



STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES (SIMS)  
BACHELOR OF SCIENCE IN STATISTICS AND DATA SCIENCE  
MAT 1102 DISCRETE MATHEMATICS  
END OF SEMESTER EXAMINATION

Date: 26th July 2024

Time: 08:00 am to 10:00 am

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Instructions

- Answer **QUESTION ONE** and any other **TWO** questions.
- Show all your workings in the booklet provided.

**QUESTION ONE (30 Marks)[COMPULSORY]**

- Q1).** (a) State if each of the following statements are **TRUE** or **FALSE**. Provide a brief justification for your answer.
- (i) The possible number of arrangements of the letters of the word **REMAIN** given that such arrangements must begin with **RE** is 24. (2 marks)
  - (ii) In a drawer containing a mixture of black socks and blue socks, **the minimum number of socks, picked one at a time from the drawer without looking**, that guarantee picking a pair of the same colour, is 3. (2 marks)
  - (iii) The complete graph  $K_6$  is planar. (2 marks)
  - (iv) The total number of ways of flipping 5 two sided coins is 10. (2 marks)
- (b) Simplify  $\frac{(x^2 - 4)!}{(x - 2)(x^2 - 5)!}$ . (3 marks)
- (c) By drawing the cycle graph  $C_6$ , show that graph  $C_6$  is bipartite. (3 marks)

- (d) How many numbers are there between 99 and 1000, having at least one of their digits as 7? (3 marks)
- (e) In how many ways can a group of 5 men and 4 ladies be arranged on a bench if a couple from the group request to sit next to each other? (3 marks)
- (f) Using Binomial theorem, compute  $(-0.999)^5$  to three decimal places. (3 marks)
- (g) Using Euler's formula show that the graph  $K_5$  is not planar. (3 marks)
- (h) Give the recurrence formula for computing Stirling numbers of the second kind and hence find  $S(4, 3)$ . Interpret your answer. (4 marks)

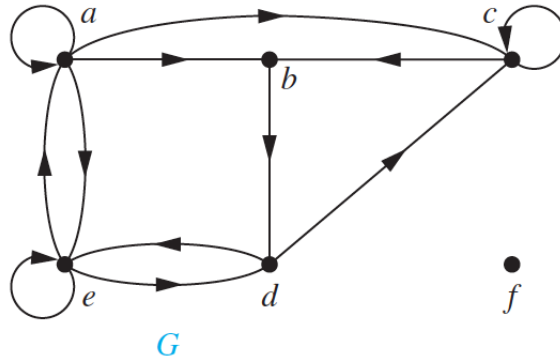
**QUESTION TWO (20 Marks)[OPTIONAL]**

- Q2).** (a) At a dinner party 6 men and 6 women sit at a round table. In how many ways can they sit if two friends request to sit next to each other? (3 marks)
- (b) A committee of 12 people is to be formed from a group of 10 gentlemen and 12 ladies. In how many ways can the committee be formed if:
- (i) at most 5 ladies are to be part of the committee? (3 marks)
- (ii) two thirds gender rule is satisfied in forming the committee? (That is, there should be not more than two-thirds of either gender in the committee) (3 marks)
- (c) The government of Kenya is planning to restart the registration of the number plates of government vehicles. If the number plate is supposed to have 6 characters with the first two characters as  $G$  and  $K$ , then followed by any three letters of the English Alphabet and the last character given by any digit.
- (i) How many number plates can be produced using this method? (3 marks)
- (ii) How much should the government budget with to produce all the number plates if the cost of producing 5 number plates is KES 15, 000. (2 marks)
- (d) Find the coefficient of  $x^5$  in the expansion of (6 marks)

$$\left(2x^2 - \frac{1}{3x}\right)^{10}.$$

QUESTION THREE (20 Marks)[OPTIONAL]

Q3). (a) Consider the following graph  $G$ , and let  $V = \{a, b, c, d, e, f\}$ .



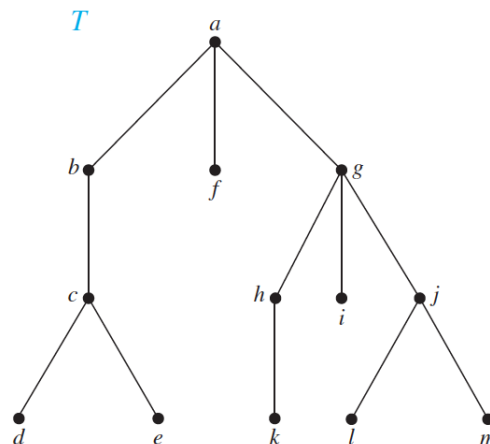
Verify that

$$\sum_{v \in V} \text{deg}^-(v) = \sum_{v \in V} \text{deg}^+(v).$$

(4 marks)

(b) Give one example of a graph that is not a tree and provide a justification for your answer. (2 marks)

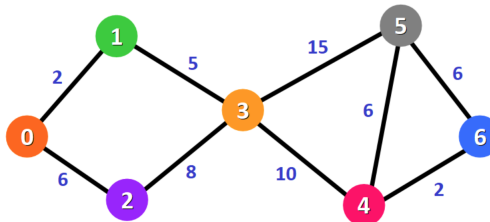
(c) Consider the following rooted tree  $T$ .



Find:

- (i) The parent of  $c$  (1 mark)
- (ii) The children of  $g$ . (2 marks)
- (iii) All ancestors of  $e$ . (2 marks)
- (iv) Draw the subtree rooted at  $g$ . (2 marks)

(d) Consider the following graph.



By applying Dijkstra's Algorithm compute the shortest path between vertices 0 and 6. (7 marks)

**QUESTION FOUR (20 Marks)[OPTIONAL]**

**Q4).** (a) Draw an undirected graph represented by the following adjacency matrix.

$$\begin{pmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{pmatrix}$$

(3 marks)

(b) Suppose that a connected simple planar graph has 300 vertices each of degree 4. Into how many regions does the representation of the graph split the plane?

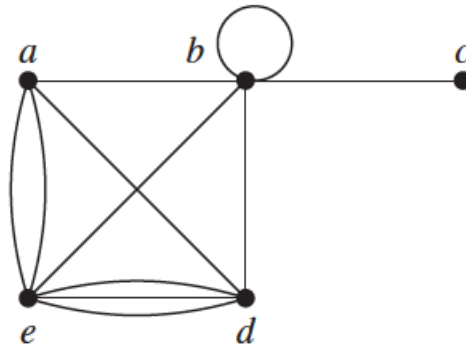
(3 marks)

(c) Draw and name each of the following graphs, and check whether they are planar or not:

(i)  $K_5$  (3 marks)

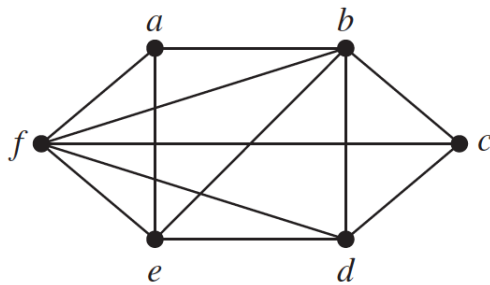
(ii)  $K_{2,2}$  (3 marks)

(d) Considering the following graph  $H$ .



$H$

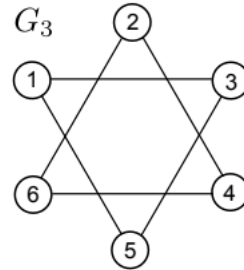
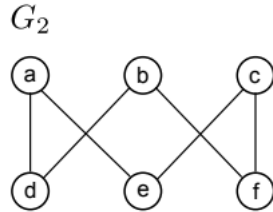
- (i) Find the degree of each vertex. (3 marks)
- (ii) Using your answer in part (i) above, verify the Handshaking Theorem. (2 marks)
- (e) Check whether the following graph  $H$  is a bipartite graph or not. (3 marks)



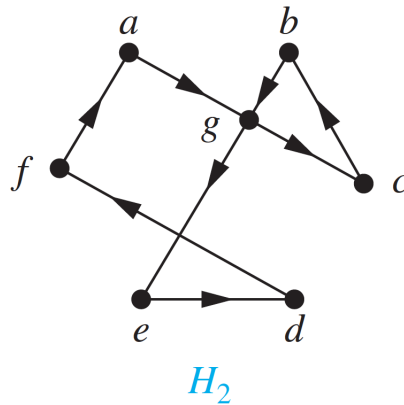
$H$

QUESTION FIVE (20 Marks)[OPTIONAL]

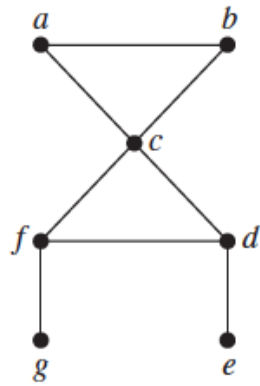
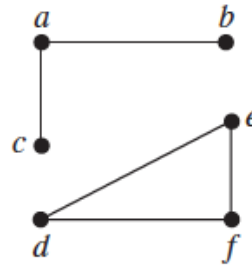
- Q5). (a) Give an example of a graph with an Hamilton path but with no Hamilton circuit. (2 marks)
- (b) Check whether the following graphs  $G_2$  and  $G_3$  are isomorphic or not. Provide an elaborate justification for your answer. (4 marks)



- (c) Find the chromatic number of each of the following graphs. (Clearly draw and show the colouring in your answer booklet).
- (i)  $K_5$  (3 marks)
- (ii)  $K_{5,5}$  (3 marks)
- (d) Check whether the following graph  $H_2$  has an Euler circuit or not. (3 marks)



- (e) Check whether each of the following graphs ( $G_1$  and  $G_2$  shown below) are connected or not. Justify your answers. (5 marks)

 $G_1$  $G_2$ 

THE END