



Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
END OF SEMESTER EXAMINATION
MAT 1102 APPLIED MATHEMATICS I

Date: October 27, 2021

Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

- (a) If $\vec{p} = 4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\vec{q} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\vec{r} = \mathbf{i} - 2\mathbf{k}$, find $(\vec{p} - 2\vec{q}) \times \vec{r}$ [3 Marks]
- (b) A packet contains 100 washers, 24 of which are brass, 36 copper and the remainder steel. One washer is taken at random, retained, and a second washer similarly drawn. Determine the probability that the first is brass and the second copper [4 Marks]
- (c) A family that owns two automobiles is selected at random.
Let $A_1 = \{\text{the older car is American}\}$ and $A_2 = \{\text{the newer car is American}\}$.
If $P(A_1) = 0.7$, $P(A_2) = 0.5$ and $P(A_1 \cap A_2) = 0.4$, compute:
- (i) $P(A_1 \cup A_2)$, (the probability that at least one car is American). [2 Marks]
 - (ii) The probability that neither car is American. [1 Marks]
- (d) The birth weights (x) in kg, of 1500 babies are summarised as follows:
- $$\sum fx = 4841, \quad \sum fx^2 = 15889.5.$$
- Compute the mean and standard deviation for birth weights. [4 Marks]
- (e) Let P , Q and R be the points at 1 on the x-axis, 2 on the y-axis and 3 on the z-axis, respectively.
- (i) Express \vec{QP} and \vec{QR} in terms of \mathbf{i} , \mathbf{j} and \mathbf{k} . [2 Marks]
 - (ii) Find the cosine of the angle PQR. [3 Marks]
- (f) Let $P(A) = 0.4$ and $P(A \text{ or } B) = 0.7$. For what value of $P(B)$ are events A and B mutually exclusive? [2 Marks]

(g) The lengths of 50 copper plugs gave the following frequency distribution:

Length, x (mm)	14.0 - 14.2	14.3-14.5	14.6-14.8	14.9-15.1
Frequency, f	2	4	9	15

Length, x (mm)	15.2 - 15.4	15.5-15.7	15.8-16.00
Frequency, f	11	6	3

(i) Calculate the mean and the standard deviation. [3 Marks]

(ii) For a full batch of 2400 plugs, calculate the number of plugs with lengths greater than 15.09 mm. [2 Marks]

(h) Differentiate between the following terminologies.

(i) Statistics and Statistic [2 Marks]

(ii) Mutually exclusive events and independent events [2 Marks]

QUESTION TWO (15 MARKS)

(a) Describe the empirical rule of a normally distributed data. [3 Marks]

(b) A sample of size 40 yields the following sorted data. Note that I have x -ed out $x_{(39)}$ (the second largest number). This fact will NOT prevent you from answering the questions below.

14.1	46.0	49.3	53.0	54.2	54.7	54.7
54.7	54.8	55.4	57.6	58.2	58.3	58.7
58.9	60.8	60.9	61.0	61.1	63.0	64.3
65.6	66.3	66.6	67.0	67.9	70.1	70.3
72.1	72.4	72.9	73.5	74.2	75.3	75.4
75.9	76.5	77.0	x	88.9		

(i) Calculate range and median of these data. [2 Marks]

(ii) Given that the mean of these data is 63.50 (exactly) and the standard deviation is 12.33, what proportion of the data lie within one standard deviation of the mean? [3 Marks]

(iii) How does your answer to (ii) compare to the empirical rule approximation? [1 Marks]

(iv) **Ralph** decides to delete the smallest observation, 14.1, from these data. Thus, Ralph has a data set with $n = 39$.

Calculate the range and median of **Ralph's** new data set. [2 Marks]

(v) Refer to (iv). Calculate the mean of **Ralph's** new data set. [2 Marks]

(c) State two disadvantages of the median, as a measure of central tendency. [2 Marks]

QUESTION THREE (15 MARKS)

- (a) If $\vec{OA} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\vec{OB} = 2\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$, determine the cosine of the angle between \vec{OA} and \vec{OB} [4 Marks]
- (b) The points $A(2, -1, 5)$, $B(5, 2, 10)$ and $D(-1, 1, 4)$, together with a point C , are the vertices of the parallelogram $ABCD$, where $\vec{AB} = \vec{DC}$. Find the position vector of C . [3 Marks]
- (c) If A is the point $(1, -1, 2)$, B is the point $(-1, 2, 2)$ and C is the point $(4, 3, 0)$, find the direction cosines of \vec{BA} and \vec{BC} , and hence show that the angle $ABC = 69^{\circ}14'$. [4 Marks]
- (d) Calculate the area of the parallelogram spanned by the vector $\vec{a} = 7\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $\vec{b} = \mathbf{i} + 0\mathbf{j} + 6\mathbf{k}$ [4 Marks]

QUESTION FOUR (15 MARKS)

- (a) State and explain 3 approaches of assigning probabilities [3 Marks]
- (b) State the multiplicative law for non-independent events. [2 Marks]
- (c) A chain of video stores sells three different brands of videocassette recorders (VCRs). Fifty percent of its VCR sales are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a one-year warranty on parts and labour. It is known that 25% of brand 1's VCRs require warranty repair work, whereas the corresponding percentages for brand 2 and 3 are 20% and 10%, respectively.
- (i) What is the probability that a randomly selected purchaser has bought a brand 1 VCR that will need repair while under warranty? [2 Marks]
- (ii) What is the probability that a randomly selected purchaser has a VCR that will need repair while under warranty? [2 Marks]
- (iii) If a customer returns to the store with a VCR that needs warranty repair work, what is the probability that it is a brand 1 VCR? A brand 2 VCR? A brand 3 VCR? [6 Marks]

QUESTION FIVE (15 MARKS)

(a) The lengths, in millimetres, of 40 bearings were determined with the following results:

16.6	18.7	13.6	17.3	15.3	16.4	18.3	16.6	16.3	19.0
17.2	15.3	14.2	15.8	18.0	16.4	16.7	18.4	15.8	17.3
17.3	15.1	19.3	16.9	18.2	17.0	16.8	14.7	15.6	18.9
17.7	16.2	14.9	18.3	16.8	17.4	17.2	15.9	17.9	15.6

(i) Group the data into six equal width classes between 13.5 and 19.4 mm and hence the frequency distribution. [3 Marks]

(ii) Calculate (i) the mean, (ii) the variance. [4 Marks]

(b) Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ be two vectors in \mathbb{R}^3 , and let θ be the angle the two vectors form when their feet are place together. Show that the cross product of \mathbf{u} and \mathbf{v} is given as follows: [4 Marks]

$$\mathbf{u} \times \mathbf{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$

(c) Calculate the dot product of the vectors $\mathbf{a}=(1,2,4)$ and $\mathbf{b}=(4,-5,6)$. Do the vectors form an acute angle, right angle or obtuse angle? [4 Marks]

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