School of Computing and Engineering Sciences

Bachelor of Science in Computer Science

CNS 1104 Discrete Mathematics

End of Semester Examination

Date: 22nd Dec 2021 **Duration: 2 Hours**

Instructions:

(i) Answer Question One and Any other Two questions.

(ii) Show all your workings clearly.

Question 1 (30 Marks)

(a) Giving a reason in each case, determine whether each of the following sets is countable or uncountable.

(i) $A = \{\frac{1}{q} | q \in \mathbb{N}\}$ [2 Marks]

(ii) $B = \{y \in \mathbb{Q} | -72 \leq y \leq 36\}$ [2 Marks]

(iii) $C = \{(x, y) | x \in \mathbb{N},\ y \in \mathbb{Z}\}$ [2 Marks]

(b) Let A, B, C be any three arbitrary sets. For each of the following, draw a Venn diagram and shade the area representing the given set.

(i) $A \cup (B \cap C)$ [2 Marks]

(ii) $A - (B \cap C)$ [2 Marks]

(c) A survey of 500 internet users produced the following information: 285 visit soccer sites, 195 visit hockey sites and 115 visit national geographic sites, 45 visit both soccer and national geographic sites, 70 visit both soccer and hockey sites, 50 visit both hockey and national geographic sites while 50 do not visit any of the three sites.

(i) How many people in the survey visit all the three sites. [3 Marks]

(ii) How many visit exactly one site. [2 Marks]

(d) Consider the quantified predicate: $\forall y \neq 0\ (y^3 \neq 0)$.

(i) What does this statement mean in the domain of real numbers? [2 Marks]

(ii) Express it in English and logic using conditional statement. [2 Marks]

(e) Use the principle of Mathematical induction to prove that for $n \geq 1$,

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

[5 Marks]

(f) Test the following function to check if it is injective:

$$f : [0, \infty) \to \mathbb{R}, \quad f(x) = x^3$$

[3 Marks]

(g) Let the functions $g(x) = 3x^2 + 7$ and $h(x) = 2x^3 + 3x + 5$. Find the composition function

$$(g \circ h)(x)$$

[3 Marks]

Question 2 (20 Marks)

(a) Let $A = \{1, 2, 3, 4, 5\}$ be a set. Define a set of ordered pairs ,R, if R is the relation $\leq$ from A to A. [3 Marks]

(b) (i) Find the relation R defined on A by the following matrix:

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

for $A = \{a, b, c, d, e\}$ [3 Marks]

(ii) Let $A = \{1, 2, 3\}$ and R be a relation defined on set A as

$$R = \{(1, 1), (2, 2), (3, 3)(1, 2), (2, 1), (2, 3), (3, 2)\}$$

Prove that R is transitive. [5 Marks]

(c) Construct a truth table for the following compound proposition. Is it a contingency?

$$(\neg p \vee q) \vee \neg(p \wedge q)$$

[4 Marks]

(d) Prove if the following propositions are logically equivalent.

$$p \wedge (q \vee r) \quad and \quad (p \wedge q) \vee (p \wedge r)$$

[5 Marks]

Question 3 (20 Marks)

(a) (i) Draw a graph with the following incidence matrix

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

[4 Marks]

(ii) Find the adjacency matrix for the graph in (a) above. [4 Marks]

(b) Prove by the principles of mathematical induction that for all $n \geq 1$

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$$

[4 Marks]

(c) Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$ and $C = \{x, y, z\}$. Consider the relation aRb, from A to B, and bSc, from B to C defined respectively by

$$R = \Big\{(1, a), (2, b), (3, a), (3, b), (3, d)\Big\}$$

$$S = \Big\{(b, x), (b, z), (c, y), (d, z)\Big\}$$

Find the matrix of the composition relation, $M_{R \circ S}$. [4 Marks]

(d) An IT department hires 25 programmers to handle system programming jobs and 40 programmers for applications programming. Of those hired, 10 will be expected to perform jobs on both types. How many programmers must be hired? [4 Marks]

<u>Question 4 (20 Marks)</u>

(a) Determine whether or not each of the following relations is a function.

(i) $R_1 = \Big\{(2, 4), (3, -7), (6, 10)\Big\}$. [1 Mark]

(ii) $R_2 = \Big\{(-1, 8), (4, -7), (-1, 6), (0, 0)\Big\}$. [1 Mark]

(iii) $R_3 = \Big\{(6, 10), (-7, 3), (0, 4), (6, -4)\Big\}$. [1 Mark]

(iv) $R_4 = \Big\{(2, 1), (9, 10), (-8, 1), (-4, 10)\Big\}$. [1 Mark]

(b) Find the inverse of the following functions

(i)

$$g(x) = x^3 + 1$$

[2 Marks]

(ii)

$$h(x) = \frac{2x}{x-3}$$

[3 Marks]

(c) Determine whether the sequence $\{a_n\}$ is a solution of the recurrence relation

$$a_n = 2a_{n-1} - a_{n-2}, \quad for \quad n = 2, 3, 4, \cdots$$

where $a_n = 2^n$ for every non-negative integer n. [4 marks]

(d) Suppose that a person invests \$ 30, 000 in a savings account at a bank yielding 16% p.a with interest compounded annually.

(i) Write a recurrence relation and initial conditions that define the amount in the account at time n. [4 marks]

(ii) How much will be in the account after 20 years? [3 marks]

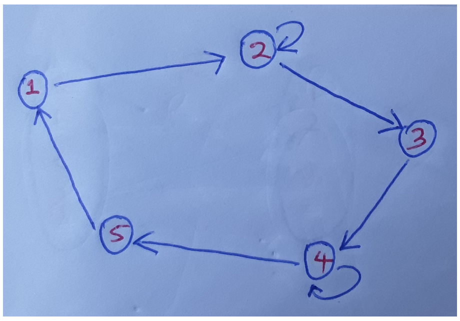
Question 5 (20 Marks)

(a) In how many different ways can the letters of the word '$MATHEMATICS'$' be arranged such that the vowels must always come together? [5 Marks]

(b) In a survey of 130 people, the following data were collected: 106 people subscribed to the newspaper, 29 people subscribed to magazines, and 17 people were members of a mail CD club. Seventeen subscribed to both the newspaper and the magazines, 5 people subscribed to magazines and were members of a CD club, and 10 people subscribed to the newspaper and were members of a mail CD club. Three people subscribed to both the newspaper and magazines and were members of a mail CD club. Make and fill in a Venn diagram to illustrate this situation. [5 Marks]

(c) Express the following statement using predicates and quantifiers:
"Every student in this class has visited either *Kuala Lumpur* or *Changi*" [4 Marks]

(d) (i) Find R as a set of ordered pairs from the digraph below:



[3 Marks]

(ii) Consider a relation, $R = \Big\{(0,1),(2,1),(1,1),(0,0),(1,2)\Big\}$ defined on the set $A = \{0,1,2\}$. Is R reflexive? Justify your answer. [3 Marks]

END