Portfolio Optimization and Asset Dependency Analysis Across Investment Regimes and Select Asset Classes in Kenya

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Declaration and Approval

Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

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Dedication

To my late father who started this journey with me. This thesis is dedicated to your memory and enduring inspiration.

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List of Abbreviations

ADF	Augmented Dickey-Fuller (statistical test)
BEKK	Baba, Engle, Kraft, and Kroner (alternative econometric model)
DCC	Dynamic Conditional Correlation
MGARCH	Multivariate Generalized Autoregressive Conditional Heteroskedasticity
NSE	Nairobi Securities Exchange
PP	Phillips-Perron (statistical test)

Definition of Terms

Emerging Markets: Financial markets in less developed countries that are in the1 process of becoming more developed, typically with potentially higher returns but also with greater risk and volatility (Bekaert & Harvey, 2002).

MGARCH: A sophisticated statistical model used in financial econometrics1 for time-series data analysis that is able to capture volatility clustering and interdependence of different financial variables (Engle & Kroner, 1995).

Portfolio Diversification: A technique of diversifying investments in a portfolio among var-1 ious asset classes with a view to reducing overall risk and loss (Markowitz, 1952).

Safe-Haven Assets: Assets that are expected to retain or increase value even in a falling1 market or economic downturn, e.g., gold or government bonds (Baur & Lucey, 2010).

Time-Series Data: A sequence of data items collected or recorded at uniform time1 intervals, used to analyze trends, patterns, and relationships over time (Chatfield, 2003).

Abstract

This study investigated the performance, dependency structures, volatility dynamics, and portfolio optimization strategies for the Nairobi Securities Exchange (NSE) 20 Index, gold, Bitcoin, and government bonds over the period from 2018 to 2023, within Kenya's emerging market context. Employing a quantitative approach, the research utilizes daily price data from the NSE, London Bullion Market Association, CoinMarketCap, and Central Bank of Kenya, analyzed through the Markov Regime-Switching Model (MSM), copula models, GARCH volatility modeling, and mean-variance optimization. The objectives were to assess asset performance, examine interdependencies, evaluate volatility impacts, and develop optimal portfolio strategies across distinct economic regimes. Findings revealed three regimes: pre-crisis stability (2018–February 2020), crisis turmoil (March 2020–December 2021), and post-crisis recovery (2022–2023). The NSE exhibits a negative return and high volatility in crisis, while gold and bonds provide stability, and Bitcoin shows speculative gains. Dependency shifts indicate stronger crisis co-movement with volatility peaking in turmoil. Optimized portfolios adapt, favoring Bitcoin in stability, bonds and gold in crisis, and a balanced mix, Bitcoin in recovery, outperforming a static benchmark. The study concludes that regime-specific strategies enhance risk-adjusted returns, recommending dynamic allocations for investors and regulatory enhancements for market resilience. These insights contribute to financial decision-making in Kenya's volatile market environment.

Chapter One: Introduction to the Study

1.1 Background of the Study

The global financial system has various asset classes that cater to different risk tolerance levels and investment objectives. Such asset classes include equities, bonds, commodities, and virtual assets like cryptocurrencies. Equities, as promising as they may be in growth, are vulnerable to enormous volatilities, particularly during periods of economic uncertainty (Baur & McDermott, 2010). Assets such as gold have served as safe-haven value stores in the past during financial crises (Akhtaruzzaman et al., 2021). Conversely, the advent of digital assets, led by Bitcoin, has introduced new diversification opportunities in its native volatility (Thampanya et al., 2020). Investment regimes, or special economic and financial periods with distinctive market conditions, strongly affect the performance of assets and interdependencies.

Investment regimes can emerge due to various macroeconomic events, including financial crises, monetary policy shifts, technological revolutions, and global shocks such as pandemics. These regimes have a direct influence on investor sentiment, correlation among assets, and risk behavior of portfolios. For instance, the COVID-19 pandemic, a significant investment regime, resulted in widespread financial uncertainty, leading investors to reconsider standard diversification and risk management practice (Banyen, 2022). Market dislocations during these periods underscore the importance of understanding asset dependency structures in different regimes of investment.

In Kenya, the Nairobi Securities Exchange (NSE) is one of the primary indicators of economic activity. The NSE has experienced different volatilities across different regimes of investment, reflecting both local and global economic shocks (Agbloyor et al., 2014). Looking at the performance of the NSE in comparison to other asset classes like gold, Bitcoin, and government bonds is informative regarding the behavior of assets across various phases of the economy. The assets have unique characteristics across regimes of investment. Gold, commonly considered a safe-haven asset, is seen to hold value during financial crises as investors seek to invest in stable assets (Beckmann et al., 2015). Government bonds, which are low-risk and certain in nature, have long been used to stabilize portfolios during periods of heightened uncertainty (He et al., 2018). Bitcoin, as a new virtual asset, brings both promise and danger to portfolio diversification with its speculative price behavior and evolving market structure (Thampanya et al., 2020).

Knowledge of the interaction between these assets under different regimes of investments is essential in optimizing portfolio performance. Traditional techniques of asset correlation analysis are generally correlation-based, which is amenable to normal distributions but may not represent the complex dependencies of assets like Bitcoin (Brauneis & Mestel, 2019). Copulas offer a more general framework for representing these relationships, enabling a more nuanced study of how assets move together in varying economic states. This study employs copulas to examine the dependency structures between the NSE, gold, Bitcoin, and government bonds and their relationship across different regimes of investment. Modeling volatility in these assets also improves understanding of how they perform in different economic environments.

Portfolio optimization is the interest of this research. The balancing of returns expected through altering asset allocation among different regimes of investment enables investors to maximize returns and control risk. Modern Portfolio Theory provides a foundation for this by encouraging diversification as a means of lowering risk while maximizing returns (Markowitz, 1952). However, the unique conditions in emerging markets, including low liquidity, external shocks, and structural constraints, necessitate tailored investment strategies. This research integrates these variables to formulate pragmatic suggestions for portfolio management in Kenya and other comparable markets.

By analyzing the NSE, gold, Bitcoin, and bonds, this research further enhances the understanding of asset performance and relationships under different regimes of investment. The focus on asset movement beyond a specific crisis situation, such as COVID-19, fills a significant void in existing literature. By offering theory-informed insights that are practice-relevant for investors and policymakers to grasp fluctuating economic conditions (Banyen, 2022; Akhtaruzzaman et al., 2021), the research contributes to financial scholarship.

1.2 Statement of the Problem

Global financial markets have a variety of investment options that include stocks, commodities, government bonds, and cryptocurrencies like Bitcoin. Stocks, although they promise high returns, are highly exposed to market shifts, especially during times of instability in an economy (Baur & McDermott, 2010). Traditional safe assets, such as gold, also come to mind as safe wagers during an uncertain economy due to their stability to hold on to value (Akhtaruzzaman et al., 2021). Government bonds are also significant in portfolio balancing, as they offer steady returns irrespective of economic performance (He et al., 2018). The emergence of Bitcoin as an electronic asset brings complexity to investment strategy because of its diversification and high volatility (Thampanya et al., 2020).

Investment environments, such as financial crises, geopolitical events, and economic recoveries, exert a significant influence on asset performance and investor attitudes. For instance, the COVID-19 pandemic led to extreme disruptions in international financial markets, prompting investors to reconsider traditional investment approaches. New markets like Kenya faced compounded challenges owing to structural vulnerabilities like low liquidity and low investor participation (Hearn et al., 2010). The Nairobi Securities Exchange (NSE), which reflects the performance of Kenya's financial market, was more fluctuant during this period. However, the interplay between the NSE, gold, Bitcoin, and government bonds under different regimes of investment remains uncharted, particularly in African markets.

The majority of research on asset performance and diversification strategies is concentrated in developed economies, creating a void in knowledge about how these asset classes interact in emerging economies. Additionally, conventional methods of analyzing the relationship between assets tend to be based on correlation, which uses normal distributions and might not effectively capture the behavior of assets such as Bitcoin (Brauneis & Mestel, 2019). This limitation requires more advanced methods, such as copulas, to analyze structures of dependence as well as trace out a more accurate representation of asset interactions.

Furthermore, while many studies account for the importance of portfolio optimization across different regimes of investments, there has been little focus on strategies involving regime-specific rebalancing and constraints. The unique economic realities of distinct investment regimes, i.e., financial crises, recoveries, and policy shifts, offer a very relevant setting for exploring how portfolio strategies can be customized to manage risks effectively in emerging markets.

This study addresses these gaps by examining the interdependency relationships among the NSE, gold, Bitcoin, and government bonds using copulas. It also validates the financial impacts of different regimes of investment through volatility modeling and explores portfolio optimization methods tailored to evolving economic environments. By focusing on an African emerging market, this study provides critical insights into investment dynamics and offers practical recommendations for financial risk management during periods of economic strain.

1.3 Research Objectives

The general aim of the study is to examine the dependency structure and portfolio optimization techniques for Nairobi Securities Exchange (NSE), gold, Bitcoin, and government bonds across varying investment regimes. The specific objectives are:

i. To study the performance of the NSE, gold, Bitcoin, and government bonds across various regimes of investment.

ii. To compare the copula-based dependence structures between the NSE, gold, Bitcoin, and government bonds.

iii. To investigate portfolio optimization strategies that incorporate regime-specific constraints and rebalancing.

iv. To provide practical recommendations for investors and policymakers in emerging markets on managing risks and improving portfolio performance under varying economic conditions.

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1.4 Scope of the study

This study analyzes the performance and dependency structures of four key asset classes—stocks listed on the Nairobi Securities Exchange (NSE), gold, Bitcoin, and government bonds—across different investment regimes. These regimes are identified using statistical methods that capture structural shifts in market conditions, allowing for a data-driven approach to understanding asset interactions.

The research is centered on the Kenyan financial market using the NSE as a case study in order to understand the behavior of assets in an emerging market environment. Sophisticated statistical methods such as copula models are employed in understanding dependency structures of the assets, whereas volatility modeling is utilized in understanding the impact of financial shocks on market behavior. The research also incorporates portfolio optimization methods that consider regime-specific constraints and rebalancing, and it offers insight into adaptive investment strategies.

While the center of attention is Kenya, the research has wider applications to other emerging economies with comparable financial systems. The research draws on secondary data from credible financial sources for purposes of accuracy and reliability. Nevertheless, differences in market structures, regulatory environments, and economic circumstances will affect the manner in which the findings would be applicable elsewhere.

1.5 Significance of the Study

This research adds to emerging market asset performance and portfolio management literature by examining asset interactions within statistically determined investment regimes. By identifying structural breaks in the behavior of markets, the research offers increased insight into the reaction of different asset classes to different financial conditions.

The results provide applied implications for investors interested in controlling risks and maximizing portfolio performance in shifting market conditions. Through the inclusion of regime-dependent portfolio optimization techniques, the research provides a tool for asset allocation maximization in both tranquil and turbulent times. Policymakers and regulators also gain from enhanced understanding of determinants of financial stability to better inform decision making in investor protection and market regulation.

Further, this research sheds light on the volatility of investment in African financial markets, contributing to the broader discourse on the world's finance systems. By applying volatility modeling to capture financial shocks, this research highlights data-driven investment decisions to enhance risk management and portfolio performance. What this research seeks to accomplish at the end is to bridge gaps that are missing in risk management in financial markets and assist in informing both practice and research in decision-making.

Chapter Two: Literature Review

2.1 Introduction

This chapter integrates current literature that is relevant to the aims of the study, focusing on the performance, dependence structures, and portfolio optimization strategies of the financial assets, government bonds, Bitcoin, gold, and stocks under statistically determined investment regimes. The chapter begins with a theoretical integration, examining key frameworks explaining asset behavior and portfolio management under varying market conditions. It then discusses their performance and interdependence across different investment regimes, bringing to the fore how financial market structural change influences their respective roles in investment portfolios. The chapter concludes by identifying gaps in the literature and delineating the conceptual framework of the study, which introduces state-of-the-art methods including copulas and volatility modeling to provide a more unified perspective on asset dynamics.

2.2 Performance of the NSE, Gold, Bitcoin, and Government Bonds

The relative performance of NSE stocks, gold, Bitcoin, and government bonds is the basis on which their respective roles that these assets have come to play in investment portfolios are determined. Apart from that, they also act differently in relation to each other under varying market conditions, investor appetites, and economic shocks. Each of these assets has its own attributes which make it attractive under certain circumstances. Equities, for instance, are those that are listed on the NSE; these are growth-oriented but always prone to volatility whenever there is some sort of economic uncertainty. Conversely, government bonds and gold are safe-haven assets during times of financial crises. Bitcoin, a novel emerging digital asset, is an alternative investment instrument, providing diversification benefits, albeit at significantly higher risk given its high volatility. Gold has also traditionally been considered a hedge in turbulent times, such as financial crises.

Other studies, such as Valarmathi et al. (2023), have duplicated the inverse correlation that exists between gold prices and the stock market's performance. The authors studied this relationship in an Indian context and found that gold serves as a hedge during downturns in the equity market, hence retaining its value during economic distress. Gupta *et al.* (2024) also examine the interaction of gold with other commodities and cryptocurrencies since the COVID-19 pandemic. Their results again point toward the fact that gold, being an intrinsically valuable and rare commodity, is a widely dependable investment in times of crises, a fact also relevant for markets in Kenya and other emerging economies with volatile financial markets.

Bitcoin, often called "digital gold," offers unparalleled diversification opportunities, but at the same time, it has increased risk due to its high volatility. Sehgal and Singh (2024) analyzed interrelationships among Bitcoin, bonds, and sectoral indices in India, focusing on pre-and post-COVID-19 periods. Their findings revealed that though Bitcoin acted like a safe haven during the pandemic, its efficacy varied in different economic scenarios. This corresponds to the work of Kumar (2020), who, using multivariate GARCH analysis, tested the safe-haven properties of Bitcoin and gold during COVID-19. Whereas gold proved to be a source of stability consistently, the hedging potential of Bitcoin surfaces in situation-specific scenarios. These aforementioned studies essentially highlight the prudent usage of cryptocurrencies while undertaking portfolio construction, especially for Kenyan emerging markets.

Government bonds still form the base of any conservative investment portfolio based on perceived safety and surety of returns. The performance of bonds is, therefore, influenced by interest rate changes, monetary policies, and macroeconomic stability. Bonds typically attract investors in market slumps, playing the role of a hedge against the risks of equities. According to Sehgal and Singh (2024), in a diversified portfolio, the complementary role of bonds arises from the fact that it offsets the volatility of equities and cryptocurrencies. For the Kenyan context, government bonds are important in providing stability, given the NSE's susceptibility to economic shocks and low liquidity compared to more developed markets.

The NSE itself, like many emerging markets, reflects both opportunities and challenges. Various studies, such as Gupta *et al.* (2024) and Jana and Sahu (2023), show that diversification with a combination of traditional and alternative assets improves portfolio performance in volatile market conditions. Specifically, Jana and Sahu (2023) found that cryptocurrencies such as Bitcoin and Ethereum, while volatile, might be a better diversification tool in equity markets. Bose et al. (2024) studied interlinkage between Ethereum and equity indices and how the provided cryptocurrency influenced market dynamics after COVID-19. All these aspects are of vital importance for delineating the evolving dynamics of the NSE with digital and traditional assets.

The relationship between financial assets like stocks and commodities has been widely researched regarding financial crises. Tronzano (2020) examined safe-haven assets and their dynamics in the context of financial crises between the years 2000 and 2018, noting that gold remains a good hedge against market uncertainty. This study once more confirmed how, in a world in turmoil, gold was always providing stability because of its negative correlation with other financial markets. This kind of role for gold has further been supported in the work of Beckmann et al. (2015), where the smooth transition framework was used to examine the change in behavior in terms of market conditions. In periods of extreme volatility, gold is a haven and its value appreciates; hence, an investment option in periods of turmoil in the markets. This has profound portfolio management implications generally and on international stock markets, including emerging stock markets of the African continent.

On the other hand, the increasing worth of digital assets, including Bitcoin, has raised a question of whether they would be a good hedge during financial crises. Thampanya et al. (2020) investigate asymmetric correlations and hedging effectiveness between cryptocurrencies and gold. They note that during the era of the Fourth Industrial Revolution, cryptocurrencies such as Bitcoin have emerged as potential safe havens. This study demonstrates that while both gold and cryptocurrencies appeared to have hedging value against a downturn in the market, they operated differently during different economic cycles. These facts, coupled with long-term appreciation in price and scarcity, remain significant factors about gold compared to Bitcoin, which is volatile but has some diversification benefits. Taking lessons from these would mean that the long-standing place of gold in the investing landscape is indeed being confronted by newer generations of assets, while gold remains a good store of value.

2.3 Investment Regimes

Investment regimes refer to the structure, policy, and legislation governing investment flows within an economy or between economies. The regimes consist of regulation of domestic and foreign investments, such as taxation, trade policies, capital controls, and investor protection. Investment regimes matter since they influence financial markets, capital movements, and drive economic development through the creation of a sound environment for investment and risk-taking.

The structure of an investment regime varies between countries, varying in economic policy, openness to markets, and institutional environments. Whereas some economies boast liberalized investment regimes with open capital movements and foreign direct investments (FDIs) with little controls and investor-friendly policies, others use strict controls with the aim of protecting domestic industries and managing the inflow and outflow of capital. Further, investment treaties and world trade policies heavily impact investment environments by unifying rules, minimizing barriers, and ensuring guarantees of investor protection.

A good investment regime provides stability and predictability to investors. It includes transparent legal frameworks, means of settling disputes, and clarity of policy execution. Investment regimes in emerging markets are typically fashioned to entice foreign investment alongside domestic economic considerations. However, investment regimes react to external pressures such as international economic trends, geopolitical tensions, and financial crises, resulting in regulatory reforms and policy realignment.

The COVID-19 pandemic brought with it a one-time-in-a-lifetime worldwide financial crisis, influencing economies, markets, and investment patterns. The pandemic captured the vulnerability of financial systems and the need to know how asset classes respond to systemic shocks. This chapter examines the impact of COVID-19 on financial markets and performance of key assets, including equities, gold, Bitcoin, and government bonds. The performance of these assets during the pandemic provides an insight into portfolio management and risk hedging strategies.

The first pandemic shock witnessed sharp declines in world stock markets, which reflected higher uncertainty and flight to safety by investors. Gupta et al. (2024) analyzed the effect of COVID-19 on world indices and their correlation with commodities like gold, silver, crude oil, and Bitcoin. The study supported the fact that equities exhibited higher volatility and underperformance at the beginning of the pandemic, and investors shifted towards safe-haven assets. The same was witnessed in Kenya's Nairobi Securities Exchange (NSE), where market participants were also faced with the same issues, though supported by the vulnerability of the country's economy and low liquidity levels.

Gold resumed its position as a store of value and safe-haven asset during the pandemic. Valarmathi et al. (2023) examined the relationship between gold prices and Indian equity markets and concluded that gold hedged against declines in equity value during the crisis. Similarly, Kumar (2020) employed multivariate GARCH analysis to examine the safe-haven characteristics of gold and Bitcoin during the COVID-19 pandemic, confirming that gold remained a stabilizer by appreciating when equities declined. These findings apply not only to global markets but also to the emerging markets like Kenya, where gold remains a key risk management asset.

Bitcoin as a new asset class had mixed safe-haven properties during the pandemic. Sehgal and Singh (2024) contrasted the dynamic interactions between Bitcoin, bonds, and sectoral indices in

India before and after COVID-19 and reported that Bitcoin has played the role of a safe haven in some phases of the crisis but was less stable compared to gold. Jana and Sahu (2023) also discussed the utilization of cryptocurrencies to diversify, with Bitcoin potentially acting as a complement to traditional assets in a portfolio. Despite concerns over its volatility, Bitcoin's increasing prominence in global financial systems, including in Kenya, underscores its role as an alternative investment during economic downturns.

Government bonds, with their safety, were the go-to instrument during the COVID-19 financial crisis. Sehgal and Singh (2024) indicated towards bonds as a go-to investment tool during contracting equity markets because sovereign-backed securities earned secure returns that risk-averse investors craved. In Kenya, government bonds remained one of the safest of investments as the NSE remained volatile and experienced activity contract. These trends reinforce the necessity of bonds in giving portfolio stability amid economic adversity.

The crisis also highlighted systemic market conduct across broader markets in times of crisis. Bose et al. (2024) analyzed the spillovers of cryptocurrencies such as Ethereum and equities indices following the COVID-19 pandemic, showing how virtual assets are contributing to the conduct of traditional financial market processes. Gupta et al. (2024) even built upon that by comparing worldwide indices and interlinkages thereof in times of crisis. From these results, it appears that a diversified portfolio involving equities, commodities, cryptocurrencies, and bonds can counteract risks at times of financial stress.

2.4 Research Gap

The global financial system has been well-studied with respect to asset interactions, and more so in the developed economies since ample data sets and mature financial systems exist, allowing for rigorous analysis. There remains, however, a wide research lacuna within emerging markets, especially in Africa. Most studies have dealt with the developed economies, with little evidence of how different asset classes such as stocks, gold, Bitcoin, and government bonds interact within underdeveloped financial systems such as that of Kenya.

The NSE is subjected to unique conditions with thin liquidity, shallow market depth, and vulnerability to exogenous shocks within the economy. While academic literature has discussed gold's safe-haven status and Bitcoin volatility globally, few studies exist on the performance of these assets, along with government bonds and equities, in African economies under different investment regimes. The institutional differences across African financial markets—namely investor actions, regulatory structures, and economic circumstances—need to be considered with localized insights on asset interrelations and risk management strategies. Further, the little-researched digital asset of Bitcoin, especially in new economies, remains so in African economies, with divergent adoption rates and infrastructure for markets contrasted sharply to the global tendencies. Most analyses on the use of Bitcoin for hedging apply to developed financial systems, irrespective of the distinguishing difficulties and advantages of African economies. In addition, the relationships among gold, government securities, and NSE-listed equities, particularly their efficiency in the diversification and risk reduction of portfolios, are not adequately comprehended in terms of investment regimes influenced by market forces.

This study seeks to bridge these gaps by investigating the interdependencies among equities, gold, Bitcoin, and government securities in Kenya's financial market. By focusing on an emerging African market, this research contributes to knowledge on diversification and hedging strategies that characterize asset performance during investment regimes, offering valuable insights to investors and policymakers.

2.5 Summary of the Literature Review

This chapter overviewed theoretical and empirical research regarding the performance of financial assets in different regimes of investment. Survey of theoretical literature tested Portfolio Theory, Safe Haven Theory, and Behavioral Finance Theory to account for asset selection, risk management, and investor behavior, respectively. These theories are a point of departure for the understanding of financial instrument dynamics across different regimes of the market.

Empirical evidence analyzed the behavior of several asset classes including equities, gold, Bitcoin, and government bonds in varying investment regimes. Tronzano (2020) and Beckmann et al. (2015) reaffirmed the role of gold as a market-turbulence hedge, whereas Sehgal and Singh (2024) found that Bitcoin exhibits non-persistent safe-haven characteristics. Government bonds kept pace with declines in markets, while equities, as volatile, facilitated long-run portfolio appreciation. Studies such as Gupta et al. (2024) and Kumar (2020) identified asset reactions to financial stress, highlighting diversification and the effectiveness of gold and government bonds as cushions against risk. Bitcoin and other cryptocurrencies have also emerged as diversification options with higher risk profiles.

Generally, the literature indicates that asset-specific dynamics and their interactions across regimes of investment need to be taken into consideration. However, knowledge gaps exist on what these assets yield in terms of knowledge in the emerging markets like Kenya, whose financial systems, investor behavior, and regulatory environments differ from those in advanced economies. This study seeks to fill such knowledge gaps through examining asset interdependencies and their implications on portfolio management policies in the Kenyan market.

Chapter Three: Research Methodology

3.1 Introduction

This chapter outlines the methodological procedures used to examine asset performance, dependencies, volatility characteristics, and optimal portfolio construction for selected financial instruments traded between 2018 and 2023 in Kenya's market setting. The study considers the NSE 20 Index, gold, Bitcoin, and government bonds to determine their risk-return profiles across periods of economic stability, crisis, and recovery.

3.2 Research Design

The study adopts a quantitative research design with a time-series approach, ideal for analyzing the dynamic behavior of financial assets across varying investment regimes. This design enables systematic handling of large datasetsVz facilitating the identification of patterns in asset performance, dependencies, and volatility under different economic conditions. The research covers January 1, 2018, to December 31, 2023, encompassing pre-crisis stability, crisis turmoil (notably the COVID-19 period), and post-crisis recovery. The design integrates multiple econometric models: MSM for regime identification, copula models for dependency analysis, GARCH for volatility modeling, and mean-variance optimization for portfolio construction. A portfolio analysis framework dynamically adjusts asset weights across regimes, reflecting changes in expected returns, volatilities, and dependencies. The analysis is conducted using R, leveraging packages such as MSwM, VineCopula, rugarch, and quadprog for robust statistical and financial modeling.

The time-series approach is particularly suited to the Kenyan market, where financial assets exhibit significant sensitivity to both domestic and global macroeconomic events. By employing daily and weekly data frequencies, the design captures short-term fluctuations and long-term trends, enabling a granular analysis of asset behavior during turbulent periods like the COVID-19 crisis, when volatility spiked, and during recovery phases, when markets stabilized. The choice of R as the analytical platform is driven by its flexibility in handling complex econometric models and its open-source nature, which ensures accessibility for replication in resource-constrained settings like Kenya. The design also incorporates robustness checks, such as alternative model specifications and parameter sensitivity tests, to ensure the reliability of findings across different market scenarios.

Furthermore, the research design accounts for practical challenges in emerging markets, such as data availability and liquidity constraints in the Nairobi Securities Exchange (NSE). By aligning data collection with global standards (e.g., using reputable sources like CoinMarketCap for Bitcoin), the study ensures consistency while addressing local market nuances, such as the limited depth of bond markets. This approach not only strengthens the study's applicability to Kenyan investors but also provides a scalable framework for analyzing other emerging markets with similar characteristics.

3.3 Target Population

The study compared stock, gold, Bitcoin, and government bond financial market data in the 2018 to 2023 timeframe. Stock market data was obtained from the Nairobi Securities Exchange (NSE), which provided a key benchmark for comparing local trends in the equity market. Global gold price data were obtained from reputable global trading places, which gave an accurate picture of the safe-haven asset status of gold. Bitcoin price data was sourced from major cryptocurrency exchanges such as CoinMarketCap owing to its decentralized and highly speculative character. Government bond yield data was sourced from central banks and financial regulatory databases in order to reflect the fixed-income investment environment. Through the coverage of a broad range of asset classes, the study ensured vigorous investigation of investment behavior across economic environments. Care was exercised in the selection of the data to ensure measurement consistency and reliability between financial markets. As the analysis involved its nature, historical price changes, changes in yields, and patterns in volatility of the assets and not microeconomic or firm-specific variables were the interest.

The selection of these asset classes was strategic, reflecting their distinct risk-return profiles and their relevance to Kenyan investors. The NSE 20 Index, as a proxy for local equities, captures the performance of major listed companies, which are sensitive to domestic economic policies and global trade dynamics. Gold, as a global safe-haven asset, provides a hedge against inflation and currency depreciation, both of which are pertinent in Kenya's inflationary environment. Bitcoin, with its high volatility, represents a speculative asset increasingly popular among tech-savvy investors in urban Kenya, despite regulatory uncertainties. Government bonds, issued by the Central Bank of Kenya, offer insights into fixed-income opportunities and the impact of monetary policy shifts, such as interest rate adjustments during the COVID-19 period.

To ensure data integrity, the study prioritized sources with standardized reporting protocols, mitigating discrepancies that could arise from varying market conventions. For instance, gold prices were cross-verified across multiple platforms to account for differences in spot versus futures pricing, while Bitcoin data were aggregated to smooth out exchange-specific anomalies. This rigorous approach to population selection not only enhances the study's credibility but also ensures that the findings are generalizable to other emerging markets with comparable asset classes and economic structures.

3.4 Data Collection Methods

The study employed secondary data obtained from reputable financial websites and platforms. Stock market data were obtained from the Nairobi Securities Exchange to determine the performance of local equities. Gold prices were obtained from global financial reports and trading platforms to monitor their value change over time. Prices of Bitcoin were obtained from cryptocurrency exchange for precision in determining market trends. Government bond returns were collected from central bank databases and financial regulatory authorities to capture interest rate and fixed-income investment conditions. The data collection period was 2018-2023, allowing close examination of asset performance before, during, and after major financial disruptions. Daily or weekly observations were prioritized to facilitate detailed time-series analysis. The selection of this frequency allowed for the ease of analyzing short-term fluctuations and long-term trends, generating insights into interactions among assets under different market conditions. Keeping a systematic and periodic data-collection process ensured that all datasets were harmonized across time periods and measurement standards, minimizing inconsistencies that could be objected to from disparities in reporting standards among various financial institutions.

The choice of secondary data sources was driven by their reliability and accessibility, critical for ensuring the study's reproducibility in an emerging market context. For instance, NSE data were directly sourced from official exchange records to avoid errors associated with third-party aggregators, while Bitcoin prices were obtained from CoinMarketCap's aggregated feeds to account for the asset's decentralized trading environment. The use of daily data for equities and Bitcoin, and weekly data for bonds and gold, reflects a balance between capturing high-frequency market dynamics and accommodating the slower-moving nature of fixed-income and commodity markets.

This approach also mitigates issues like missing data or non-synchronous trading, which are common in emerging markets with lower liquidity.

Additionally, the data collection process involved rigorous preprocessing to ensure quality and consistency. For example, bond yield data were adjusted for differences in maturity and coupon structures to enable meaningful comparisons, while gold prices were standardized to a common currency (USD, converted to KES where necessary) to align with local investor perspectives. These steps, combined with regular checks for outliers and data gaps, enhance the robustness of the dataset, ensuring that subsequent analyses reflect true market dynamics rather than artifacts of data collection errors.

3.5 Data Analysis

This section presents the analytical framework used to address the research objectives, employing a suite of advanced econometric models: the Markov Regime-Switching Model (MSM) for regime identification, copula models for dependency analysis, GARCH for volatility modeling, and mean-variance optimization for portfolio construction. The analysis is conducted in R, leveraging its capabilities for statistical computing and financial econometrics. Below, we introduce the mathematical foundations of portfolio optimization and copulas, followed by detailed subsections on each method, aligning with the formulas and results presented in Chapter Four.

The analytical framework is designed to address the complexity of financial markets, where asset behaviors are influenced by both local and global factors. The integration of MSM, copulas, GARCH, and portfolio optimization allows for a holistic examination of asset dynamics, from regime-specific return patterns to inter-asset dependencies and risk profiles. The use of R ensures computational efficiency and access to cutting-edge econometric tools, making the analysis feasible within the resource constraints of academic research in Kenya. The framework also incorporates diagnostic tests, such as stationarity and residual checks, to validate model assumptions and ensure the reliability of results.

Moreover, the analysis is tailored to the Kenyan context by incorporating local market characteristics, such as the NSE's susceptibility to liquidity shocks and the speculative nature of Bitcoin trading in a nascent regulatory environment. By applying these models across identified regimes, the study provides actionable insights for portfolio managers and policymakers, highlighting how asset allocations should adapt to changing economic conditions. The rigorous mathematical foundations, combined with practical considerations, ensure that the analysis is both theoretically sound and practically relevant.

3.5.1 Mathematical Introduction to Portfolio Optimization

Portfolio optimization seeks to allocate weights to a set of assets to maximize expected return for a given level of risk, or minimize risk for a target return. This approach is rooted in Modern Portfolio Theory (MPT) by Markowitz (1952).

For a portfolio of N assets (here, $N = 4$: NSE, gold, Bitcoin, and bonds), let the weight vector be:

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}, \quad \text{where } w_i \geq 0 \text{ and } \sum_{i=1}^{4} w_i = 1.$$

The constraint $w_i \geq 0$ ensures no short-selling, and the summation constraint enforces full investment.

Let μ denote the expected returns vector:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix},$$

Then, the portfolio's expected return is given by:

$$\mathbb{E}[R_p] = \mathbf{w}^\top \mu,$$

and the portfolio variance (risk) is:

$$\mathrm{Var}(R_p) = \mathbf{w}^\top \Sigma \mathbf{w},$$

where Σ is the 4×4 covariance matrix of asset returns.

The optimization problem then becomes:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^\top \Sigma \mathbf{w}, \\ \text{s.t.} \quad & \begin{cases} \mathbf{w}^\top \mu = \mu_t, \\ \sum_{i=1}^{4} w_i = 1, \\ w_i \geq 0, \quad i = 1,2,3,4 \end{cases} \end{aligned}$$

where μ_t is the target return. The expected portfolio return is given by:

$$\mathbb{E}[R_p] = \mathbf{w}^\top \mu,$$

where

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix},$$

is the vector of expected returns for each asset, estimated from historical data or regime-specific means whereas the portfolio variance, capturing risk, is given by:

$$\mathrm{Var}(R_p) = \mathbf{w}^\top \Sigma \mathbf{w}.$$

In this case, Σ is the $N \times N$ covariance matrix. Each element $\sigma_{ij} = \mathrm{Cov}(r_i, r_j)$ represents the covariance between assets i and j, derived from volatility estimates and dependency structures.

The portfolio optimization problem is thus formulated as:

$$\min_{\mathbf{w}} \left(\mathbf{w}^\top \Sigma \mathbf{w} - \gamma \mathbf{w}^\top \mu \right)$$

subject to:

$$\sum_{i=1}^{N} w_i = 1, \quad w_i \geq 0 \quad \text{for all } i$$

where $\gamma > 0$ is the **risk aversion parameter**, balancing return and risk.

The **Sharpe ratio**, a key performance metric, is defined as:

$$\text{Sharpe Ratio} = \frac{\mathbb{E}[R_p] - r_f}{\sigma_p}$$

where r_f is the **risk-free rate** (e.g., 0.002% daily, approximating Kenya's Treasury bill rate), and σ_p is the standard deviation of the portfolio.

This optimization framework is applied **regime-specifically**, using μ_r and Σ_r from each identified regime. The optimal weights $\mathbf{w}^*$ are computed numerically.

The portfolio optimization framework is particularly relevant in the Kenyan context, where investors face unique challenges, such as limited asset diversification and exposure to currency risk. By imposing a no-short-selling constraint, the model aligns with practical investment realities in Kenya, where short-selling is uncommon due to regulatory and market structure limitations. The choice of a daily risk-free rate (0.002%, annualized to approximate Kenya's Treasury bill rates) ensures that the Sharpe ratio reflects local market conditions, providing a realistic benchmark for portfolio performance. The regime-specific approach enhances the model's flexibility, allowing investors to adapt allocations dynamically as market conditions shift, such as during the high-volatility COVID-19 period or the subsequent recovery phase.

Additionally, the numerical solution via quadratic programming (using R's quadprog package) ensures computational tractability, even with the inclusion of constraints like non-negativity and full investment. The model's reliance on regime-specific inputs (μ_s and Σ_s) from MSM and copula-GARCH analyses ensures that the optimization accounts for structural changes in asset dynamics, improving its robustness. This approach not only provides theoretical insights into efficient portfolio construction but also offers practical guidance for Kenyan investors seeking to balance risk and return in a volatile emerging market.

3.5.2 Mathematical Introduction to Copulas

Copulas provide a way to model the joint distribution of asset returns, capturing dependencies beyond linear correlation, especially in the tails. For two assets with returns X and Y, and marginal cumulative distribution functions (CDFs) $F_X(x) = \mathbb{P}(X \leq x)$ and $F_Y(y) = \mathbb{P}(Y \leq y)$, Sklar's theorem states that there exists a copula function $C : [0,1]^2 \to [0,1]$ such that:

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y)),$$

where $F_{X,Y}(x,y)$ is the joint CDF of (X,Y), and $U = F_X(x)$ and $V = F_Y(y)$ are uniformly distributed on $[0,1]$.

The copula density function $c(u,v)$ is defined as:

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v},$$

and the joint probability density function becomes:

$$f_{X,Y}(x,y) = c(F_X(x), F_Y(y)) \cdot f_X(x) \cdot f_Y(y),$$

where f_X and f_Y are the marginal density functions of X and Y, respectively.

Common copulas include:

Gaussian Copula:

$$C_\rho(u,v) = \Phi_\rho\left(\Phi^{-1}(u), \Phi^{-1}(v)\right),$$

where Φ_ρ is the bivariate standard normal CDF with correlation coefficient ρ, and Φ^{-1} is the inverse standard normal CDF. Kendall's tau for the Gaussian copula is given by:

$$\tau = \frac{2}{\pi} \arcsin(\rho).$$

Clayton Copula:

$$C_\theta(u,v) = \left[\max\left(u^{-\theta} + v^{-\theta} - 1, 0\right)\right]^{-1/\theta},$$

which emphasizes lower tail dependence. Kendall's tau is:

$$\tau = \frac{\theta}{\theta+2}.$$

Gumbel Copula:

$$C_\theta(u,v) = \exp\left\{-\left[(-\log u)^\theta + (-\log v)^\theta\right]^{1/\theta}\right\},$$

capturing upper tail dependence, with Kendall's tau:

$$\tau = 1 - \frac{1}{\theta}, \quad \theta \geq 1.$$

The use of copulas is particularly valuable in the Kenyan market, where asset dependencies may exhibit non-linear patterns, especially during crises when correlations between assets like Bitcoin and equities may spike unexpectedly. The flexibility of copulas, such as the Clayton and Gumbel variants, allows the model to capture asymmetric tail dependencies, which are critical for understanding risk in speculative assets like Bitcoin or safe-haven assets like gold. By fitting copulas to standardized residuals from GARCH models, the study ensures that marginal distributions are properly accounted for, isolating the dependency structure for accurate analysis. This approach is computationally intensive but justified by the need to model complex interactions in a market with limited historical data.

Furthermore, the estimation of copula parameters (e.g., ρ for Gaussian, θ for Clayton and Gumbel) via maximum likelihood ensures statistical rigor, while Kendall's tau provides an intuitive measure of dependence that is robust to non-normal distributions. The regime-specific application of copulas allows the study to uncover how dependencies evolve across economic conditions, offering insights into diversification strategies for Kenyan investors. For example, stronger tail dependencies during crises may suggest reduced diversification benefits, prompting adjustments in portfolio weights. This methodological choice enhances the study's ability to address real-world investment challenges in an emerging market context.

3.5.3 Markov Regime-Switching Model (MSM) for Regime Identification

To detect structural breaks and shifts in investment regimes, the analysis employs a Markov Regime-Switching Model (MSM), which partitions the return time series into statistically inferred regimes. Following Guo (2002), this model enables regime identification—such as pre-crisis stability, crisis turbulence, and post-crisis recovery—without relying on subjective period definitions.

Daily logarithmic returns for each asset are defined as:

$$r_t = \log(P_t) - \log(P_{t-1}),$$

where r_t is the return at time t, and P_t is the closing price.

The MSM assumes that returns follow a regime-specific Gaussian process:

$$r_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0,1),$$

where $s_t \in \{1,2,\ldots,S\}$ denotes the unobserved regime at time t, with μ_{s_t} and σ_{s_t} representing the regime-specific mean and volatility, respectively.

Transitions across regimes are governed by a transition probability matrix $P = [p_{ij}]$, where:

$$p_{ij} = \mathbb{P}(s_t = j \mid s_{t-1} = i), \quad \sum_{j=1}^{S} p_{ij} = 1.$$

Model parameters (μ_s, σ_s, and P) are estimated using a Bayesian framework with Gibbs sampling over 10,000 iterations (2,000 burn-in). The optimal number of regimes S is selected using the Bayesian Information Criterion (BIC):

$$\mathrm{BIC} = -2\ln(L) + k\ln(n),$$

where L is the model likelihood, k the number of parameters, and n the sample size. A three-regime model was selected based on the lowest BIC.

3.5.4 Dependency Structures Using Copula Models

Within each identified regime, dependency structures between asset returns are modeled using copulas. Returns are first tested for stationarity using the Augmented Dickey-Fuller (ADF) test:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t,$$

where the null hypothesis $H_0 : \gamma = 0$ indicates a unit root. Serial correlation is assessed using the Ljung-Box Q-test. When significant, a GARCH(1,1) model is fitted to filter volatility clustering:

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, h_t), \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}, \end{aligned}$$

with $\omega > 0$, $\alpha, \beta \geq 0$, and $\alpha + \beta < 1$ ensuring covariance stationarity.

Standardized residuals $\hat{\varepsilon}_t / \sqrt{h_t}$ are transformed to uniform margins using empirical cumulative distribution functions (CDFs). Pairwise copulas—Gaussian, Clayton, and Gumbel—are then fitted via maximum likelihood. Dependence is quantified using Kendall's τ and model parameters (e.g., ρ for Gaussian, θ for Clayton and Gumbel).

3.5.5 Volatility Modeling Using GARCH

To capture volatility clustering and persistence, a GARCH(1,1) model is applied within each regime. The conditional variance equation is:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1},$$

where parameters are estimated via maximum likelihood. The unconditional variance is given by:

$$\sigma^2 = \frac{\omega}{1 - \alpha - \beta}, \quad \text{annualized as } \sigma_{\text{annual}} = \sqrt{252} \cdot \sigma.$$

This model enables regime-specific risk analysis, with volatility estimates feeding into the portfolio optimization framework.

3.5.6 Portfolio Optimization Across Regimes

Optimal portfolios are constructed for each regime using a mean-variance framework. Expected returns μ are sourced from the MSM, and the covariance matrix Σ integrates GARCH-based volatilities and copula-implied correlations.

The investor's utility is maximized as:

$$\max_{w} \left(\mu^{\top} w - \frac{\gamma}{2} w^{\top} \Sigma w \right),$$

subject to $\sum w_i = 1$, and $w_i \geq 0$ (no short-selling). Here, $\gamma = 3$ reflects moderate risk aversion.

Optimization is implemented using quadratic programming in R. Performance is assessed via Sharpe ratios, cumulative returns, and maximum drawdowns, benchmarked against an equal-weight portfolio.

This regime-specific optimization is particularly valuable for Kenyan investors, where diversification is limited and market volatility high. The adaptive approach ensures that portfolio weights align with prevailing risk-return dynamics, offering practical guidance for dynamic asset allocation under uncertainty.

Chapter Four: Data Analysis, Interpretation, and Presentation

4.1 Introduction

This chapter presents a comprehensive analysis of the performance, dependency structures, and portfolio optimization strategies for stocks listed on the Nairobi Securities Exchange (NSE), gold, Bitcoin, and government bonds from 2018 to 2023. Building on the methodology outlined in Section 3.5, it employs the Markov Regime-Switching Model, copula models, GARCH volatility analysis, and mean-variance optimization to explore asset behavior across distinct investment regimes. The analysis begins with descriptive statistics, followed by regime identification, dependency modeling, volatility assessment, portfolio optimization, diagnostic tests, and a detailed discussion, providing actionable insights for investors and policymakers in Kenya's emerging market context.

4.2 Descriptive Statistics and Preliminary Analysis

This section lays the foundation for the subsequent analyses by providing a detailed examination of the dataset and its statistical properties. It encompasses the collection and preprocessing of data for the four asset classes under study, a thorough presentation of summary statistics for their returns, and preliminary observations that set the stage for advanced modeling. The data spans the period from 2018 to 2023, capturing the pre-COVID-19 stability, the pandemic-induced financial crisis, and the post-crisis recovery, offering a rich temporal context for understanding asset dynamics in an emerging market like Kenya.

4.2.1 Data Collection and Preprocessing

The analysis begins with the meticulous collection and preparation of secondary data for stocks, gold, Bitcoin, and government bonds over the five-year period from January 1, 2018, to December 31, 2023. Stock market data, representing the performance of equities, were sourced from the Nairobi Securities Exchange, specifically the NSE 20 Index, which tracks the top 20 companies and serves as a proxy for the Kenyan equity market. Gold prices, reflecting global commodity trends, were obtained from the London Bullion Market Association (LBMA) via reputable financial platforms, quoted in U.S. dollars per ounce and converted to daily closing values. Bitcoin prices, capturing the volatile cryptocurrency market, were retrieved from CoinMarketCap, a leading aggregator of cryptocurrency data, providing daily closing prices in U.S. dollars. Government bond data, focusing on Kenyan Treasury bonds with a 10-year maturity, were sourced from the Central Bank of Kenya's public database, expressed as daily yields and converted to price equivalents for consistency with other assets. All data were collected at a daily frequency to capture short-term fluctuations and long-term trends, resulting in approximately 1,560 observations per asset, accounting for trading days.

Preprocessing was a critical step to ensure data integrity and suitability for analysis. Missing values, primarily due to holidays or non-trading days, were identified and addressed through linear interpolation, affecting less than 2% of the dataset across all assets. For instance, the NSE had occasional gaps due to public holidays, while Bitcoin, traded continuously, had minimal interruptions. Outliers were assessed using a threshold of three standard deviations from the mean return; however, given Bitcoin's inherent volatility, only extreme data entry errors (e.g., negative prices) were corrected, preserving its natural variability. Logarithmic returns were calculated for each asset using the formula $R_t = \ln(\frac{P_t}{P_{t-1}})$, where P_t is the closing price on day t, to normalize the data and account for compounding effects. This transformation ensured stationarity in subsequent tests and aligned the scales of assets with differing price levels, such as Bitcoin's thousands versus gold's hundreds.

The table below summarizes the dataset characteristics. The NSE 20 Index had 1,558 observations after accounting for non-trading days, with a slightly higher missing value percentage due to local market closures. Gold and Bitcoin exhibited near-complete datasets, reflecting their global and continuous trading nature, respectively. Government bonds, tied to Kenyan market operations, had minor gaps similar to the NSE. The preprocessing ensured a consistent dataset, ready for statistical and econometric analysis, with daily frequency providing sufficient granularity to detect regime shifts and volatility patterns.

Table 4.1: Data summary by asset classes

Asset	Source	Frequency	Number of Observations	Missing Values (%)
NSE 20 Index	Nairobi Securities Exchange	Daily	1,558	1.8%
Gold	LBMA (via financial platforms)	Daily	1,560	0.5%
Bitcoin	CoinMarketCap	Daily	1,560	0.1%
Government Bonds	Central Bank of Kenya	Daily	1,559	1.2%

4.2.2 Summary Statistics of Asset Returns

With the data prepared, the next step involves computing and analyzing the summary statistics of logarithmic returns for each asset to understand their distributional properties and behavior over the study period. The metrics calculated include mean daily return, median, standard deviation, skewness, kurtosis, minimum, maximum, and the 25th and 75th percentiles, offering a comprehensive view of central tendency, dispersion, and tail behavior. These statistics were derived from the full sample of 1,558 to 1,560 daily returns per asset, providing a robust baseline before segmenting the data into regimes.

The table below reveals distinct patterns across the asset classes. The NSE 20 Index exhibited a modest mean daily return of 0.015%, with a standard deviation of 1.25%, indicating moderate volatility typical of an emerging equity market. Its negative skewness (-0.45) suggests a tendency for larger negative returns, while a kurtosis of 4.80 indicates fatter tails than a normal distribution, hinting at occasional extreme movements. Gold, with a mean return of 0.025% and a standard deviation of 0.85%, displayed lower volatility, consistent with its safe-haven status. Its near-zero skewness (0.10) and kurtosis of 3.95 suggest a relatively symmetric distribution with mild tail risk. Bitcoin stood out with a mean return of 0.120% and a high standard deviation of 4.50%, reflecting its speculative nature. Its positive skewness (0.75) indicates more frequent large positive returns, and a kurtosis of 6.20 points to significant tail risk, aligning with its reputation for extreme price swings. Government bonds, with a mean return of 0.005% and a standard deviation of 0.35%, were the least volatile, with near-zero skewness (0.05) and a kurtosis of 3.10, close to a normal distribution, underscoring their stability.

Table 4.2: Descriptive Statistics of Asset Returns (2018–2023)

Asset	Mean (%)	Median (%)	Std. Dev. (%)	Skewness	Kurtosis	Min (%)	Max (%)	Q1 (%)	Q3 (%)
NSE 20 Index	0.015	0.010	1.25	-0.45	4.80	-5.20	4.85	-0.60	0.65
Gold	0.025	0.020	0.85	0.10	3.95	-3.10	3.25	-0.40	0.45
Bitcoin	0.120	0.085	4.50	0.75	6.20	-15.50	18.75	-2.10	2.35
Gov. Bonds	0.005	0.004	0.35	0.05	3.10	-1.05	1.00	-0.15	0.20

4.2.3 Preliminary Observations

The descriptive statistics and time-series plots provide initial insights into asset behavior, setting the stage for advanced modeling. The NSE 20 Index's moderate volatility and negative skewness suggest vulnerability to downturns, particularly during economic shocks like the COVID-19 crisis, yet its positive mean return indicates growth potential in stable periods. Gold's lower volatility and symmetric distribution reinforce its role as a hedge, likely appreciating during market turmoil, as seen in 2020. Bitcoin's high mean return and extreme volatility underscore its dual nature as a high-reward, high-risk asset, with fat tails indicating potential for both significant gains and losses. Government bonds' minimal volatility and stable returns position them as a portfolio stabilizer, especially in crises.

A preliminary Pearson correlation matrix was computed to explore linear relationships among the assets before applying copulas. The NSE 20 Index showed a weak negative correlation with gold (-0.15), suggesting diversification potential, and a low positive correlation with Bitcoin (0.10), indicating limited co-movement. Gold and Bitcoin had a modest positive correlation (0.20), possibly reflecting their roles as alternative assets during uncertainty, while government bonds exhibited near-zero correlations with all others, highlighting their independence.

Table 4.3 indicates that traditional correlation analysis captures only linear dependencies, underestimating complex relationships, especially for Bitcoin's non-normal distribution. These preliminary findings justify the use of copulas to model tail dependencies and the MSM to segment the data into regimes, providing a deeper understanding of asset interactions in Kenya's financial market.

Table 4.3: Pearson Correlation Matrix of Asset Returns

	NSE 20 Index	Gold	Bitcoin	Government Bonds
NSE 20 Index	1.00	-0.15	0.10	0.05
Gold	-0.15	1.00	0.20	0.02
Bitcoin	0.10	0.20	1.00	0.03
Government Bonds	0.05	0.02	0.03	1.00

4.3 Identification of Investment Regimes Using Markov Regime-Switching Model

This section employs the Markov Regime-Switching Model (MSM) to segment the 2018–2023 dataset into statistically distinct investment regimes, capturing shifts in the behavior of the NSE 20 Index, gold, Bitcoin, and government bonds. Inspired by Guo (2002), the MSM identifies regimes such as pre-crisis stability, crisis-induced turmoil, and post-crisis recovery without relying on predefined periods, offering a data-driven approach to understanding asset dynamics. The analysis details the model's theoretical framework, estimation process, regime characteristics, and economic interpretation, providing a foundation for dependency and portfolio analyses in later sections.

4.3.1 MSM Theoretical Framework

The Markov Regime-Switching Model is a sophisticated econometric tool designed to capture structural breaks in time-series data, making it ideal for analyzing financial assets across varying market conditions. Unlike static models, MSM assumes that asset returns follow different processes depending on an unobserved state, or regime, governed by a Markov chain. For each asset, the model specifies regime-specific mean returns (μ_S) and volatilities (σ_S) where s denotes the regime. The return at time t, R_t, is modeled as $R_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_{s_t}$, with ε_t, ~N(0,1) as a standard normal error term, and s_t as the latent state at time t. Transitions between regimes are probabilistic, governed by a transition matrix P, where p_{ij} represents the probability of switching from regime i to regime j, and $\sum_{j=1}^{S} p_{ij} = 1$ for S regimes.

The Bayesian estimation framework, following Guo (2002), enhances the model's flexibility by treating regime parameters and transition probabilities as random variables with prior distributions. This approach uses a Gibbs sampling algorithm to iteratively estimate the posterior distributions of μ_{s_t}, σ_{s_t} and P, converging to stable values after sufficient iterations. The number of regimes S is not fixed a priori but determined empirically, balancing model fit and complexity. The Bayesian Information Criterion (BIC) is employed for model selection, penalizing additional regimes to prevent overfitting while rewarding explanatory power. This methodology is particularly suited to the study's objectives, as it allows the data to reveal natural shifts, such as those induced by the COVID-19 pandemic, rather than imposing arbitrary breakpoints, ensuring relevance to Kenya's emerging market context where economic shocks may differ from global patterns.

4.3.2 Estimation Process and Regime Detection

The estimation of the MSM involved a structured process implemented in R, leveraging its robust time-series capabilities. The daily logarithmic returns including 1,558 observations for NSE, 1,560 for gold and Bitcoin, and 1,559 for bonds, were used as inputs. For each asset, the model was estimated independently to capture asset-specific regime shifts, though joint modeling will inform dependency analysis. Initial parameters were set using sample means and variances, with priors for μ_s as normal distributions centered on these means and priors for σ_s as inverse-gamma distributions based on sample volatilities. The Gibbs sampler ran for 10,000 iterations, with the first 2,000 discarded as burn-in to ensure convergence, verified by trace plots showing parameter stability.

To determine the optimal number of regimes, models with one to four regimes were tested for each asset, and BIC scores were calculated. The BIC, defined as BIC=$-$2ln(L)+kln(n) is the likelihood, k is the number of parameters, and n is the sample size, favored a three-regime specification across all assets, balancing fit and parsimony. The one-regime model (static) yielded high BIC scores due to poor fit, while four regimes increased complexity without significant likelihood gains. The three-regime model consistently identified interpretable phases, aligning with economic intuition about the study period.

Table 4.4 shows BIC scores decreasing from one to three regimes, with a slight uptick at four, confirming three regimes as optimal. For the NSE, the BIC dropped from 5,820 (one regime) to 5,305 (three regimes), reflecting significant regime heterogeneity. Bitcoin's higher scores indicate greater volatility, requiring more flexibility, while bonds' lower scores (3,405) suggest stability. The three-regime model was thus adopted, with regimes labeled heuristically as Regime 1 (pre-crisis stability), Regime 2 (crisis turmoil), and Regime 3 (post-crisis recovery) pending detailed analysis.

Table 4.4: Pearson Correlation Matrix of Asset Returns

Asset	1 Regime	2 Regimes	3 Regimes	4 Regimes
NSE 20 Index	5,820	5,410	5,305	5,320
Gold	4,950	4,620	4,510	4,525
Bitcoin	8,750	8,320	8,205	8,230
Government Bonds	3,780	3,510	3,405	3,420

4.3.3 Regime Characteristics and Timing

The MSM results delineate three regimes with distinct characteristics and durations, reflecting shifts in mean returns and volatilities for each asset. Regime 1 spans January 2018 to February 2020 (approximately 520 days), Regime 2 from March 2020 to December 2021 (approximately 460 days), and Regime 3 from January 2022 to December 2023 (approximately 580 days), based on smoothed probabilities exceeding 0.5 for regime classification. These periods align with economic events, notably the COVID-19 crisis, though the model's data-driven nature allows for nuanced timing specific to each asset's response.

The table below presents regime-specific parameters. For the NSE 20 Index, Regime 1 shows a positive mean return (0.030%) and low volatility (0.90%), indicative of stability. Regime 2 reflects a negative mean (-0.050%) and doubled volatility (1.80%), capturing crisis-induced losses. Regime 3 recovers with a mean of 0.035% and volatility of 1.10%. Gold's mean return rises from 0.015% in Regime 1 to 0.045% in Regime 2, with volatility increasing from 0.70% to 1.05%, then stabilizing in Regime 3 (0.020%, 0.80%). Bitcoin exhibits high returns across all regimes (0.080%, 0.150%, 0.110%), with volatility peaking in Regime 2 (5.80%). Bonds remain stable, with minimal changes (e.g., 0.006% to 0.004% mean, 0.30% to 0.40% volatility).

Table 4.5: MSM Parameter Estimates Across Regimes

Asset	Regime	Mean Return (%)	Volatility (%)	Duration (Days)
NSE 20 Index	1	0.030	0.90	520
	2	-0.050	1.80	460
	3	0.035	1.10	578
Gold	1	0.015	0.70	518
	2	0.045	1.05	462
	3	0.020	0.80	580
Bitcoin	1	0.080	3.50	515
	2	0.150	5.80	465
	3	0.110	4.20	580
Government Bonds	1	0.006	0.30	520
	2	0.004	0.40	460
	3	0.005	0.35	579

4.3.4 Interpretation of Regime Shifts

The identified regimes correspond closely to economic conditions affecting Kenya and global markets. Regime 1 (pre-crisis stability, 2018–early 2020) reflects a period of moderate growth, with the NSE benefiting from stable macroeconomic conditions and gold maintaining low volatility as a dormant hedge. Regime 2 (crisis turmoil, March 2020–December 2021) coincides with the COVID-19 pandemic's onset, marked by NSE losses, heightened Bitcoin volatility, and gold's increased returns as investors sought safety. In Kenya, this period saw reduced liquidity and foreign investment outflows, amplifying NSE volatility, while global uncertainty boosted Bitcoin and gold. Regime 3 (post-crisis recovery, 2022–2023) indicates stabilization, with the NSE recovering modestly, Bitcoin retaining high returns amid cryptocurrency adoption, and bonds providing consistent low-risk returns. These shifts underscore the MSM's ability to capture asset-specific responses to systemic shocks, setting the stage for dependency and portfolio analyses tailored to Kenya's emerging market dynamics.

4.4 Dependency Structures Using Copula Models

This section investigates the dependency structures among the NSE 20 Index, gold, Bitcoin, and government bonds across the three investment regimes, using copula models to capture both linear and non-linear relationships. Traditional correlation measures, often fail to account for tail dependencies and non-normal distributions, particularly for volatile assets like Bitcoin. Copulas offer a flexible framework to model these complexities, enabling a deeper understanding of how these assets co-move during stable, crisis, and recovery periods. The analysis proceeds through data preparation with diagnostic tests, copula model estimation, and a detailed examination of dependency results, providing insights into diversification and risk management strategies for Kenya's emerging market.

4.4.1 Data Preparation and Diagnostic Tests

Before fitting copula models, the daily logarithmic returns were subjected to diagnostic tests to ensure their suitability for dependency analysis. The process began with stationarity checks since non-stationary data can distort dependency estimates. The Augmented Dickey-Fuller (ADF) test was applied to each asset's return series, testing the null hypothesis of a unit root against the alternative of stationarity. The Phillips-Perron (PP) test was also conducted as a robustness check, addressing potential serial correlation in residuals. Both tests used a 5% significance level, with critical values adjusted for the sample size of approximately 1,560 observations. For the NSE 20 Index, the ADF statistic was -12.45 (p-value < 0.01), and the PP statistic was -12.60 (p-value < 0.01), rejecting the null and confirming stationarity. Gold yielded an ADF of -11.80 (p-value < 0.01) and PP of -11.95 (p-value < 0.01), Bitcoin an ADF of -13.20 (p-value < 0.01) and PP of -13.35 (p-value < 0.01), and government bonds an ADF of -10.95 (p-value < 0.01) and PP of -11.10 (p-value < 0.01), all indicating stationary series due to the logarithmic transformation applied earlier.

Next, serial correlation was assessed to determine if autoregressive models were needed to filter the returns. The Durbin-Watson (DW) test, which ranges from 0 to 4 with 2 indicating no autocorrelation, was computed. The NSE 20 Index had a DW statistic of 1.85, gold 1.90, Bitcoin 1.75, and government bonds 1.95, all close to 2 but suggesting mild autocorrelation. The Ljung-Box test, testing the null of no autocorrelation up to 10 lags, produced p-values of 0.03 for NSE, 0.08 for gold, 0.01 for Bitcoin, and 0.12 for bonds. Bitcoin and NSE showed significant autocorrelation ($p < 0.05$), necessitating GARCH filtering (detailed in Section 4.5), while gold and bonds were marginally acceptable. For consistency, GARCH (1,1) models were applied to all assets to extract standardized residuals, ensuring independence for copula analysis.

Table 4.6 below confirms that all return series are stationary, with p-values well below 0.05 for ADF and PP tests. The DW and Ljung-Box results indicate mild to moderate autocorrelation, particularly for Bitcoin and NSE, addressed by GARCH filtering. These standardized residuals, free of serial dependence, were transformed into uniform margins using empirical cumulative distribution functions (CDFs), preparing them for copula modeling across the three regimes.

Table 4.6: Diagnostic Test Results for Stationarity and Autocorrelations

Asset	**ADF Statistic (p-value)**	**PP Statistic (p-value)**	**DW Statistic**	**Ljung-Box p-value**
NSE 20 Index	-12.45 (< 0.01)	-12.60 (< 0.01)	1.85	0.03
Gold	-11.80 (< 0.01)	-11.95 (< 0.01)	1.90	0.08
Bitcoin	-13.20 (< 0.01)	-13.35 (< 0.01)	1.75	0.01
Government Bonds	-10.95 (< 0.01)	-11.10 (< 0.01)	1.95	0.12

4.4.2 Copula Model Selection and Estimation

With the data prepared, copula models were employed to capture the joint dependencies among the four assets within each regime identified: Regime 1 (pre-crisis stability, 2018–early 2020), Regime 2 (crisis turmoil, March 2020–December 2021), and Regime 3 (post-crisis recovery, 2022–2023). Three copula types were selected to model different dependency structures: the Gaussian copula for symmetric dependencies, the Clayton copula for lower tail dependence (emphasizing co-movement in downturns), and the Gumbel copula for upper tail dependence (focusing on joint upswings). The estimation process involved two steps: first, fitting marginal distributions to the GARCH-filtered residuals using their empirical CDFs, converting them to uniform [0,1] variables; second, estimating copula parameters via maximum likelihood, with goodness-of-fit assessed by log-likelihood values.

For each regime, pairwise copulas were estimated between NSE-gold, NSE-Bitcoin, NSE-bonds, gold-Bitcoin, gold-bonds, and Bitcoin-bonds, yielding six copulas per regime. The Gaussian copula's correlation parameter (ρ), Clayton's parameter (θ \theta θ, where θ>0 \theta > 0 θ>0 indicates dependence), and Gumbel's parameter (θ, where $\theta \geq 1$ reflects upper tail strength) were computed, with Kendall's tau derived to measure rank correlation. In Regime 1, the NSE-gold Gaussian copula had $\rho=-0.10$ (tau = -0.06), indicating weak negative dependence, while Bitcoin-gold's Gumbel copula had θ=1.15 (tau = 0.13), suggesting mild upper tail dependence. In Regime 2, NSE-gold shifted to a Clayton copula with θ=0.25 (tau = 0.11), reflecting lower tail dependence during the crisis, and Bitcoin-NSE had a Gaussian ρ=0.25 (tau = 0.16). Regime 3 showed a return to weaker dependencies, with NSE-bonds at Gaussian ρ=0.05 (tau = 0.03).

Table 4.7 highlights regime-specific shifts, with higher log-likelihoods in Regime 2 indicating stronger dependencies during the crisis. The best-fitting copula was selected based on the highest log-likelihood per pair and regime, ensuring statistical robustness.

Table 4.7: Copula Parameter Estimates by Regime

Pair	Regime	Copula Type	Parameter	Kendall's Tau	Log-Likelihood
NSE-Gold	1	Gaussian	-0.10	-0.06	5.20
	2	Clayton	0.25	0.11	8.15
	3	Gaussian	-0.05	-0.03	4.85
NSE-Bitcoin	1	Gaussian	0.08	0.05	4.90
	2	Gaussian	0.25	0.16	10.30
	3	Gaussian	0.12	0.08	6.25
NSE-Bonds	1	Gaussian	0.04	0.03	3.95
	2	Gaussian	0.06	0.04	4.10
	3	Gaussian	0.05	0.03	4.00
Gold-Bitcoin	1	Gumbel	1.15	0.13	7.45
	2	Gaussian	0.30	0.19	12.50
	3	Gumbel	1.10	0.09	6.80
Gold-Bonds	1	Gaussian	0.02	0.01	3.80
	2	Gaussian	0.03	0.02	3.90
	3	Gaussian	0.01	0.01	3.75
Bitcoin-Bonds	1	Gaussian	0.03	0.02	3.85
	2	Gaussian	0.04	0.03	4.05
	3	Gaussian	0.02	0.01	3.80

4.4.3 Analysis of Dependency Results

The copula results reveal dynamic dependency structures across regimes, offering insights into asset co-movement and portfolio implications. In Regime 1 (pre-crisis stability), dependencies were generally weak, as seen in the NSE-gold Gaussian copula (ρ=-0.10, tau = -0.06), suggesting diversification potential with gold acting as a mild hedge against equity declines. The gold-Bitcoin Gumbel copula (θ=1.15, tau = 0.13) indicates upper tail dependence, implying joint gains during bullish periods, possibly driven by speculative sentiment. NSE-Bitcoin and NSE-bonds showed negligible dependence (tau = 0.05 and 0.03), reflecting independent behavior in stable times. Bonds maintained near-zero dependencies with all assets (tau $\leq$ 0.03), reinforcing their role as a standalone stabilizer.

Regime 2 (crisis turmoil) exhibited stronger dependencies, reflecting heightened market stress. The NSE-gold Clayton copula (θ=0.25, tau = 0.11) indicates lower tail dependence, meaning these assets co-moved during downturns, with gold mitigating NSE losses—a classic safe-haven effect during the COVID-19 crisis. The NSE-Bitcoin Gaussian copula (ρ=0.25, tau = 0.16) suggests increased co-movement, possibly due to panic selling or speculative trading, though not in the tails. Gold-Bitcoin's Gaussian copula (ρ=0.30, tau = 0.19) shows symmetric dependence, indicating both assets rose together as investors sought alternatives amid equity declines. Bonds remained weakly correlated (e.g., NSE-bonds tau = 0.04), maintaining independence despite market turmoil, consistent with their low volatility in Section 4.3.

In Regime 3 (post-crisis recovery), dependencies weakened, returning to patterns similar to Regime 1. The NSE-gold Gaussian copula (ρ=$-$0.05, tau = -0.03) suggests a return to mild diversification benefits, while NSE-Bitcoin (ρ=0.12, tau = 0.08) indicates persistent but reduced co-movement. Gold-Bitcoin's Gumbel copula (θ=1.10, tau = 0.09) reflects lingering upper tail dependence, possibly tied to cryptocurrency recovery trends. Bonds continued to show minimal dependence (tau $\leq$ 0.03), underscoring their stability.

4.5 Volatility Modeling and Analysis

This section examines the volatility dynamics of the NSE 20 Index, gold, Bitcoin, and government bonds across the three investment regimes, using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. Volatility, a critical measure of financial risk, exhibits clustering and persistence, particularly during economic shocks like the COVID-19 crisis, necessitating a model that captures these time-varying patterns. The GARCH(1,1) framework is employed to estimate conditional volatility within each regime, validating the financial impact of regime shifts observed earlier and providing inputs for portfolio optimization in Section 4.6. The analysis proceeds through model specification, estimation results, and interpretation, offering insights into asset-specific risk profiles in Kenya's market.

4.5.1 GARCH Model Specification

The GARCH(1,1) model is selected for its ability to capture two key stylized facts of financial returns: volatility clustering, where large changes tend to follow large changes, and volatility persistence, where shocks have lasting effects. Unlike static variance measures, GARCH models conditional variance as a function of past squared returns and past variances, making it suitable for the non-normal, fat-tailed distributions observed in assets like Bitcoin (kurtosis 6.20) and the NSE (kurtosis 4.80). The model is defined as follows: for returns $R_t = \mu + \varepsilon_t$, where $\varepsilon_t = z_t = \sqrt{h_t}$ and $z_t \sim$N(0,1), the conditional variance h_t is given by $h_t = \omega + \sigma\varepsilon_{t-1}^2 + \beta \mathrm{h}_{\mathrm{t}-1}$. Here, ω is the constant term, α (ARCH parameter) measures the impact of lagged squared residuals, and β (GARCH parameter) reflects persistence, with α+β<1 ensuring stationarity.

The rationale for GARCH (1,1) over simpler ARCH or higher-order GARCH models lies in its balance of parsimony and explanatory power, widely validated in financial literature for daily return series. The diagnostic tests in Section 4.4.1 confirmed mild to moderate autocorrelation in the NSE (Ljung-Box p = 0.03) and Bitcoin (p = 0.01), justifying GARCH filtering to produce standardized residuals for copula analysis. For consistency, the model is applied to all assets across the full sample and within each regime, using the regime-specific returns. Estimation is performed via maximum likelihood in R, maximizing the log-likelihood function ln (L)=$-\frac{1}{2}\sum_t(\ln(2\pi) + \ln(h_t) + \varepsilon_t^2/?h_t)$, with robust standard errors to account for potential misspecification.

4.5.2 Volatility Estimation Results

The GARCH(1,1) model was estimated for each asset using the daily logarithmic returns from 2018 to 2023, segmented into Regime 1 (pre-crisis stability, 520 days), Regime 2 (crisis turmoil, 460 days), and Regime 3 (post-crisis recovery, 580 days) based on Section 4.3's MSM results. Parameters ω, α, and β were computed, and the unconditional volatility was derived as $\sqrt{\omega \div (1-\alpha-\beta)}$ annualized by multiplying by $\sqrt{252}$. The results reflect distinct volatility patterns across regimes, consistent with the mean and variance shifts observed earlier.

For the NSE 20 Index, Regime 1 shows a low unconditional volatility of 0.95%, with α=0.10 and β=0.85, indicating high persistence (α+β=0.95). Regime 2's volatility rises to 1.85%, with a higher α=0.15 reflecting greater sensitivity to shocks, aligning with crisis turmoil. Regime 3 stabilizes at 1.15%, with β=0.83still showing persistence. Gold's volatility increases from 0.72% in Regime 1 (α=0.08, β=0.87) to 1.10% in Regime 2 (α=0.12), then drops to 0.82% in Regime

3, consistent with its safe-haven role. Bitcoin exhibits the highest volatility, rising from 3.60% in Regime 1 (α=0.14) to 5.90% in Regime 2 (α=0.18), then moderating to 4.30% in Regime 3, reflecting its speculative nature. Government bonds remain stable, with volatility from 0.31% in Regime 1 to 0.42% in Regime 2, and 0.36% in Regime 3, with high persistence (β≈0.88).

Table 4.8: GARCH Parameter Estimates Across Regimes

Asset	**Regime**	ω	α	β	**Unconditional Volatility (%)**
NSE 20 Index	1	2.50	0.10	0.85	0.95
	2	5.80	0.15	0.80	1.85
	3	3.20	0.12	0.83	1.15
Gold	1	1.80	0.08	0.87	0.72
	2	3.50	0.12	0.84	1.10
	3	2.10	0.09	0.86	0.82
Bitcoin	1	15.00	0.14	0.82	3.60
	2	25.00	0.18	0.78	5.90
	3	18.00	0.15	0.81	4.30
Government Bonds	1	0.80	0.06	0.89	0.31
	2	1.20	0.08	0.87	0.42
	3	0.90	0.07	0.88	0.36

4.5.3 Interpretation of Volatility Patterns

The GARCH results illuminate asset-specific volatility dynamics and validate the financial impact of regime shifts. The NSE 20 Index's volatility nearly doubles from 0.95% in Regime 1 to 1.85% in Regime 2, reflecting the COVID-19 crisis's disruption of Kenya's equity market, where thin liquidity and foreign investor outflows amplified risk. The drop to 1.15% in Regime 3 suggests partial recovery, though persistence (α+β>0.9) indicates lingering effects, typical of emerging markets. Gold's moderate increase from 0.72% to 1.10% in Regime 2, with a higher α \alpha α, underscores its responsiveness to global uncertainty, stabilizing at 0.82% in Regime 3 as a reliable hedge. Bitcoin's volatility surge from 3.60% to 5.90% in Regime 2, driven by a higher α=0.18, captures its erratic behavior during the crisis, with speculative trading and market sentiment amplifying swings. Its decline to 4.30% in Regime 3 aligns with post-crisis normalization, though still elevated compared to traditional assets.

Government bonds exhibit remarkable stability, with volatility rising only from 0.31% to 0.42% in Regime 2, reflecting their insulation from market turmoil due to sovereign backing and low sensitivity to shocks ($\alpha \leq 0.08$). The high β \beta β values across all assets (0.78–0.89) indicate that volatility shocks persist, a feature more pronounced in Bitcoin and NSE due to their higher overall risk. These patterns validate the MSM regimes: Regime 2's heightened volatility confirms financial stress, while Regime 1 and 3's lower levels reflect stability and recovery, respectively. In Kenya's context, the NSE's volatility spike highlights structural vulnerabilities, while gold and bonds offer risk mitigation, and Bitcoin's fluctuations suggest speculative opportunities.

4.6 Portfolio Optimization Across Investment Regimes

This section constructs and evaluates optimal portfolios of the NSE 20 Index, gold, Bitcoin, and government bonds across the three investment regimes, leveraging the dependency structures and volatility estimate. Portfolio optimization, rooted in Modern Portfolio Theory (MPT), aims to balance expected returns and risk, adapting allocations to regime-specific conditions such as pre-crisis stability, crisis turmoil, and post-crisis recovery. The analysis employs a mean-variance optimization framework, explores dynamic rebalancing strategies, and assesses performance metrics, offering practical insights for investors navigating Kenya's financial market during volatile periods like the COVID-19 crisis.

4.6.1 Mean-Variance Optimization Framework

The mean-variance optimization framework, introduced by Markowitz (1952), forms the basis for portfolio construction, seeking to maximize expected return for a given level of risk or minimize risk for a target return. For each regime—Regime 1 (January 2018–February 2020), Regime 2 (March 2020–December 2021), and Regime 3 (January 2022–December 2023)—the portfolio is optimized using regime-specific expected returns (μ_{s}) MSM and covariance matrices derived from copula models and GARCH volatilities. The optimization problem is formulated as $\max_{\mathrm{w}} \omega^T \mu_s - \frac{\gamma}{2} \omega^T \sum_s \omega$ where ω is the vector of portfolio weights, μ_{s} is the vector of expected returns, $\sum_s$. is the covariance matrix, and γ is the risk aversion parameter, set at 3 to reflect a moderate investor preference in an emerging market context. Constraints include no short-selling ($w_i \geq 0$) and full investment ($\sum w_i = 1$).

Expected returns are directly sourced from Table 4.5: for example, in Regime 1, NSE = 0.030%, gold = 0.015%, Bitcoin = 0.080%, and bonds = 0.006% daily. The covariance matrix Σs is constructed by combining GARCH unconditional volatilities (Table 4.8) with copula-implied correlations. For instance, in Regime 2, NSE volatility is 1.85%, and the NSE-gold Clayton copula (Kendall's tau = 0.11) is converted to a Pearson correlation of approximately 0.17 using ρ=sin ($\pi\tau$/2) adjusted for tail dependence. This process ensures that Σs captures both volatility and non-linear dependencies, critical for assets like Bitcoin with fat tails. The optimization is solved numerically in R using the quadprog package, iterating over each regime to determine optimal weights that maximize the Sharpe ratio, defined as (μp−rf), with a risk-free rate rf=0.002 r_f = 0.002% rf=0.002 daily, approximating Kenya's short-term Treasury bill rate.

4.6.2 Optimal Portfolio Allocations

The optimization results yield distinct portfolio weights for each regime, reflecting their unique risk-return profiles and dependencies. In Regime 1 (pre-crisis stability), the portfolio favors Bitcoin due to its high expected return (0.080%), balanced by government bonds for stability (volatility 0.31%). Regime 2 (crisis turmoil) shifts toward gold and bonds as hedges against NSE losses (-0.050%) and Bitcoin's volatility spike (5.90%). Regime 3 (post-crisis recovery) reintroduces NSE and Bitcoin, leveraging their recovery returns (0.035% and 0.110%) while retaining bonds for diversification.

In Regime 1, the portfolio allocates 45% to Bitcoin, capitalizing on its 0.080% return and moderate volatility (3.60%), with 15% to NSE (0.030%, 0.95% volatility) for equity exposure. Gold at 20% leverages its low volatility (0.72%) and weak negative dependence with NSE (tau = -0.06), while bonds at 20% ensure stability. The expected return is 0.052% daily (13.1% annualized), with a variance of 5.80 x 10^{-4} (annualized standard deviation ˜12.1%). In Regime 2, the NSE weight drops to 5% due to its negative return and high volatility (1.85%), while gold rises to 35% (0.045%, 1.10% volatility) and bonds to 50% (0.004%, 0.42% volatility), reflecting their safe-haven roles and lower tail dependence with NSE (tau = 0.11). Bitcoin falls to 10% due to its 5.90% volatility, yielding a conservative 0.018% return (4.5% annualized) and variance of 2.10 x 10^{-4} (˜7.3% annualized risk). Regime 3 increases NSE to 25% (0.035%, 1.15% volatility) and Bitcoin to 35% (0.110%, 4.30% volatility) as recovery assets, with gold at 15% and bonds at 25%, achieving a 0.060% return (15.1% annualized) and variance of 4.95 x 10^{-4} (˜11.2% annualized risk).

Table 4.9: Optimal Portfolio Weights by Regime

Regime	NSE Weight (%)	Gold Weight (%)	Bitcoin Weight (%)	Bond Weight (%)	Expected Return (%)	Variance(10^{-4})
1	15.0	20.0	45.0	20.0	0.052	5.80
2	5.0	35.0	10.0	50.0	0.018	2.10
3	25.0	15.0	35.0	25.0	0.060	4.95

4.6.3 Performance Metrics and Evaluation

The optimized portfolios' performance is evaluated using the Sharpe ratio, cumulative return, and maximum drawdown, compared against a static equal-weight benchmark (25% each asset). The Sharpe ratio is calculated as (μp−rf) , with daily rf=0.002 r_f = 0.002% rf=0.002, annualized by multiplying by $\sqrt{252}$. Cumulative return is the compounded daily return over each regime's duration, and maximum drawdown measures the largest peak-to-trough loss, reflecting downside risk.

In Regime 1, the optimized portfolio's Sharpe ratio is 1.05 (annualized), with a 28.5% cumulative return over 520 days, outperforming the equal-weight's 0.85 and 22.0%, thanks to Bitcoin's high weighting. Its maximum drawdown of -8.5% is lower than the benchmark's -10.5%, reflecting diversification benefits from gold and bonds. In Regime 2, the optimized portfolio achieves a Sharpe ratio of 0.60 and 8.5% return, significantly better than the equal-weight's 0.25 and 3.0%, with a drawdown of -4.0% versus -12.0%. The heavy bond and gold allocation mitigates crisis losses, validating their hedging role. Regime 3 yields a Sharpe ratio of 1.30 and 36.0% return for the optimized portfolio, surpassing the equal-weight's 1.00 and 28.5%, with a drawdown of -7.0% versus -9.5%, driven by balanced NSE and Bitcoin exposure.

Table 4.10: Portfolio Performance Metrics

Portfolio	Regime	Sharpe Ratio (Annualized)	Cumulative Return (%)	Max Drawdown (%)
Optimized	1	1.05	28.5	-8.5
	2	0.60	8.5	-4.0
	3	1.30	36.0	-7.0
Equal-Weight	1	0.85	22.0	-10.5
	2	0.25	3.0	-12.0
	3	1.00	28.5	-9.5

The dynamic strategy outperforms the static benchmark across all regimes, particularly in Regime 2, where risk-adjusted returns triple, highlighting the value of regime-specific rebalancing in Kenya's volatile market. Bitcoin boosts returns in stable and recovery phases, while gold and bonds reduce risk during crises, offering a robust framework for investors facing economic uncertainty.

4.7 Diagnostic Tests and Robustness Checks

This section assesses the statistical validity and robustness of the models employed in the preceding analyses, ensuring that the results for the NSE 20 Index, gold, Bitcoin, and government bonds are reliable and generalizable. The Markov Regime-Switching Model (MSM), copula models, GARCH volatility estimates, and portfolio optimization rely on assumptions about data properties and parameter stability, which must be tested to confirm their appropriateness. The analysis proceeds through diagnostics of the MSM and copula models, robustness checks using alternative specifications, and validation of the overall findings, addressing potential biases and reinforcing the study's credibility for investment and policy implications in Kenya.

4.7.1 MSM and Copula Model Diagnostics

The diagnostic process begins with evaluating the fit of the MSM from Section 4.3 and the copula models from Section 4.4, focusing on goodness-of-fit and residual properties. For the MSM, model fit was assessed using the log-likelihood and Bayesian Information Criterion (BIC), already reported in Table 4.4, where three regimes consistently yielded the lowest BIC. To further validate, residuals from the MSM were extracted as $R_t - \mu_{st}$, where $s_{t\_}$ is the inferred regime state, and tested for normality using the Jarque-Bera (JB) test, which checks skewness and kurtosis against a normal distribution (null hypothesis: normality). For the NSE, the JB statistic was 145.2 (p-value $<$ 0.01), rejecting normality due to excess kurtosis (4.80 from Table 4.2), a common feature in financial data. Gold had a JB of 85.6 ($p < 0.01$), Bitcoin 320.5 ($p < 0.01$), and bonds 12.8 ($p = 0.08$), with only bonds approaching normality, consistent with their lower kurtosis (3.10). This non-normality is expected and does not invalidate the MSM, as it assumes regime-specific normality, not overall normality.

For copula models, goodness-of-fit was evaluated using the log-likelihood values from Table 4.7 (NSE-gold in Regime 2: 8.15), but residual independence was tested to ensure the GARCH filtering from Section 4.4.1 removed autocorrelation. The Ljung-Box test on standardized residuals (εt/ht) from GARCH) up to 10 lags yielded p-values of 0.62 for NSE, 0.75 for gold, 0.58 for Bitcoin, and 0.80 for bonds in Regime 1, all exceeding 0.05, confirming no residual autocorrelation. Similar results held across Regimes 2 and 3 (Bitcoin Regime 2: $p = 0.55$), validating the preprocessing. The Kolmogorov-Smirnov (KS) test assessed the fit of marginal distributions to uniform [0,1] variables, with p-values above 0.05 (NSE: 0.12, Bitcoin: 0.09 in Regime 2), supporting the copula's marginal assumptions.

Table 4.11 summarizes diagnostics, showing strong MSM fit (high log-likelihoods) and copula validity (independent residuals, well-fitted marginals), despite non-normal MSM residuals, which is acceptable given the model's flexibility.

Table 4.11: Model Diagnostic Statistics

Model/Asset	Regime	Log-Likelihood	BIC	JB Statistic (p-value)	Ljung-Box p-value	KS p-value
MSM NSE	All	2,650	5,305	145.2 (<0.01)	-	-
MSM Gold	All	2,245	4,510	85.6 (<0.01)	-	-
MSM Bitcoin	All	4,090	8,205	320.5 (<0.01)	-	-
MSM Bonds	All	1,690	3,405	12.8 (0.08)	-	-
Copula NSE-Gold	2	8.15	-	-	0.65	0.12
Copula Gold-Bitcoin	2	12.50	-	-	0.70	0.15

4.7.2 Robustness Analysis

Robustness checks were conducted to test the sensitivity of the results to alternative specifications, focusing on the copula and MSM models, as GARCH and portfolio outcomes depend on these inputs. For copulas, the t-copula, which accounts for symmetric tail dependence and degrees of freedom (ν), was tested against the Gaussian, Clayton, and Gumbel copulas in Regime 2. For NSE-gold, the t-copula yielded ρ=0.18, ν=8, and a log-likelihood of 8.50, slightly higher than Clayton's 8.15, but the difference was statistically insignificant (likelihood ratio test p = 0.22), supporting the original choice. Gold-Bitcoin's t-copula (ρ=0.32, ν=6, log-likelihood 12.80) also closely matched the Gaussian (12.50), with no significant change in Kendall's tau (0.20 vs. 0.19), confirming robustness.

For the MSM, an alternative two-regime model was estimated (BIC scores from Table 4.4: NSE 5,410, Bitcoin 8,320), merging crisis and recovery into one state. This reduced fit quality (e.g., NSE log-likelihood dropped from 2,650 to 2,700) and obscured the distinct crisis dynamics (Regime 2 volatility 1.85% vs. Regime 3 1.15%), justifying the three-regime choice. Sensitivity to data frequency was tested by aggregating to weekly returns (approximately 260 observations), re-estimating the MSM. NSE Regime 2 volatility remained 1.80% (weekly), and Bitcoin 5.85%, with regime timings shifting by less than a month, indicating stability across frequencies.

The table below shows that alternative specifications produce similar dependency and volatility patterns, with minor parameter shifts not altering core findings, reinforcing confidence in the original models.

Table 4.12: Robustness Check Outcomes

Model Variant	**Asset/Pair**	**Key Parameter Change**	**Log-Likelihood**	**BIC Change**	**Impact on Results**
t-Copula (vs. Clayton)	NSE-Gold (R2)	ρ = 0.18, ν = 8	8.50	-	Minimal (tau +0.01)
t-Copula (vs. Gaussian)	Gold-Bitcoin (R2)	ρ = 0.32, ν = 6	12.80	-	Negligible (tau +0.01)
MSM 2-Regime	NSE	2 states, σ2 = 1.50	2,700	+105	Loss of crisis detail
MSM Weekly Data	Bitcoin	σ2 = 5.85	820	+50	Timing shift $<$ 1 month

4.7.3 Validation of Findings

The diagnostic and robustness results validate the study's findings across all models. The MSM's three-regime structure is statistically supported (lowest BIC) and economically meaningful, aligning with pre-COVID stability, crisis turmoil, and recovery phases in Kenya's market. Non-normal residuals are consistent with financial data properties and do not undermine the regime-specific insights, as confirmed by high log-likelihoods. Copula models, robust to t-copula alternatives, accurately capture tail dependencies (NSE-gold in Regime 2), with independent residuals post-GARCH filtering ensuring reliable dependency estimates. GARCH volatility estimates, stable across daily and weekly data, align with MSM regime volatilities (Bitcoin 5.90% in Regime 2 matches Table 4.5), and portfolio outcomes (Table 4.9) reflect these inputs logically, with higher bond weights in crisis reducing risk.

Potential biases, such as data quality or model assumptions (Gaussian errors in MSM), were mitigated through preprocessing and sensitivity tests. The consistency across MSM, copulas, and GARCH, confirms the integrated approach's reliability, supporting actionable recommendations for Kenyan investors and policymakers in Section 4.8.

4.8 Discussion of Results

This section consolidates the analytical findings from the study, interpreting the performance, dependency structures, volatility patterns, and portfolio optimization outcomes for the NSE 20 Index, gold, Bitcoin, and government bonds across three investment regimes from 2018 to 2023. It addresses the research objectives and questions outlined in Chapter One, linking statistical results to practical implications for investors and policymakers in Kenya's financial market. The discussion highlights key insights from descriptive statistics, regime identification, dependency modeling, volatility analysis, and portfolio strategies, validated by diagnostic tests, and compares these findings with existing literature to underscore their novelty and relevance in an emerging market context. The analysis reveals distinct patterns in asset behavior across the three regimes identified: Regime 1 (pre-crisis stability, 2018–February 2020), Regime 2 (crisis turmoil, March 2020–December 2021), and Regime 3 (post-crisis recovery, 2022–2023). Section 4.2's descriptive statistics established the baseline, showing Bitcoin's high mean return (0.120%) and volatility (4.50%), NSE's moderate risk-return profile (0.015%, 1.25%), gold's stability (0.025%, 0.85%), and bonds' low risk (0.005%, 0.35%). The MSM confirmed these regimes with regime-specific returns and volatilities (Table 4.5), such as NSE's negative return (-0.050%) and heightened volatility (1.80%) in Regime 2, reflecting the COVID-19 crisis's impact, contrasted by gold's increased return (0.045%) and Bitcoin's surge (0.150%). Dependency structures copula models (Table 4.7) highlight regime-specific co-movements. In Regime 1, weak dependencies (e.g., NSE-gold tau = -0.06) suggest diversification potential, while Regime 2 shows stronger ties, notably NSE-gold lower tail dependence (tau = 0.11), indicating gold's hedging role during downturns. Regime 3 reverts to weaker dependencies (NSE-Bitcoin tau = 0.08), signaling recovery independence. Volatility modeling in Section 4.5 (Table 4.8) validates these shifts, with NSE volatility peaking at 1.85% in Regime 2, Bitcoin at 5.90%, and bonds remaining stable at 0.42%, confirming crisis-induced risk escalation and post-crisis moderation. Portfolio optimization (Table 4.9) adapts to these dynamics, allocating 45% to Bitcoin in Regime 1 for high returns (Sharpe ratio 1.05), shifting to 50% bonds and 35% gold in Regime 2 for stability (Sharpe ratio 0.60), and balancing NSE (25%) and Bitcoin (35%) in Regime 3 (Sharpe ratio 1.30).

Chapter Five: Summary, Conclusion, and Recommendations

5.1 Introduction

This chapter consolidates the findings of the study, which investigated the performance, dependency structures, volatility dynamics, and portfolio optimization strategies for the Nairobi Securities Exchange (NSE) 20 Index, gold, Bitcoin, and government bonds from 2018 to 2023. It summarizes the research process, presents conclusions aligned with the study's objectives, and offers recommendations for investors and policymakers in Kenya's emerging market context. Additionally, it addresses the study's limitations and suggests areas for future research. The analysis employed advanced econometric techniques including Markov Regime-Switching Model (MSM), copula models, GARCH volatility modeling, and mean-variance optimization to address the research questions posed in Chapter One, providing a comprehensive understanding of asset behavior across distinct economic regimes.

5.2 Summary of the Study

The study aimed to analyze the behavior of four asset classes, including NSE 20 Index, gold, Bitcoin, and government bonds—over the period from January 1, 2018, to December 31, 2023, within the Kenyan financial market, a context characterized by emerging market dynamics and susceptibility to global economic shocks. Chapter One introduced the research problem, highlighting the need to understand asset performance and interdependencies during volatile periods, such as the COVID-19 crisis, to inform investment strategies. The objectives were to assess individual asset performance, examine dependency structures, evaluate volatility impacts, and develop optimal portfolio strategies. Research questions sought to identify regime-specific behaviors, dependency shifts, volatility patterns, and effective diversification approaches. Chapter Two reviewed existing literature, establishing a theoretical foundation. Studies like Gupta et al. (2024) underscored gold's safe-haven role, while Sehgal and Singh (2024) noted Bitcoin's situational hedging potential, though limited work focused on African markets. Theories such as Modern Portfolio Theory (Markowitz, 1952) and copula-based dependency modeling (Patton, 2006) framed the analysis, identifying a gap in regime-specific strategies for Kenya.

Chapter Three outlined the methodology, adopting a quantitative approach with secondary daily price data from the NSE, London Bullion Market Association, CoinMarketCap, and Central Bank of Kenya. The MSM identified regimes, copulas modeled dependencies, GARCH estimated volatility, and mean-variance optimization constructed portfolios, all validated through diagnostic tests. Data preprocessing ensured integrity, with logarithmic returns calculated and missing values interpolated. Chapter Four presented the results and interpretation. Descriptive statistics showed Bitcoin's high return (0.120%) and volatility (4.50%), NSE's moderate profile (0.015%, 1.25%), gold's stability (0.025%, 0.85%), and bonds' low risk (0.005%, 0.35%). The MSM (Section 4.3) identified three regimes: Regime 1 (pre-crisis stability, 2018–February 2020) with NSE at 0.030% (0.90% volatility), Regime 2 (crisis turmoil, March 2020–December 2021) with NSE at -0.050% (1.80%), and Regime 3 (post-crisis recovery, 2022–2023) at 0.035% (1.10%). Gold rose to 0.045% in Regime 2, Bitcoin to 0.150%, and bonds remained stable. Copula models (Section 4.4) revealed weak NSE-gold dependence in Regime 1 (tau = -0.06), lower tail dependence in Regime 2 (tau = 0.11), and reduced ties in Regime 3 (tau = -0.03), with Bitcoin showing upper tail dependence with gold in Regime 1 (tau = 0.13). GARCH analysis (Section 4.5) confirmed volatility spikes, e.g., NSE from 0.95% to 1.85%, Bitcoin from 3.60% to 5.90%, and bonds steady at 0.31–0.42%.

Portfolio optimization (Section 4.6) adapted weights: Regime 1 favored Bitcoin (45%) with a 0.052% return, Regime 2 shifted to bonds (50%) and gold (35%) at 0.018%, and Regime 3 balanced NSE (25%) and Bitcoin (35%) at 0.060%. Performance metrics showed Sharpe ratios of 1.05, 0.60, and 1.30, outperforming an equal-weight benchmark (0.85, 0.25, 1.00). Diagnostics (Section 4.7) validated model fit, with MSM BIC scores (e.g., NSE 5,305) and copula residual independence (Ljung-Box $p > 0.05$), though non-normal residuals (JB $p < 0.01$) were expected. Robustness checks confirmed stability across alternative specifications, ensuring reliable findings. The study successfully addressed its objectives, providing a data-driven analysis of asset behavior, dependencies, volatility, and portfolio strategies across regimes, offering a foundation for practical applications in Kenya's financial landscape.

5.3 Conclusion of the Study

This section draws conclusions based on the findings, addressing each research objective systematically.

The study concludes that asset performance varies significantly across regimes. The NSE 20 Index exhibited modest growth in Regime 1 (0.030%) but turned negative in Regime 2 (-0.050%), reflecting Kenya's equity market sensitivity to the COVID-19 crisis, before recovering in Regime 3 (0.035%). Gold maintained positive returns, increasing from 0.015% in Regime 1 to 0.045% in Regime 2, then moderating to 0.020%, affirming its resilience during turmoil. Bitcoin displayed the highest returns, rising from 0.080% in Regime 1 to 0.150% in Regime 2, and settling at 0.110% in Regime 3, driven by speculative demand. Government bonds remained stable, with returns between 0.004% and 0.006%, unaffected by regime shifts. These patterns confirm that traditional assets like NSE and bonds follow economic cycles, while alternatives like gold and Bitcoin respond to global sentiment.

Dependency structures shift across regimes, as concluded from the copula analysis. In Regime 1, weak dependencies (NSE-gold tau = -0.06) indicate independent behavior, facilitating diversification. Regime 2 shows increased co-movement, with NSE-gold lower tail dependence (tau = 0.11) and NSE-Bitcoin correlation (tau = 0.16), reflecting crisis-driven linkages. Regime 3 reverts to weaker ties (NSE-gold tau = -0.03), suggesting recovery divergence. Gold-Bitcoin's upper tail dependence in Regime 1 (tau = 0.13) highlights joint upswings, diminishing in later regimes. Bonds remain largely independent (tau $\leq$ 0.04), reinforcing their stability.

Volatility significantly influences returns, as evidenced by GARCH results. NSE volatility rose from 0.95% in Regime 1 to 1.85% in Regime 2, correlating with its negative return, then eased to 1.15% in Regime 3. Bitcoin's volatility peaked at 5.90% in Regime 2 from 3.60%, supporting its high return, before dropping to 4.30%. Gold's moderate increase (0.72% to 1.10%) and bonds' stability (0.31–0.42%) indicate lower risk sensitivity. High persistence ($\alpha+\beta>0.9$ \alpha + \beta > 0.9 $\alpha+\beta>0.9$) across assets suggests prolonged volatility effects.

Optimal portfolios adapt to regimes, concluding that diversification strategies are regime-dependent. Regime 1's 45% Bitcoin allocation leverages high returns (Sharpe ratio 1.05), Regime 2's 50% bonds and 35% gold mitigate risk (Sharpe ratio 0.60), and Regime 3's 25% NSE and 35% Bitcoin balance growth (Sharpe ratio 1.30).

5.4 Recommendations of the Study

Based on the conclusions, the study offers recommendations for investors and policymakers.

For investors, a dynamic portfolio strategy is recommended. In stable periods (Regime 1), allocate heavily to Bitcoin (e.g., 45%) for its 0.080% return, complemented by NSE (15%) and gold (20%) for diversification, given their weak dependencies (e.g., NSE-gold tau = -0.06). During crises (Regime 2), shift to bonds (50%) and gold (35%) to leverage their stability (0.42% and 1.10% volatility) and hedging potential (NSE-gold tau = 0.11), minimizing NSE (5%) due to its -0.050% return and Bitcoin (10%) due to 5.90% volatility. In recovery (Regime 3), balance NSE (25%) and Bitcoin (35%) for growth (0.035% and 0.110%), retaining bonds (25%) for stability. This approach, outperforming a static strategy (e.g., Regime 2 drawdown -4.0% vs. -12.0%), optimizes returns while managing risk in Kenya's volatile market.

For policymakers, enhancing market resilience is key. The NSE's Regime 2 volatility (1.85%) suggests improving liquidity through incentives for domestic institutional investors and modernizing trading infrastructure. Gold's hedging role (0.045% in Regime 2) supports promoting commodity-linked instruments, while Bitcoin's volatility (5.90%) necessitates regulatory frameworks, such as trading guidelines and investor education, to harness its potential (35% in Regime 3) safely. Bonds' stability (0.31–0.42%) recommends maintaining competitive yields and accessible markets, anchoring confidence. Real-time monitoring tools, inspired by MSM, could detect regime shifts, enabling timely interventions to stabilize Kenya's financial system.

5.5 Limitations of the Study

The study faced several limitations. First, the reliance on secondary data from the NSE, LBMA, CoinMarketCap, and Central Bank of Kenya assumes accuracy, yet minor gaps (<2%) were interpolated, potentially introducing slight bias. Second, the MSM's three-regime structure, while statistically optimal (BIC 5,305 for NSE), may oversimplify complex market transitions, as a two-regime model (BIC 5,410) was less distinct but simpler. Third, copula models assumed GARCH-filtered residuals captured all volatility dynamics, yet non-normal residuals (JB $p < 0.01$) suggest unmodeled tail risks, particularly for Bitcoin (kurtosis 6.20). Fourth, the portfolio optimization used a fixed risk aversion parameter (γ=3), which may not reflect all investor preferences in Kenya's diverse market. Finally, the focus on four assets excludes other classes (e.g., real estate), limiting generalizability within Kenya's financial landscape.

5.6 Areas of Further Study

Future research could address these limitations. Investigating additional assets, such as real estate or foreign equities, could broaden portfolio options for Kenyan investors. Extending the MSM to dynamic regime numbers or incorporating macroeconomic variables (e.g., inflation) might refine regime detection. Exploring advanced copulas (e.g., vine copulas) could capture higher-dimensional dependencies, improving tail risk modeling. Sensitivity analyses varying risk aversion in portfolio optimization could tailor strategies to diverse investor profiles. Finally, a comparative study with other African markets (e.g., Nigeria) could contextualize Kenya's findings, enhancing regional financial insights.

References

Adham, S. A., Abd Elaal, M. K., & Malaka, H. M. (2016). Gaussian copula regression application. In *Journal International Mathematical Forum* (Vol. 11, pp. 1053-1065).

Agbloyor, E. K., Abor, J., Adjasi, C. K., & Yawson, A. (2014). Private capital flows and economic growth in Africa: The role of domestic financial markets. *Journal of International Financial Markets, Institutions, and Money, 30*, 137–152.

Aielli, G. P. (2013). Dynamic conditional correlation: on properties and estimation. *Journal of Business & Economic Statistics*, *31*(3), 282-299.

Akhtaruzzaman, M., Boubaker, S., & Sensoy, A. (2021). Financial contagion during COVID–19 crisis. *Finance Research Letters, 38*, 101604. https://doi.org/10.1016/j.frl.2020.101604

Banyen, T. (2022). Behavioral Drivers of Stock Market Participation: Insights from Ghanaian Investors. *Journal of Business and Economic Options*, *5*(2), 1-13.

Bekaert, G., & Harvey, C. R. (2002). Research in emerging markets finance: Looking to the future. Emerging Markets Review, 3(4), 429–448. https://doi.org/10.1016/S1566-0141(02)00044-8

Baur, D. G., & McDermott, T. K. (2010). Is gold a safe haven? International evidence. *Journal of Banking & Finance*, *34*(8), 1886-1898.

Baur, D. G., & Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. Financial Review, 45(2), 217–229. https://doi.org/10.1111/j.1540-6288.2010.00244.x

Beckmann, J., Berger, T., & Czudaj, R. (2015). Does gold act as a hedge or a safe haven for stocks? A smooth transition approach. *Economic Modelling*, *48*, 16-24.

Bose, R., Nagarkar, J., Malik, S., & Bharti, N. (2024). Unveiling the interlinkage between Ethereum and Nifty indices: impact of cryptocurrency on Indian equity markets post Covid-19. *Cogent Economics & Finance*, *12*(1), 2359599.

Boudt, K., Danielsson, J., & Laurent, S. (2013). Robust forecasting of dynamic conditional correlation GARCH models. *International Journal of Forecasting*, *29*(2), 244-257.

Brauneis, A., & Mestel, R. (2019). Cryptocurrency-portfolios in a mean-variance framework. *Finance Research Letters*, *28*, 259-264.

Cermak, V. (2017). Can bitcoin become a viable alternative to fiat currencies? An empirical analysis of bitcoin's volatility based on a GARCH model. *An Empirical Analysis of Bitcoin's Volatility Based on a GARCH Model (May 2, 2017)*.

Chatfield, C. (2003). The analysis of time series: An introduction (6th ed.). Chapman and Hall/CRC.

Chu, J., Chan, S., & Zhang, Y. (2021). Bitcoin versus high-performance technology stocks in diversifying against global stock market indices. *Physica A: Statistical Mechanics and its Applications*, *580*, 126161.

Clarke, R., De Silva, H., & Thorley, S. (2011). Minimum-variance portfolio composition. *Journal of Portfolio Management*, *37*(2), 31.

Engle, R. F., & Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. Econometric Theory, 11(1), 122–150. https://doi.org/10.1017/S0266466600009063

Gupta, S. K., Fegade, A., Padhye, P., Mohture, A., Kalshetti, P., & Murthy, Y. S. (2024). Evaluating the effect of covid-19 on major world indices: in relation to gold, silver, crude oil, and bitcoin. *Academy of Marketing Studies Journal*, *28*(2).

He, Z., O'Connor, F., & Thijssen, J. (2018). Is gold a Sometime Safe Haven or an Always Hedge for equity investors? A Markov-Switching CAPM approach for US and UK stock indices. *International Review of Financial Analysis*, *60*, 30-37.

Hearn, B., Piesse, J., & Strange, R. (2010). The role of the stock market in the provision of Islamic development finance: Evidence from Sudan. *Emerging Markets Review, 11*(2), 141–153. https://doi.org/10.1016/j.ememar.2010.02.002

Henriques, I., & Sadorsky, P. (2018). Can bitcoin replace gold in an investment portfolio?. *Journal of Risk and Financial Management*, *11*(3), 48.

Hileman, G., & Rauchs, M. (2017). 2017 global cryptocurrency benchmarking study. *Available at SSRN 2965436*.

Ibe, R., & Pibowei, B. (2022). Liquidity and volatility in African stock exchanges: An empirical analysis. *African Journal of Finance, 24*(3), 205–222.

Ibe, U. G., & Pibowei, W. E. (2022). The Effect of Financial Leverage on Corporate Financial Performance of Dangote Cement PLC (2010-2021). *Available at SSRN 3874639*.

Jana, S., & Sahu, T. N. (2023). Can diversification be improved by using cryptocurrencies? Evidence from Indian equity market. *Journal of Financial Economic Policy*, *15*(6), 551-573.

Kumar, A. S. (2020). Testing safe haven property of bitcoin and gold during covid-19: evidence from multivariate GARCH analysis. *Economics Bulletin*, *40*(3), 2005-2015.

Mathis, C. (2017). Three Trends in Middle Market Private Equity. *The Journal of Private Equity*, 16-18.

Sehgal, S., & Singh, J. (2024). Dynamic Interrelationships among Bitcoin, Bonds, and Sectoral Indices in India: Evidence from Pre-and Post-COVID-19. *Financial Internet Quarterly*, *20*(3), 42-57.

Thampanya, N., Nasir, M. A., & Huynh, T. L. D. (2020). Asymmetric correlation and hedging effectiveness of gold & cryptocurrencies: From pre-industrial to the 4th industrial revolution[2730?]. *Technological Forecasting and Social Change*, *159*, 120195.

Thampanya, N., Nasir, M. A., & Huynh, T. L. D. (2020). Hedging cryptocurrencies with gold and the role of economic policy uncertainty. *Applied Economics Letters, 27*(9), 804–808. https://doi.org/10.1080/13504851.2019.1646873

Tronzano, M. (2020). Safe-haven assets, financial crises, and macroeconomic variables: Evidence from the last two decades (2000–2018). *Journal of risk and financial management*, *13*(3), 40.

Valarmathi, A., Umamaheswari, S., & Lakshmi, M. R. (2023). A Study on Dynamic Relationship between Gold Price and Stock Market Price in India. *Journal of Survey in Fisheries Sciences*, *10*(3S), 6529-6539.

Appendices

Appendix A: Similarity Report

12% Overall Similarity

The combined total of all matches, including overlapping sources, for each database.

Filtered from the Report

- Bibliography
- Quoted Text

Match Groups

159 Not Cited or Quoted 11%
Matches with neither in-text citation nor quotation marks

20 Missing Quotations 1%
Matches that are still very similar to source material

0 Missing Citation 0%
Matches that have quotation marks, but no in-text citation

0 Cited and Quoted 0%
Matches with in-text citation present, but no quotation marks

Top Sources

9%	Internet sources
7%	Publications
10%	Submitted works (Student Papers)

Integrity Flags

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Our system's algorithms look deeply at a document for any inconsistencies that would set it apart from a normal submission. If we notice something strange, we flag it for you to review.

A Flag is not necessarily an indicator of a problem. However, we'd recommend you focus your attention there for further review.

Match Groups

- **159** Not Cited or Quoted 11%
  Matches with neither in-text citation nor quotation marks
- **20** Missing Quotations 1%
  Matches that are still very similar to source material
- **0** Missing Citation 0%
  Matches that have quotation marks, but no in-text citation
- **0** Cited and Quoted 0%
  Matches with in-text citation present, but no quotation marks

Top Sources

9% Internet sources
7% Publications
10% Submitted works (Student Papers)

Top Sources

The sources with the highest number of matches within the submission. Overlapping sources will not be displayed.

1	Internet	su-plus.strathmore.edu	2%
2	Internet	erepository.uonbi.ac.ke	1%
3	Internet	www.coursehero.com	<1%
4	Internet	erepository.uonbi.ac.ke:8080	<1%
5	Internet	link.springer.com	<1%
6	Internet	mpra.ub.uni-muenchen.de	<1%
7	Submitted works	WorldQuant University on 2025-04-08	<1%
8	Submitted works	Kumasi Polytechnic on 2025-03-03	<1%
9	Submitted works	Technological University Dublin on 2025-03-23	<1%
10	Internet	www.mdpi.com	<1%

Appendix B: Ethical Clearance Confirmation

Strathmore
UNIVERSITY

4th April 2025

Ms Mwakio Yvonne,
yvonne.mwakio@strathmore.edu

Dear Ms Mwakio,

RE: Portfolio Optimization and Asset Dependency Analysis Across Investment Regimes and Select Asset Classes in Kenya

This is to inform you that SU-ISERC has reviewed and **approved** your above **SU-masters** proposal. Your application reference number is **SU-ISERC2769/25.** The approval period is from **4th April 2025 to 3rd April 2026.**

This approval is subject to compliance with the following requirements:

i. Only approved documents including (informed consents, study instruments, MTA) will be used.
ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by SU-ISERC.
iii. Death and life-threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to SU-ISERC within 72 hours of notification.
iv. Any changes anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to SU-ISERC within 72 hours.
v. Clearance for the export of biological specimens must be obtained from relevant institutions.
vi. Submission of a request for renewal of approval at least 60 days prior to the expiry of the approval period. Attach a comprehensive progress report to support the renewal.
vii. Submission of an executive summary report within 90 days of completion of the study to SU-ISERC.

Before commencing your study, you will be expected to obtain a research license from National Commission for Science, Technology, and Innovation (NACOSTI) https://research-portal.nacosti.go.ke/ and obtain other clearances needed.

Yours sincerely,

Mr Ambrose Rachier,
Chairperson; SU-ISERC

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Appendix C: Data Collection Form

Section 1: Asset Information

Asset Type	Asset Name	Market Source	Ticker/Symbol	Data Frequency (Daily/Weekly/Monthly)
Stock				
Gold				
Bitcoin				
Government Bonds				

Section 2: Date and Time Information

Date	Day of Week	Time (if applicable)	Trading Volume	Asset Type (Stock/Gold/Bitcoin/Bond)
(MM/DD/YYYY)				
(MM/DD/YYYY)				

Section 3: Financial Data (Prices, Returns, etc.)

Date	Opening Price (USD)	Closing Price (USD)	Daily High (USD)	Daily Low (USD)	Return (%)	Volatility Measure	Notes
(MM/DD/YYYY)							
(MM/DD/YYYY)							