



**STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES**  
**END OF SEMESTER EXAMINATION FOR BACHELOR OF BUSINESS**  
**SCIENCE: ACTUARIAL SCIENCE, FINANCIAL ECONOMICS AND**  
**FINANCIAL ENGINEERING**  
**BSE 2205: INTERMEDIATE ECONOMETRICS**

Date: 4<sup>th</sup> December, 2024

Time: 2.5 hours

**Instructions**

1. This examination consists of **Five** questions.
2. Answer **Question One**(Compulsory) and **any other two** questions.

**Question 1**

- (a) Consider the matrices  $X'X = \begin{bmatrix} 1000 & 2000 \\ 2000 & 9000 \end{bmatrix}$  and  $X'Y = \begin{bmatrix} 5000 \\ 8000 \end{bmatrix}$ . Use this matrix to answer the following questions:
- (i) What is the number of observations used to derive these matrices {2 marks}
  - (ii) What is  $\sum_{i=1}^n x_i^2$ ? {2 mark}
  - (iii) What is  $\sum_{i=1}^n x_i y_i$ ? {2 marks}
  - (iv) What is the underlying model being estimated using these matrices? {2 marks}
  - (v) Suppose the  $\sum_{i=1}^n x_i^2 = 4,000$ , would it be possible to estimate the parameters in 1a(iv)? If so why? {2 marks}
- (b) To estimate  $\hat{\beta}_{OLS}$  we make several assumptions, one of these assumptions is that  $E[UU'] = \sigma^2 I_n$ . State the assumption(s) implied by this expression and with the help of appropriate algebra prove that indeed  $E[UU'] = \sigma^2 I_n$  {5 marks}

- (c) Eugene has entered some data on Stata. The resulting matrix in data view is  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ .

He seeks to estimate the model,  $y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i$ . Required:

- (i) Construct the matrix  $X$  that Stata uses to estimate this model {3 marks}
- (ii) Given  $X$ , what problems will Eugene experience in estimating the model {3 marks}
- (iii) State the three specifications of the model that Eugene can estimate to overcome the challenges in 1c(ii) above {3 marks}

- (d) Using a well labelled diagram distinguish between residuals, errors and bias {3 marks }
- (e) Matrix A shows observations on the age, weight and height of three individuals
- $$\begin{bmatrix} 10 & 20 & 100 \\ 2 & 6 & 30 \\ 35 & 70 & 161 \end{bmatrix}$$
- use it to answer the following questions:
- (i) What is the age of the second individual? {1 mark}
- (ii) What is the height of the third individual? {1 mark}
- (iii) What is weight of the first individual? {1 mark}

[30 marks]

### Question 2

Consider the model  $y_i = \beta_1 + \beta_2 x_{2i} + \epsilon_i$  where  $Y' = [4 \ 3 \ -5 \ -6]$  and  $X' = [2 \ 1 \ -1 \ -2]$ . Required:

- (a) Find  $\hat{\beta}_{OLS}$  {3 marks}
- (b) Show that  $\hat{\beta}_{OLS}$  is unbiased {3 marks}
- (c) Using the findings in 2(b) derive the expression for  $var(\hat{\beta}_{OLS})$  {3 marks}
- (d) Using the expression in 2(c), find  $var \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$  if  $\hat{\epsilon}'\hat{\epsilon} = 3.6$  {3 marks}
- (e) Estimate the standard errors associated with  $\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$  {2 marks}
- (f) Estimate t-statics associated with  $\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}$  {2 marks}
- (g) If the tabulated t statistics at 2 degrees of freedom and at 1%, 5% and 10% level of significance are 9.925, 4.303 and 2.920 respectively, what asterisks would be associated with each of the parameters should these results be output using the `outreg2` command in Stata? {2 marks}
- (h) Construct a table showing how the output from `outreg2` command would look like {2 marks}

[20 marks]

### Question 3

You are given the following data sampling process  $y_i = \beta_1 + \beta_2 x_{2i} + \epsilon_i$  where:

$$\epsilon_i = \sqrt{x_{2i}} * u_i$$

$$u_i \stackrel{iid}{\sim} N(0, 1)$$

and  $x_{2i}$  is a non-stochastic positive variable.

- (a) Show that this model is heteroskedastic {2 marks}
- (b) If the empirical information is  $Y = [4 \ 2 \ 5 \ 7]'$  and  $x_{2i} = [1 \ 1 \ 4 \ 4]'$ . Estimate  $\hat{\beta}_{OLS}$  {2 marks}

- (c) What are the characteristics of  $\hat{\beta}_{OLS}$ ? {2 marks}
- (d) Discuss how you would transform the data so that you could remove the heteroskedasticity {2 marks}
- (e) Now estimate the model with the empirical information given in section (b), but by GLS. {4 marks}
- (f) Show that in this case  $var(\hat{\beta}_{GLS}) = \begin{bmatrix} \frac{10}{9} & -\frac{4}{9} \\ -\frac{4}{9} & \frac{5}{18} \end{bmatrix}$  {2 marks}
- (g) Supply the robust standard errors that can be used to test the significance of  $\beta_1$  and  $\beta_2$  in 2(c) above {2 marks}
- (h) The figure below shows edited stata output detailing results for a heteroscedasticity test. Use it to test whether the reference estimation was heteroscedastic. {2 marks}

```
. hettest

Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
Variable: Fitted values of price

H0: Constant variance

      chi2(1) = 105.75
Prob > chi2 = 0.0000
```

- (i) Benji ran the following regression  $Price_i = \beta_1 + \beta_2 \text{lotsize} + u_i$ . If this regression was heteroscedastic and price and lot-size are positively related, sketch the distribution of price around the line of best fit. {2 marks}

[20 marks]

#### Question 4

Consider the following data sampling process

$$Y_t = \beta x_t + \epsilon_t \text{ where}$$

$$\epsilon_t = 0.6\epsilon_{t-1} + U_t$$

$$U_t \stackrel{iid}{\sim} N(0, 1)$$

You are told that  $x$  is exogenous and are also given the following matrices:

$$X'X = \begin{bmatrix} 20 & 10 \\ 10 & 10 \end{bmatrix}', X'y = \begin{bmatrix} 86.6 \\ 68.4 \end{bmatrix}', X'\Psi X = \begin{bmatrix} 72.5 & 36.25 \\ 36.25 & 32.55 \end{bmatrix}', X'\Psi^{-1}X = \begin{bmatrix} 5.75 & 2.875 \\ 2.875 & 3.8125 \end{bmatrix}'$$

$$\text{and } X'\Psi^{-1}y = \begin{bmatrix} 25.475 \\ 25.29375 \end{bmatrix}' \text{ where } \sigma^2\Psi \text{ is } E(\epsilon\epsilon')$$

Required:

- (a) Assume that  $\epsilon_t \sim N(\mu_\epsilon, \sigma_\epsilon^2)$  for every  $t$ . Show that  $\mu_\epsilon = 0$  and  $\text{var}(\epsilon_t) = \frac{25}{16}$  {3 marks}
- (b) What is the shape and dimension of  $\Psi$ ? (You don't have to write it out in full) {4 marks}

- (c) Estimate  $\beta_1$  and  $\beta_2$  using OLS {3 marks}
- (d) Discuss the characteristics of  $\hat{\beta}_{OLS}$  {5 marks}
- (e) Estimate the true value of variance-covariance matrix of  $\hat{\beta}_{OLS}$  {3 mark}
- (f) Test the null hypothesis that  $\beta_2 = 0$  using your OLS estimator of  $\beta_2$  {2 marks}

[20 marks]

### Question 5

Consider the model  $y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \epsilon_i$ . Required:

- (a) State the OLS assumption that would be violated if  $x_{2i}$  was endogenous {3 marks}
- (b) What implication would the violation in 5(a) have on the distribution of  $y_i$ ? {3 marks}
- (c) What would be the consequences of the endogeneity of  $x_{2i}$  on the estimation of  $\hat{\beta}_{OLS}$ ? {5 marks}
- (d) One of the methods of dealing with endogeneity requires that we understand how to derive  $\hat{\beta}_{OLS}$  but with the generalized method of moments (GMM). Derive  $\hat{\beta}_{GMM}$  for the equation above {5 marks}
- (e) With the help of a relevant empirical example, suggest the solution for endogeneity implied in 5(d) above {4 marks}

[20 marks]

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