Strathmore Institute of Mathematical Sciences
BSc. Financial Engineering
End of Semester Examination
BSM 3220 - Optimization Methods in Finance

Date: $11^{th} January$ 2022 Time: 2 Hours

Instruction

1. Answer **QUESTION ONE** and any other **TWO QUESTIONS**

QUESTION ONE [30 Marks]

a) A standard linear programming problem is one of the form

$$Max\ \mathbf{C}^T\mathbf{x} : A\mathbf{x} \leq b$$

where $\mathbf{x}, \mathbf{C} \in \mathbb{R}^n$, $b \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m\times n}$.

i. Give the dual of the primal problem above. [1 Mark]

ii. When is the primal problem above is feasible? [1 Mark]

iii. When is a feasible solution to the problem optimal? [1 Mark]

b) Given that $Q \geq 0$, give the Karush-Kuhn-Tucker conditions for the problem: [2 Marks]

$$Min\ \frac{1}{2}x^TQx + C^Tx,\ \ such\ that\ \ Ax = 0.$$

c) True or false, given a primal problem **P** and its dual **D** one of the following applies:

i. Both **P** and **D** are infeasible. [1 Mark]

ii. If **P** is unbounded then **D** is infeasible. [1 Mark]

iii. If **D** is unbounded then **P** is infeasible. [1 Mark]

d) Given the non-linear programming problem $Min\ f(x)$ subject to $h_k(x) = 0$ and $g_j(x) = 0$ where h and g are differentiable with $k = 1, 2, \cdots k$ and $j = 1, 2, \cdots j$. Provide the Kuhn-Tucker conditions for this non-linear programming problem. [3 Marks]

e) From (d) above, find the Kuhn-Tucker conditions given $f(T) = (a-1)^2 + b^2$ subject to $g(T) = -a + b^2$ where $T^T = [a \quad b]$. [3 Marks]

f) Max $f(x_1, x_2) = 4x_1 + 3x_2$ subject to $g(x_1, x_2) = 2x_1 + x_2 \leq 10$ and $x_1, x_2 \geq 0$ using Lagrange multipliers. [3 Marks]

g) State the dual of the following linear programming problem: [1 Mark]

$$\begin{aligned} Min\ A &= -12a + 10b + 2c \quad such\ that \\ -4a + b - 8c &\geq 1 \\ -a + b + 12c &\geq 3 \\ a, b, c &\geq 0 \end{aligned}$$

h) Given any linear programming problem, list the only three possible outcomes. [3 Marks]

i) Solve the problem by making use of the Kuhn-Tucker conditions. Exhaust the four cases that arise. [9 Marks]

$$\begin{aligned} Max\ f(x, y) &= xy + x^2 \quad such\ that \\ x^2 + y &\leq 2 \\ y &\geq 1 \end{aligned}$$

QUESTION TWO [20 Marks]

a) A pottery company in Busia county manufactures three products namely X, Y and Z. Each of the product require processing on three machines, Turning, Milling and Grinding. Product X requires 10 hours of turning, 5 hours of milling and 1 hour of grinding. Product Y requires 5 hours of turning, 10 hours of milling and 1 hour of grinding, and Product Z requires 2 hours of turning, 4 hours of milling and 2 hours of grinding. In the coming planning period, 2700 hours of turning, 2200 hours of milling and 500 hours of grinding are available. The profit contribution of X, Y and Z are K.sh. 10, K.Sh.15 and K.Sh. 20 per unit respectively. Find the optimal product mix to maximize the profit. Use simplex method. [10 Marks]

b) Use the Karush-Kuhn-Tucker conditions to [10 Marks]

$$\begin{aligned} Maximize\, f(p,q) &= pq \quad such\, that \\ q^2 + p &\leq 2 \\ q, p &\geq 0 \end{aligned}$$

QUESTION THREE [20 Marks]

a) The production manager of Mabati rolling company has arranged for a job-training programme at a ratio of one for every ten trainees. The training programme lasts for one month. From file records it has been found that out of 10 trainees hired, only seven complete the programme successfully. (The unsuccessful trainees are released). Trained machinists are also needed for machining. The company's requirement for the next three months is as follows:
January: 100 machinists, February: 150 machinists and March: 200 machinists. In addition, the company requires 250 trained machinists by April. There are 130 trained machinists available at the beginning of the year.
Pay roll cost per month is:
Each trainee 400 dollars per month.
Each trained machinist (machining or teaching) 700 per month and
Each trained machinist who is idle: 500 per month. **Formulate** a linear programming problem to help produce the minimum cost of hiring and training schedule and meet the company's requirement. [5 Marks]

b) Mr. Kibet is a poultry farmer in Iten and exports his hens to some English outlets in UK. Old hens can be bought for 2 pounds each but young ones costs 5 pounds each. The old hens lay 3 eggs per week and the young ones lay 5 eggs per week. Each egg costs 0.30 pounds. A hen costs 1 Pound per week to feed. If the financial constraint is to spend 80 Pounds per week for hens and the capacity constraint is that total number of hens cannot exceed 20 hens and the objective is to earn a profit more than 6 pounds per week, find the optimal combination of hens. Use graphical method. [7 Marks]

c) East African breweries limited manufactures two types of beers X and Y on facilities, A, B, C, D, E, and F having production capacities as under. If the profit contribution of beer X is K.Sh. $40/=$ per unit and that of Y is K.Sh. $30/=$ per unit, find the optimal product mix for maximising the profit.(Use simplex method) [8 Marks]

Facilities	Production capacity to produce
A	100 of X OR 150 of Y
B	80 of X OR 80 of Y
C	100 of X OR 200 of Y
D	120 of X OR 90 of Y
E	60 of X only (Testing facility for beer X)
F	60 of Y only.(Testing facility for beer Y)

QUESTION FOUR [20 Marks]

a) Use the KKT conditions to [5 Marks]

$$\begin{aligned} Minimize\ f(x,y) = x + y \quad such\ that \\ x^2 + y^2 &= 1 \end{aligned}$$

b) Give the KKT conditions for the problem: $Min \quad -Log(K+x)$ where k is a constant, subject to $x \geq 0$ and $1^T x = 1$. [5 Marks]

c) Use the KKT conditions to

$$\begin{aligned} Minimize\ x^2 + y^2 - 14x - 6y - 7 = (x-7)^2 + (y-3)^2 - 65 \\ such\ that \\ x + y &\leq 2 \\ x + 2y &\leq 3 \end{aligned}$$

. [7 Marks]

d) Obtain the dual of the following LPP: [3 Marks]

$$
\begin{aligned}
Maximize\; 3x + 2.5y \quad & such\; that \\
4.44x &\leq 100 \\
6.67y &\leq 100 \\
4x + 2.86y &\leq 100 \\
3x + 6y &\leq 100 \\
x &\geq 0
\end{aligned}
$$

QUESTION FIVE [20 Marks]

a) True or false:

i. Dual of primal is dual [1 Mark]

ii. Primal of dual is primal [1 Mark]

b) Given the problem below, solve both the primal and the dual and compare your results. [18 Marks]

$$
\begin{aligned}
Maximize\; Z = X_A + X_B + X_C + X_D + X_E &\leq 10 \quad such\; that \\
X_B + X_C + X_D &\geq 4 \\
0.6X_A + 0.6X_B - 0.4x_C - 0.4X_D + 3.6X_E &\leq 0 \\
4X_A + 10X_B - X_C - 2X_D - 3X_E &\leq 0 \\
X_A, X_B, X_C, X_D, X_E &\geq 0
\end{aligned}
$$