Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

END OF SEMESTER EXAMINATION

MAT1201: MATHEMATICS II

DATE: 20/03/2025 Time: 16:00-18:00

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE [30 MARKS]

(a) An estimate for $\int_0^{\pi/2} \cos x \, dx$ is required.

i. Use Simpson's rule with two strips to show that this estimate is $\frac{\pi}{12}\left(1+2\sqrt{2}\right)$.

[3 Marks]

ii. Find the exact value using algebraic integration.

[1 Mark]

iii. Find the percentage error in the estimate obtained using Simpson's rule.

[1 Mark]

(b) Given that $A=\begin{pmatrix} 5 & -2 & 1 \\ 3 & -1 & 2 \\ -1 & 3 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} -7 & 5 & -3 \\ -5 & 6 & -7 \\ -1 & -13 & 1 \end{pmatrix}$.

i. Determine BA. Hence A^{-1}. **[3 Marks]**

ii. Use results in (i) above, to solve the following simultaneous equations.

$$5x-2y+z=-7$$
$$3x-y+2z=-3$$
$$3y-x+z=0$$

[3 Marks]

(c) A variable complex number $z = x + jy$ is such that the amplitude of $\frac{z-1}{z+1} = \frac{\pi}{4}$. Show that the locus of z is a circle whose centre represents the complex number j.

[3 Marks]

(d) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{pmatrix}$ by using elementary row operations.

[3 Marks]

(e) Given that $z_1 = 4 + 3j$, $z_2 = 3 - 2j$, $z_3 = -4 + 10j$ and $z = \frac{z_1 + z_2 z_3}{z_2 + z_3}$. Evaluate $(5 + 15j)z$ leaving the answer in exponential form.

[4 Marks]

(f) Given that $M = \begin{pmatrix} 2 & -1 \\ 2 & 5 \end{pmatrix}$.Find the eigen values and the corresponding eigen vectors of M.

[5 Marks]

(g) Find the value of the integral $\int_0^{1+j} (x - y + jx^2)\,dz$, along the straight line from $z = 0$ to $z = 1 + j$. **[4 Marks]**

QUESTION TWO [15 MARKS]

(a) Find all the values of $(\sqrt{3} + j)^{\frac{1}{5}}$. When they are represented on the Argand diagram by points, show that they form the vertices of a regular pentagon.

[5 Marks]

(b) The eigen vectors of a matrix A corresponding to the eigen values are $1, 1, 3$ are $[1 \quad 0 \quad -1]^T$, $[0 \quad 1 \quad -1]^T$, and $[1 \quad 1 \quad 0]^T$ respectively .Find A.

[5 Marks]

(c) ABC Piston Company wishes to produce three types of pistons: type X, type Y and type Z. To manufacture a type X piston requires 2 minutes each on machine I and II, and 3 minutes on machine III. A type Y piston requires 2 minutes on machine I, 3 minutes on machine II and 4 minutes on machine III. A type Z piston requires 3 minutes on machine I, 4 minutes on machine II and 3 minutes on machine III. There are $3\frac{1}{2}$ hours available on machine I, $4\frac{1}{2}$ hours available on machine II and 5 hours available on machine III.

i. Use the information provided to form a system of simultaneous equations.

[2 Marks]

ii. Apply Cramer's rule to calculate the number of pistons of each type should the company make in order to use all available the available time.

[3 Marks]

QUESTION THREE [15 MARKS]

(a) Verify Cayley Hamilton theorem and find the inverse of the matrix $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$.

[6 Marks]

(b) Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

[9 Marks]

QUESTION FOUR [15 MARKS]

(a) Use De Moivres theorem to show that

$$\cos^7\theta = \frac{1}{64}\left(\cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta\right)$$ **[5 Marks]**

(b) Using Cauchy Riemann equations, show that $f(z) = z^3$ in the entire $z - plane$.

[5 Marks]

(c) Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}\, dz$ where C is $|z - j| = 2$.

[5 Marks]

QUESTION FIVE [15 MARKS]

(a) Find an exact expression for $\int_0^k e^{2x/k}\, dx$. **[3 Marks]**

(b) Use Simpson's rule to show that an approximate value of this integral is $\frac{k}{6}\left(1 + 4e + e^2\right)$.

[4 Marks]

(c) By equating your answers to (a) and (b) , find an approximate numerical value for e.

[4 Marks]

(d) Show that the corresponding approximation found using the trapezium rule is $e = 3$.

[4 Marks]

END