Strathmore
U N I V E R S I T Y

Strathmore Institute of Mathematical Sciences
BBS Actuarial Science, BBS Finance, BBS Financial Economics & BBS
Financial Engineering
END OF SEMESTER EXAMINATION

BSA 3108:- THEORY OF FINANCE

Date: 13th August, 2020   Time: 1300 - 1600 Hours

INSTRUCTIONS

1. This is an Open Book Examination

2. There are FIVE questions in this examination

3. Attempt QUESTION ONE and ANY OTHER TWO questions

4. Question one is COMPULSORY and carries 30 marks

5. All the other questions carry 20 marks each

6. Show all *your workings* and clearly define all *notations* used

7. Originality of your responses will be thoroughly checked. Collaboration with classmates is probihibited

QUESTION 1   -   COMPULSORY

(a) Imagine a two-state world with a financial market. In the market, there is a stock with current price 100. In the next period, the stock either rises to 110 with a 60% probability or declines to 90 with 40% probability. We also have a European call option on the stock maturing next period with strike price 88 and current price of 10.

    i. Is the market complete? *Explain.*   [2 marks]

    ii. *Show* that under no arbitrage, this option price implies a risk free rate of 5%   [5 marks]

    iii. *Calculate* the risk-neutral probabilities implied in this set up   [5 marks]

(b) Investors who seek to earn unusual (abnormal) positive returns through their investments in equities are often looking for news and events that will influence a stock's price. The semi-strong form of the efficient market hypothesis (Fama, 1970) supposes that as a result, this information

is incorporated into a stock's price almost as soon as it becomes public. Event studies are used as a formal statistical test to determine if and when new information causes a price change in stocks.

Recently, Safaricom Plc., a Kenyan telecommunications company announced a new CEO who would take office as per the schedule shown in the table below:

Item	Date
Announcement date	24 October 2019
Effective date	1 April 2020

Safaricom Plc provides integrated telecommunication services, including mobile and fixed voice, short messaging service (SMS), data, Internet and M-PESA, a service to send and receive money or pay for goods and services through a mobile phone. It also provides financial services and enterprise solutions to businesses and the public sector. The Company's voice services include national and international roaming services. It offers mobile handsets, mobile broadband modems, routers, tablets and notebooks. Its converged business solutions include fixed voice service for corporate and Small and Medium-sized Enterprises (SMEs), mobile solutions and fixed data for homes and businesses using fixed lease lines, and Internet solutions for enterprises and hosted services, such as cloud hosting, domains and security services. The Company offers bill payment, bulk payments and dividend payments as well.

i. Formulate an event study that can be used to study the effect of new CEO announcment on the performance of the Safaricom Plc stock price [12 marks]

ii. The change in CEO coincided many other external events such as the Covid-19 pandemic among others. How would you adjust your analysis in *(i)* above to take into account the impact of such other events. [6 marks]

Total for Question 1: 30 marks

QUESTION 2   -   OPTIONAL

There are two equally likely states of the world. In state 1, your endowment of wealth will be 12; in state 2, it will be 3. You are an expected utility maximizer with a utility function of the form $u(w) = \sqrt{\omega}$ where ω denotes your final wealth. You can buy and sell state-contingent claims to wealth at prices 1/3 for one unit of state-1 wealth and and 2/3 for one unit of state-2 wealth.

(a) Write down the expression for your expected utility.

(b) Write down the budget constraint for your choices of final wealth quantities ω_1 and ω_2 in the two states.

(c) Work out the optimal choices of ω_1 and ω_2.

(d) Would your results in $(i - iii)$ above change if you exhibited logarithmic utility of wealth instead? What argument would you give for the difference in the results of the two cases if any?

Total for Question 2: 20 marks

QUESTION 3 - OPTIONAL

(a) The optimization problem in Mean Variance Portfolio Optimization (MVO) entails either finding the portfolio ω that has the highest expected return for a given level of risk as measured by portfolio variance or finding the portfolio ω that has the smallest risk that achieves a target return as shown below:

$$\min_{\omega} \sigma^2_{p,\omega} = w'\Sigma w \ s.t.$$

$$\mu_{p,\omega} = w'\mu = \mu^0_p = target \ return$$
$$\mathbf{w'1} = 1$$

Using matrix algebra, work out the optimal portfolio ω that has the smallest risk that achieves the target return   [8 marks]

(b) Your classmate has sent you the following workings for deriving the security market line. Your classmate further clarifies that the analysis is carried out considering an investor who invests ω in a risky asset i and $(1-\omega)$ in the market portfolio M.

Beginning of your class mate's security market line workings:

The expected return on any investment in a risky asset and a risky asset can be expressed as:

$$E(R_p) = \omega E(R_i) + (1-\omega) E(R_m)$$

where $E(\cdot)$ represents expected returns. The standard deviation for the same portfolio is expressed as:

$$SD(R_P) = [\omega^2\sigma^2 + (1-\omega)^2 \sigma^2_m + 2\omega(1-\omega)\sigma_i\sigma_m\rho_{im}]^{\frac{1}{2}}$$
$$= [\omega^2\sigma^2_i + (1-\omega)^2 \sigma^2_m + 2\omega(1-\omega)\sigma_{im}]^{\frac{1}{2}}$$

Where ρ_{iM} and σ_{iM} is the correlation and covariance between the returns on asset i and returns on the market portfolio, M respectively. The slope of the line is obtained as usual.

The derivative of $E(R_p)$ with respect to ω is (evaluated at $\omega = 0$ is:

$$\frac{dE(Rp)}{d\omega} = E(Ri) - E(Rm)$$

And the derivative of $SD(R_p)$ with respect to ω is obtained through the chain rule to give

$$\frac{dSD(R_P)}{d\omega} = \frac{1}{2} \left[w^2 \ \sigma^2_i + (1-w)^2 \sigma^2_M + 2\alpha(1-\alpha)\sigma_{iM}\right]^{-\frac{1}{2}}$$

When, $w = 0$ have. $\frac{dSD(Rp)}{d\omega} = \frac{\sigma 2m}{\sigma m}$ Therefore, the slope of the risk-return trade-off function evaluated at the tangency of the efficient frontier to the market portfolio

$$\frac{dE(Rp)/d\omega}{dSD(Rp)/d\omega} = \frac{E(Ri) - E(Rm)}{\sigma^2_m}$$

This slope is equal to that of the capital market line

$$\frac{E\left(Rm\right) - Rf}{\sigma_m}$$

As such

$$\frac{E\left(R_i\right) - E\left(R_m\right)}{\sigma_m^2} = \frac{E\left(Rm\right) - Rf}{\sigma_m}$$

Hence

$$E\left(R_i\right) = R_f + \left[E\left(R_m\right) - R_f\right]\beta_i$$

Which is the security market line or the capital asset pricing model.
End of your class mate's security market line workings:

Evaluate the above workings and *identify at least three mistakes* in the workings. *Justify* your identified mistakes and suggest an appropriate correction for each. [12 marks]

Total for Question 3: 20 marks

QUESTION 4 - OPTIONAL

  (a) Using a suitable example, evaluate how Expected Shortfall (ES) performs over Value at Risk in satisfying each of the following attributes of a coherent risk measure.

     i. Translation Invariance   [3 marks]

     ii. Monotonicity   [3 marks]

     iii. Sub-additivity   [3 marks]

     iv. Positive Homogeneity   [3 marks]

  (b) Critique the validity of the following statements

     i. Intuitively, the CML in a "more risk averse economy" should be steeper, in view of its risk/return trade-off interpretation   [4 marks]

     ii. The frontier of the economy where asset returns are more correlated and where diversification opportunities are thus lower is contained inside the efficient frontier of the economy where assets are less correlated.   [4 marks]

Total for Question 4: 20 marks

QUESTION 5 - OPTIONAL

The *Binomial Option Pricing Model (BOPM)* provides a powerful tool for understanding *arbitrage pricing theory* and *probability theory*. In fact we use the model to study useful facts regarding the *Fundamental Theorem of Arbitrage Pricing (FTAP)*.

The foregoing statements yield three key reasons *(a - c below)* that have been known to underpin the use of the BOPM. Illustrate each of the uses using suitable workings.

(a) The BOPM clearly outlines the concept of *arbitrage pricing* and its relation to *risk neutral pricing*   [9 marks]

(b) The model is used in practice because with sufficient number of steps, it provides a good, computationally tractable approximation to continuous time models   [6 marks]

(c) Within the binomial model we can develop the theory of *conditional expectations* and *martingales* which are at the heart of continuous time models   [5 marks]

Total for Question 5: 20 marks

END!