Strathmore
UNIVERSITY

Strathmore Institute of Mathematical Sciences (SIMS)
End of Semester Examination for the Degree of Bachelor of Business Science in Financial Economics/Financial Engineering
BSF 4130: Foundations of Asset Pricing

DATE:10th September 2021 **Time: 2 Hours**

Instructions

- **This examination consists of FIVE questions.**
- **Answer Question ONE (COMPULSORY) and any other TWO questions.**

1. (a) Why has it seemed reasonable, from the standpoint of financial theory, that stock prices are approximately a Random Walk? Does the theory suggest a Random Walk is more likely than an AR(1) (First-Order Autoregressive) process? Explain **(4 Marks)**

   (b) Ferson, Sarkissian, & Simin (2003) demonstrate that most empirical studies on asset returns' predictability suffers from **data mining** and **spurious regression bias**. Provide a brief contextual discussion of the concepts of data mining and spurious regression bias. **(6 Marks)**

   (c) "*Inference about the risk premia is equivalent to inference about the stochastic discount factor (SDF)*". Briefly explain if you agree with this statement (Hint: Mathematically show the equivalence).**(6 Marks)**

   (d) Briefly describe the following phenomenon as used in asset pricing

      (i) Momentum effect **(3 Marks)**

      (ii) Size effect **(3 Marks)**

      (iii) Value premium puzzle **(4 Marks)**

   (e) Theory specifies the capital asset pricing model (CAPM) as follows;

   $$E[R_i^e] = \beta_i[E[R_m^e]] \tag{1}$$

   where R_i^e is the excess return (over risk free rate) of asset i and R_m^e is the excess market return. Thus, theory specifies CAPM in expectational form and not in actual data. Briefly explain the two assumptions researchers make to allow empirical estimation of model (1) **(4 Marks)**

2. Suppose a typical investor solves the following problem

$$\max_{\alpha} U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})]$$
$$\text{subject to } c_t = e_t - \alpha p_t \text{ and } c_{t+1} = e_{t+1} + \alpha x_{t+1} \tag{2}$$

where c_t and c_{t+1} denotes consumption at date t and $t+1$ respectively. The other parameters in the model are; e_t (investor's endowment at period t and $t+1$ respectively), p_t (asset price at time t), α (the number of units of the asset purchased by the investor), and x_{t+1} (payoff of the asset at period $t+1$). The general functional form $U(.)$ represents the investor's utility function. An often convenient utility function used in many applications is the power utility $u(c_t) = \frac{1}{1-\gamma}c^{1-\gamma}$

(i) Derive the basic pricing equation from model (3) **(4 Marks)**

(ii) Given the basic pricing equation derived in (i) and assuming a power utility function given in model (3) demonstrate that real interest rates in the economy will be high when consumption growth is high. **(6 Marks)**

(iii) Further, use the basic pricing equation to demonstrate that, assets that pay off when times are good and consumption levels are high are less desirable than those that pay off an equivalent amount when times are bad. **(5 Marks)**

(iv) Finally, prove that given the basic pricing equation (derived in (i)), the return premium on an asset cannot exceed the product of the price of risk and the quantity of risk (for all assets priced by the stochastic discount factor) **(5 Marks)**

3. (a) Suppose w_0 denotes the amount invested at the beginning of the period, and θ_i denotes the number of shares the investor chooses to hold of asset i. Further assume that the investor has some random endowment $\tilde{y}$ at the end of the period (for example, labor income), which he consumes in addition to the end-of-period portfolio value. Letting n denote the number of assets, the investor's choice problem is:

$$\underset{\theta_i}{\text{maximize}} \quad E[u(\tilde{w})]$$
$$\text{subject to} \quad \sum_{i=1}^{n} \theta_i p_i = w_0 \qquad \text{and} \quad \tilde{w} = \tilde{y} + \sum_{i=1}^{n} \theta_i \tilde{x}_i \; i = 1, \ldots, n.$$

(i) Form the Lagrangean and solve for the first order condition with respect to θ_i . Simplify the first order condition to obtain the price of asset i (p_i) **(6 Marks)**

(i) Assuming $p_i \neq 0$, write the simplified FOC (in (ii)) in terms of the gross return of asset i ($\tilde{R}_i$), rather than price: **(3 Marks)**

(ii) Now assuming that there is another risky asset j with $p_j \neq 0$, show that the marginal value of adding a zero-cost portfolio to the optimal portfolio is zero. **(5 Marks)**

(b) The paper by Fama and MacBeth (1973) is important in empirical asset pricing, much because of their methodological innovation. State the specific empirical model tested in this paper and provide a brief description of the hypotheses tested. **(6 Marks)**

4. (a) (i) Briefly explain the concept of Complete markets and why they are important in an economy. **(4 Marks)**

   (ii) Briefly explain the main advantage of the Arrow-Debreu pricing over the capital asset pricing model **(3 Marks)**

   (b) Consider an economic environment in which there are two periods $t = 0$ (today) and $t = 1$ (next year), and two possible states $t = 1$: a good state that occurs with probability $\pi = 1/2$ and a bad state that occurs with probability $1 - \pi = 1/2$.

   Suppose, initially, that two assets trade in this economy. A risky stock for $q^s = 3$ at $t = 0$, with payoffs; $P^G = 4$ in the good state at $t = 1$, and $P^B = 3$ in the bad state at $t = 1$. And a risk-free bond sells $q^b = 0.90$ at $t = 0$ and pays off 1 unit of consumption in both states at $t = 1$.

   (i) First, find the number of shares s and the number of bonds b that an investor would have to buy or sell short in order to replicate the payoffs from a contigent claim for the good state (that is, construct a "synthetic" contigent claim for the good state). Then use your answer to compute the price q^G at $t = 0$ of the contigent claim for the good state implied by the assumption that there are no arbitrage opportunities across the markets for the stock, bond, and contigent claim. **(4 Marks)**

   (ii) Next, find the number of shares s and the number of bonds b that an investor would have to buy or sell short in order to replicate the payoffs from a contigent claim for the bad state (that is, construct a "synthetic" contigent claim for the bad state). Then use your answer to compute the price q^G at $t = 0$ of the contigent claim for the bad state implied by the assumption that there are no arbitrage opportunities across the markets for the stock, bond, and contigent claim. **(4 Marks)**

   (iii) Finally, find the number of shares s and the number of bonds b that an investor would have to buy or sell short in order to replicate the payoff from a call option that gives the holder the right, but not the obligation, to buy a share of stock at strike price $K = 3.50$ at $t = 1$. Then, use your answer to compute the price q^0 at $t = 0$ of the option implied by the assumption that there are no arbitage opportunities across the markets for the stock, bond and option. **(5 Marks)**

5. In economics and finance, a puzzle refers to the inability of standard intertemporal economic models to rationalize the statistics that characterize financial markets. In this context, discuss the following puzzles (**Hint:** Illustrate the puzzle by data, graphs or mathematical forms. Further, highlight the various attempt(s) to solve the puzzle)

   (i) Volatility puzzle (Grossman & Shiller, 1981) **(8 Marks)**

   (ii) Equity premium puzzle (Mehra and Prescott, 1985) **(12 Marks)**

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