Strathmore
UNIVERSITY

Strathmore Institute of Mathematical Sciences
BBS Finance & BBS Financial Economics
END OF SEMESTER EXAMINATION

BSA 3108:- Foundations of Asset Pricing

Date: 20th August, 2020 Time: 0900 - 1200 Hours

INSTRUCTIONS

1. This is an Open Book Examination
2. There are FIVE questions in this examination
3. Attempt QUESTION ONE and ANY OTHER TWO questions
4. Question one is COMPULSORY and carries 30 marks
5. All the other questions carry 20 marks each
6. Show all *your workings* and clearly define all *notations* used
7. Originality of your responses will be thoroughly checked. Collaboration with classmates is probihibited

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QUESTION 1 - COMPULSORY

(a) The Basic Pricing Equation is central in asset pricing. However, in studying this key equation one is faced with the question - *which is the chicken and which is the egg between asset returns and consumption?* Using suitable manipulations of the basic pricing equation below, argue whether it really matters to start with asset prices or consumption to determine the other.

$$p_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}x_{t+1}\right]$$

[8 marks]

(b) Suppose we live in a two-state world with a financial market. In the market we have a stock with current price 100. In the next period, the stock either rises to 112 or declines to 84. We also have a European call option on the stock maturing next period with strike price 98 and current price 10.

i. Is the market complete? Explain. [2 marks]

ii. Show that under no arbitrage, this option price implies a risk free rate of 5% [4 marks]

iii. Calculate the risk-neutral probabilities implied in this set up [4 marks]

(c) An investor with log utility $u(\omega_{t+1}) = ln(\omega_{t+1})$ can invest in a stock which currently has price 100. It will either go up to 130 or down to 90, with probability 1/2 of each event. (Imagine the two states as up state μ and down state d. He can also invest in a bond, which pays zero interest.

i. Work out a discount factor m_{t+1} that prices the stock and bond. [4 marks]

ii. Work out the return of the investor's optimal portfolio in the two states. [3 marks]

iii. Formulate the portfolio that gets the return in *ii* above [5 marks]

Total for Question 1: 30 marks

QUESTION 2 - OPTIONAL

Consider a risk-averse expected utility maximizer with a twice-differentiable utility-of-consequences function $u(\omega)$, where ω denotes his random final wealth.

(a) Define the investor's coefficient of absolute risk aversion [2 marks]

(b) Suppose such an invesor is risk-averse and has a given non-random initial wealth ω_0. He can invest in a riskless asset with a sure total return R_f per unit invested, or in a risky asset with a random total return R per unit invested. The random variable R has a given density function $f(R)$. The investor chooses the amount θ to be invested in the risky asset.

i. Formulate the investor's optimization problem. Assume that the optimum is interior, i.e. $(0 < \theta < \omega_0)$. [6 marks]

ii. Find the first order condition and verify that the second order condition is satisfied. [6 marks]

iii. If the investor has constant absolute risk aversion show that their optimal θ is independent of the initial wealth ω_0. [6 marks]

Total for Question 2: 20 marks

QUESTION 3 - OPTIONAL

(a) The equivalence of beta (or factor) models to the stochastic discount factor (SDF) models is hinged on the premise that "*the existence of a stochastic discount factor, m linear in the market return i.e.* $m = a + bf$, *implies that the discount factor must be a linear function of the factors.*" Even though, linear factor models dominate most empirical work because they are easier to estimate. Viewed from this point, the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT) can be seen as special cases of the discount factor models.

i. Show that the existence of a stochastic discount factor, m linear in the market return $m = a+bf$ implies the Capital Asset Pricing Model (CAPM) of the form $E(R_i) = \alpha + \beta_i\lambda$ where λ is the excess market premium, α is the risk free rate and a and b are constants. Highlight all assumptions made. [4 marks]

ii. Extend the equivalence in (i) above to the multifactor case of the form $E(R_i) = \alpha + \lambda'\beta_i$ where β_i is a vector of k factor sensitivities with each element $\beta_{ik} = \frac{cov(f_k, R_i)}{var(f_k)}$, λ' is a vector of the k excess factor premia and α is the risk free rate. Highlight all assumptions made. [6 marks]

(b) Compare the testable implications of the Sharpe-Lintner CAPM with a risk-free asset and the APT model. [6 marks]

(c) Suppose that an econometric analysis leads to the conclusion that there are two factors explaining the cross-section of expected returns. Does this mean that we can reject the CAPM? *Justify.* [4 marks]

Total for Question 3: 20 marks

QUESTION 4 - OPTIONAL

Conventional investment advice is that people saving for retirement should hold more stocks if they are young and risk-tolerant, and should hold more bonds and cash if they are older and more risk-averse. This advice has motivated reforms in retirement benefits systems including investment of retirement savings in life-cycle funds, with a declining allocation to equities and an increasing allocation to bonds as employees approach retirement.

(a) Explain the circumstances under which the age of the client should influence portfolio choice. How, if at all, can you justify the advice that older people should hold more bonds and less stocks? [12 marks]

(b) Explain how risk-aversion should influence portfolio choice. How, if at all, can you justify the advice that more risk-averse people should hold more bonds? [8 marks]

A useful starting point to responding to the issues above is Mehra (2012) - Consumption-Based Asset Pricing Models and Breeden (2015) Consumption-Based Asset Pricing, Part 1: Classic Theory and Tests, Measurement Issues, and Limited Participation. **Total for Question 4: 20 marks**

QUESTION 5 - OPTIONAL

In the introductory paragraph of their paper, Bernanke & Gertler (2001) pose a question- "In recent decades, asset booms and busts have been important factors in macroeconomic fluctuations in both industrial and developing countries. In light of this experience, how, if at all, should central bankers respond to asset price volatility?".

(a) For a country of your choice, discuss briefly (in less than 500 words) how well or poorly the central bank in the country has approached the asset pricing question posed by Bernanke & Gertler (2001). [12 marks]

(b) What recommendations would you make, and why, for the issues you have raised in a above. [8 marks]

Your arguments in (a) and (b) above should be grounded in the relevant asset pricing literature. Your responses will be graded taking into account: [1] the correctness and cogency of your facts and arguments, [2] the use you make of asset pricing theory in your reasoning, and [3] the overall organization of your essay.

Total for Question 5: 20 marks

END!