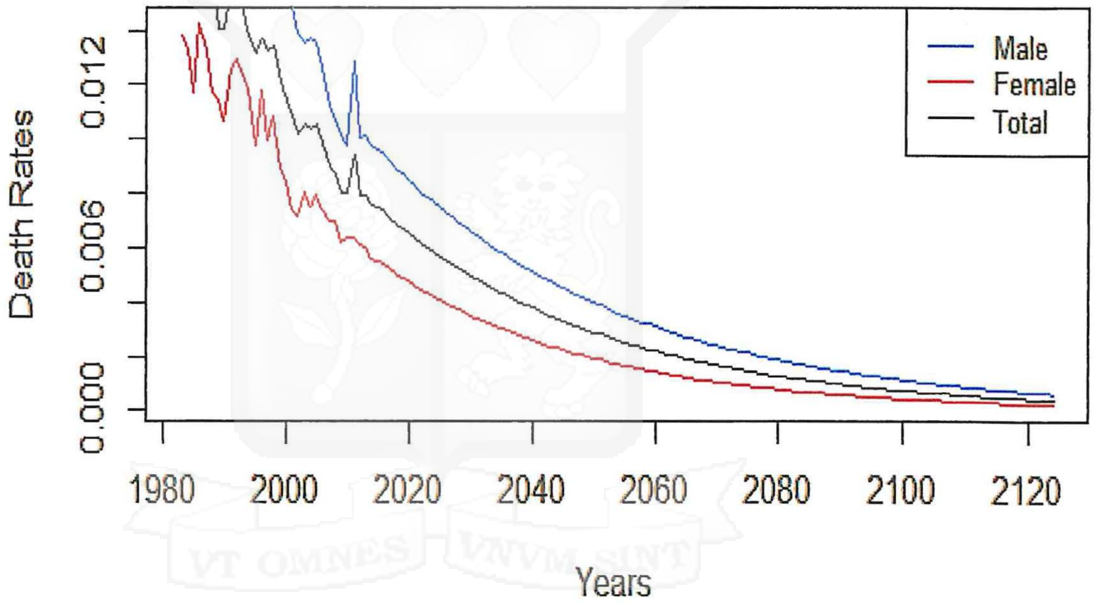
Figure 5: Projected values of K_t



As seen from the projection of the values of K_t, the mortality changes as seen in Figure 5. The Lee carter model forecast shows that mortality rates are improving. The improvement of mortality rates has the implication of increase in cost of pension in the future as a result of longevity risk (people living longer than expected).

Since the retirement age is at 65, we report the pattern of past and forecasted rates according to different population for individual aged 65. The plot below (figure 6) demonstrates clearly the improvement in mortality.

Figure 6; forecasted rates according to different population for people aged 65



4.6 Performing Actuarial Projections

In order to measure longevity risk, we calculate the actuarial present value $a_{65}^{:12}$ for the selected cohorts using the life contingencies package in R software. After getting the values e0 and actuarial present values using r coding, we transferred the values to excel to plot line graphs for representation.

The values are derived separately for males and females and finally the total population. Figure 7 to Figure 11 show the representations.

Figure 7; males' e0 relationship with increase in time

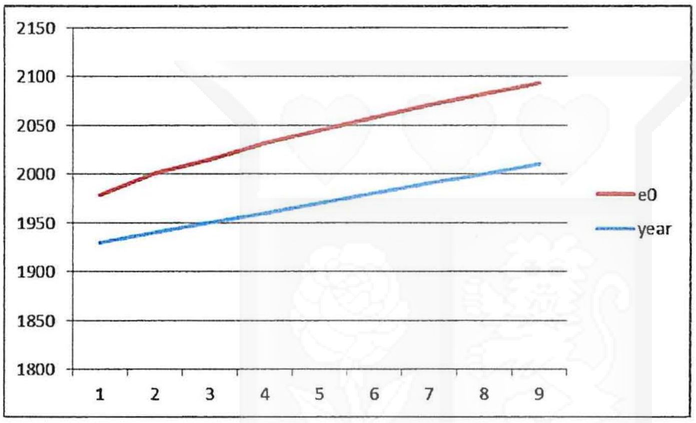


Figure 8; males' APV relationship with increase in time

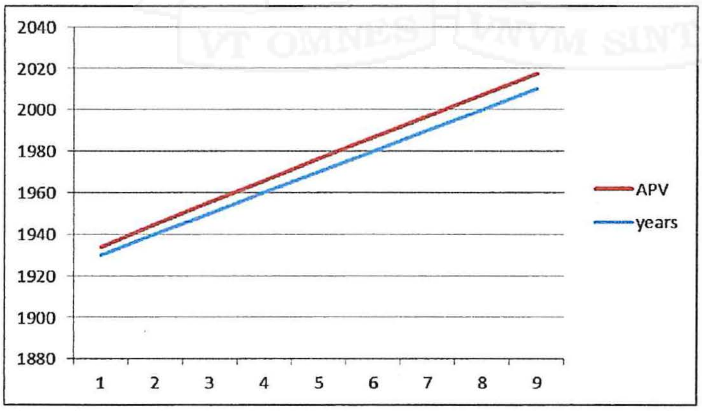


Figure 9; females' e0 relationship with increase in time

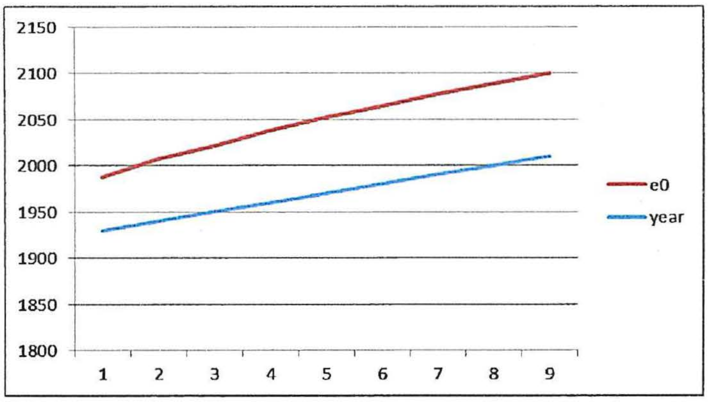


Figure 10; females' APV relationship with increase in time

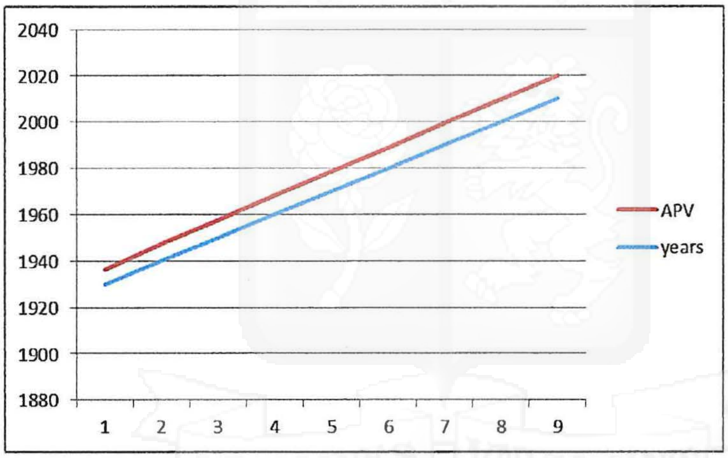


Figure 11; combined e0 relationship with increase in time

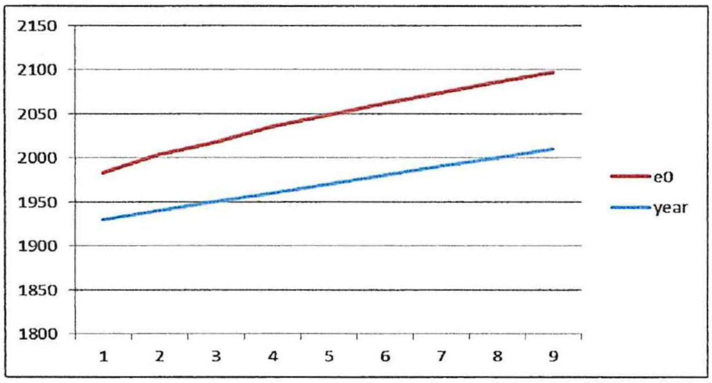


Figure 12; combined APV relationship with increase in time

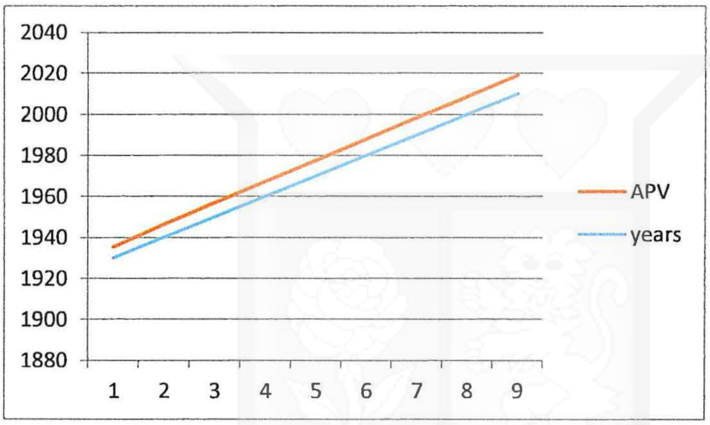


Figure 7 to Figure 12 show an increase in life expectancy and actuarial present value with increase in time. It is observed that pensions have been increasing with time as a result of declining mortality rates and the increment in life expectancy. This implies that the rise in longevity risk is as a result of general change in mortality and increase in life expectancy.

4.6.1 Calculating longevity risk premium

We calculate the growth rate in actuarial present value in the different cohorts.

Figure 13; increase in APV in males

Figure 14; increase in APV in females

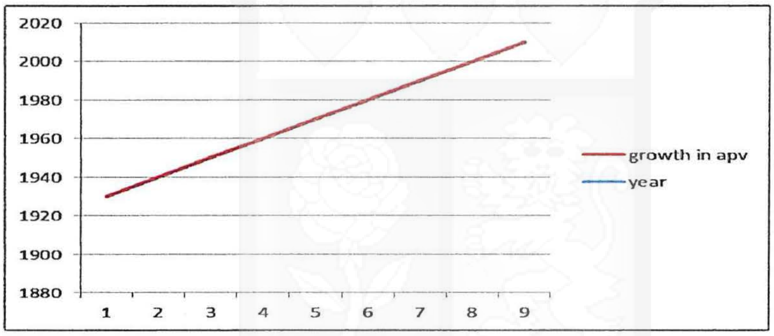
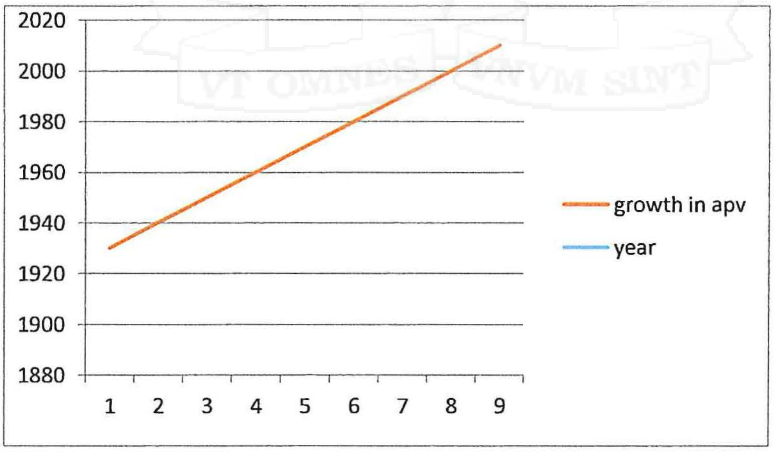


Figure 15; increase in APV in both males and females



The graphs show the increase in growth rate with increase in time. This is implies that if the pension provider does not account for this increase in growth rate, they are going to end up making losses as they will end up paying extra due to longevity risk.

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Using the standard Lee- Carter model on Israel data, we have identified a common trend of mortality changes with age. In addition we have estimated and forecasted the parameters using the ARIMA method. Finally, we have forecasted the life expectancies at birth. The results show that in deed there is a decrement in mortality rates with age and time. Furthermore, there is an increment in life expectancy with time which has led to led to the increase in actuarial present values that are used in calculating annuities.

It is therefore possible to conclude that the risk that a pensioner will live longer than expected is evident from the results and hence the existence of longevity risk is undeniable. If defined benefit pension funds choose to ignore the changes in mortality rates and not put them into consideration by still using the same life tables, they will end up overpaying annuities and suffer the risk of loss. Therefore defined benefit pension plans should reserve for longevity risk.

5.2 Limitations

A lot of credibility has been given to the Lee Carter model in the effort to model mortality rates since 1992. The Lee Carter model has proven to be robust and attempts accuracy in its predictions for forecasts. However, we cannot ignore that we have assumed constant consumption of parameters which is not practical.

Another limitation is that the model does not put the cohort effects into consideration.

5.3 Recommendations

As per the conclusion above, we have clearly proven the existence of longevity risk in defined benefit pension schemes. Therefore, we recommend the study and implementation of longevity risk management techniques.

Also, the limitation of not putting the cohort effect into consideration provides room for further study on the Lee Carter model.

List of References

Fong, J. H., Mitchell, O. S., & Koh, B. S. (2011). Longevity Risk Management in Singapore's National Pension System, *78(4), The Journal of Risk and Insurance,* 961-981.

Bütler, M., & Teppa, F. (2007). The choice between an annuity and a lump sum: Results from Swiss pension funds. *Journal of Public Economics, 91*(10), 1944-1966.

Cocco, J. F., & Gomes, F. (2011). Longevity Risk, Retirement Savings and Financial Innovation. *SSRN Electronic Journal,* Netspar Discussion Paper No. 03/2011-061.

Yang, S. S., & Huang, H. (2009). The Impact of Longevity Risk on the Optimal Contribution Rate and Asset Allocation for Defined Contribution Pension Plans. *The Geneva Papers on Risk and Insurance - Issues and Practice, 34*(4), 660-681.

Cannon, E., & Tonks, I. (2012). The Value and Risk of Defined Contribution Pension Schemes: International Evidence. *Journal of Risk and Insurance, 80*(1), 95-119.

Coughlan, G. (2015). Longevity Risk Transfer: Indices and Capital Market Solutions. *The Handbook of Insurance-Linked Securities,* 261-281.

Murtaugh, C.M., Spillman, B.C., &Warshawsky, M.J. " In Sickness & In Health: An Annuity Approach to Financing Long-Term Care & Retirement Income." Journal of Risk & Insurance 68, no. 2 (2001): 225-254.

Pang, G., & Warshawsky, M. "Optimizing the Equity-Bond-Annuity Portfolio in Retirement: The Impact of Uncertain Health Expenses," Insurance: Mathematics & Economics, 46(1) (2010), 198–209

Organization for Economic Co-operation & Development, OECD. "Private Pensions & the Financial Crisis: How to Ensure Adequate Retirement Income from Defined Contribution Pension Plans?" Working Party on Private Pensions. 2009.

Siegel, J. J. "Perspectives on the equity premium." Financial Analysts Journal 61, no. 6 (2005): 61-73.

Siegel, S. "Housing Wealth & Retirement." The Actuary (magazine). 8, no. 2. (2011): 18-23.

Sinclair, S.H. & Smetters, K.A. "Health Shocks and the Dem and for Annuities." Technical Paper Series 2004-9, Congressional Budget Office., 2004.

Wang, T. & Young, V. R. "Optimal Reversible Annuities to Minimize the Probability of Lifetime Ruin." Quantitative Finance Papers 1001.4270., 2010.

Warshawsky, M. & Hill, T. "Who Prefers Annuities? Observations about Retirement Decisions." Watson Wyatt Insider 18, no. 4 (April 2008): 12-21.

Webb, A. "Providing Income for a Lifetime: Bridging the Gap between Academic Research & Practical Advice." Center for Retirement Research, Boston College, 2009

Yaari, M. E. "Uncertain Lifetime, Life Insurance, & the Theory of the Consumer." Review of Economic Studies 32 (1965): 137-150.

Young, V. R. 2004. Optimal investment strategy to minimize the probability of lifetime ruin. North American Actuarial Journal, 8(4):105-126, 2004.

Blake, D., Cairns, A. & Dowd, K. "Pensionmetrics 2: Stochastic Pension Design During the Distribution Phase." Insurance Mathematics & Economics 33, no. 1 (2003): 29-47.

Benartzi, S., Previtero, A., & Thaler, R. H., "Annuity Puzzles". Journal of Economic Perspectives 25(4) (2011): 143-164.

Charpentier A (2012). "Actuarial Science with R 2: Life Insurance and MortalityTables."http://freakonometrics.blog.free.fr/index.php?post/2012/04/04/ Life-insurance,-with-R,-Meielisalp. Accessed: 04/11/2012.

Charpentier A, Dutang C (2013). Actuariat Avec R. http://cran.r-project.org/doc/ contrib/Charpentier_Dutang_actuariat_avec_R.pdf.

Lee R, Carter L (1992). "Modeling and Forecasting U.S. Mortality." Journal of the American Statistical Association, 87(419), 659–675. doi:10.2307/2290201.

Rob J Hyndman, Heather Booth, Leonie Tickle, John Maindonald (2011). demography: Forecasting Mortality, Fertility, Migration and Population Data. R package version 1.09-1, URL http://CRAN.R-project.org/package=demography.

Spedicato GA (2013). "The lifecontingencies Package: Performing Financial and Actuarial Mathematics Calculations in R." Journal of Statistical Software, 55(10), 1–36. URL http://www.jstatsoft.org/v55/i10/.

Pitacco, E., Denuit, M., Haberman, S., & Oliviera, A. (2009). Modelling Longevity Dynamics for Pensions and Annuity Business.

Stoto, M. A., & Arthur, W. B. (1983). An analysis of indirect mortality estimation.

Tuljapurkar, S., Li, N., and Boe, C. (1998). Mortality Change and Forecasting.

Wilmoth, J.R. 1993. "Computational Methods for Fitting and Extrapolating the Lee-Carter Model of Mortality Change." Technical Report. Depart ment of Demography. University of California, Berkeley.