Estimating Value-at-Risk Using CrashMetrics

Otao, Calvin Mochoge

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

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This Research Project has been submitted for examination with my approval as the Supervisor.

[Name of Supervisor]  [Signature]  [Date]

Strathmore Institute of Mathematical Sciences

Strathmore University
Abstract

The purpose of this study is to estimate Value-at-Risk using CrashMetrics, a methodology proposed by (Wilmott, 2006) while applying them in the Kenyan context. We compare the results from the Beta Parametric Value-at-Risk, a methodology for estimating Value-at-Risk, proposed by (Sharpe, 1964). CrashMetrics should according to (Wilmott, 2006), produce a higher loss scenario than Value-at-Risk. The dataset used includes daily returns of listed banks' stock and Nairobi All Share Index for the period beginning 2nd January 2015 to 31st December 2017. The results show that CrashMetrics indeed outperforms Value-at-Risk during periods of markets stress by providing higher value for portfolio loss than the Beta Parametric Value-at-Risk.
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1. Introduction

1.1. Background

Risk management is at the core of managing any financial organization since it determines the success or failure of any organization according to (Coleman, 2011). It is the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability or impact of unfortunate events as defined by (Hubbard, 2009). It is different from risk measurement in that, risk measurement constitutes part of the risk management process. Risk measurement is the specialized task of quantifying and communicating risk as defined by (Coleman, 2011). The most widespread risk measure is value-at-risk (VaR) used especially by banks as stated in a report by (Mehta, Neukirchen, Pfetsch, & Poppensieker, 2012).

Value-at-Risk (VaR) is a single, summary statistical measure of possible portfolio losses resulting from "normal" market movements with losses greater than the VaR being suffered only with a specified small probability as defined by (Linsmeier & Pearson, 2000).

VaR was developed in response to financial disasters of the 1980s and 1990s with the most notable being "Black Monday"¹ and obtained an increasingly important role in market risk management. However, (Dowd, 2014) suggests that its roots go way back when (Baumol, 1968) suggested a risk measure equal to \( \mu - k\sigma \), where \( \mu \) and \( \sigma \) are the mean and standard deviation of the distribution concerned, and \( k \) is a subjective parameter that reflects the user's attitude to risk.

---

¹ Black Monday refers to Monday, October 19, 1987, when stock markets around the world crashed, shedding a huge value in a very short time. The crash began in Hong Kong and spread west to Europe, hitting the United States after other markets had already declined by a significant margin. The Dow Jones Industrial Average (DJIA) fell exactly 508 points to 1,738.74 (22.61%)
JPMorgan’s RiskMetrics, which was published in 1994, led to an increased use of VaR as a measure of market risk due to the merits that VaR has.

VaR is beneficial since it is simple and easy to understand (Dowd, The Risk Measurement Revolution, 2005). One of the demerits is that when calculating VaR, one often assumes that returns on financial assets are normally distributed. Such a choice of modeling does not take into account the presence of fat tails in financial time series data. This has led to inadequate approximation when dealing with the presence of extreme price changes such as during the crisis of 2008/2009 as stated by (Oaneaa & Anghelacheb, 2015). However, during normal times, the model had performed well as shown by (Basak & Shapiro, 2001). Another demerit is that VaR is not sub-additive since it does not reduce as a portfolio is diversified and is consequently not a coherent risk measure (Artzner, Freddy, & Heath, 1999). Secondly, it does not fully represent the magnitudes of losses that exist beyond itself since it is just a percentile of a probability distribution (Zhu & Fukushima, 2005).

VaR makes assumptions that market conditions are normal and that assets are jointly normally distributed do not hold during extreme market events. Panic selling and flight-to-quality causes stock prices to fall and volatility to spike causing underlying assumptions under the VaR to collapse according to (Hua & Wilmott, 1996). This together with illiquidity in the market, makes dynamic hedging\(^2\) an unviable solution. Thus in the attempt to find a suitable method for estimating changes in portfolio value during extreme market conditions, (Hua & Wilmott, 1996) developed the CrashMetrics methodology.

\(^2\) Dynamic hedging means that the number of assets held must be continuously changed to maintain a delta-neutral position.
1.1.1. What is CrashMetrics?

CrashMetrics is a dataset and methodology for estimating the exposure of a portfolio to extreme market movements or crashes as presented by (Wilmott, 2006). It assumes that the crash is unhedgeable and then finds the worst outcome for the value of the portfolio. In calculating the CrashMetrics value, one needs to run a regression of asset price changes on index price changes. The dataset used consists of the upper and lower $n$th quantiles of the index price changes with the quantiles being determined by the desired confidence level. The coefficients explaining the relationship between the two variables are known as Crash Coefficients. By taking the upper and lower $n$th quantiles of the index price changes, one can capture the characteristics of a market crisis which are different from normal market conditions.

Normal market conditions refer to periods in which the value of portfolio will exhibit fluctuations that are rapid, but not dramatic (Zarpellon & Banca, 2014). However, there can be certain macroeconomic events that can make the players in the financial markets overreact either positively or negatively. These events may be political, economic or regulatory events. These situations are known as crashes, or crises, and are very rare. They moreover cannot be predicted nor the magnitude determined with certainty.

Crash conditions are different from normal market conditions because a crash can lead to sudden fall in market prices and gives limited chances for the portfolio to be liquidated. Moreover, during crashes there is an increase in volatility as well as a fall in asset value for all assets despite their standard correlations. This is the main feature of Crash Coefficients according to (Wilmott, 2006).
1.2. Problem Statement

Risk models are generally not designed to capture risks associated with crises and help companies manage them as argued by (Stulz, 2008). VaR, which is based on normal market conditions fails short in the event that a market crash occurs (Aymanns, Caccio, Farmer, & Tan, 2016). Ironically, it is at that moment that any risk measure should prove robust and therefore provide reliable hedging strategies. Use of VaR may lead to a gross underestimation of losses that a business stands to lose in the event that a crash occurs.

This may be a problem as seen in the financial crisis of 2008/2009 as shown by (Aymanns, Caccio, Farmer, & Tan, 2016). In the attempt to improve on VaR, we use the CrashMetrics methodology to estimate a portfolio change in the event that a market crash occurs. The results are compared to those of Beta Parametric VaR. CrashMetrics methodology should be able to provide higher loss scenarios and therefore prompt an institution to insure itself against such an occurrence.

1.3. Research Objectives

This study is aimed at evaluating a robust method of measuring the possible losses a portfolio may incur in event that a crash occurs. The major areas of concern are to:

i. To compute the Value-at-Risk using CrashMetrics methodology

ii. Compare the results from the CrashMetrics methodology to those of the Beta Parametric VaR
1.4. Research Questions

The question we seek to answer is

i. How is the CrashMetrics Methodology used in estimating Value-at-Risk?

ii. How does CrashMetrics perform as a method of estimating portfolio change compared to Beta Parametric VaR?

1.5. Significance of the Study

This study would be of value to scholars. This study will increase knowledge in risk management by contributing to a relatively under-researched alternative risk measure which is CrashMetrics.

This study would also provide regulators with a risk measure which is robust in the presence of a market crash. Many entities can adopt CrashMetrics alongside risk measures such as VaR and CoVaR\(^3\) to strengthen their risk measurements.

It is also useful from an internal risk management standpoint, providing institutions with valuable information regarding their exposures at the tail of the distribution (Gomez, Mendoza, & Nancy, 2012).

---

\(^3\) Conditional value at risk (CoVaR) performed by assessing the likelihood (at a specific confidence level) that a specific loss will exceed the value at risk.
2. Literature Review

2.1. Introduction

This chapter looks at theoretical and empirical literature on CrashMetrics. CrashMetrics is a relatively under researched area with minimal amount of literature surrounding it. Various scholars have put forward arguments promoting the use of CrashMetrics in providing a higher loss scenario.

2.2. Theoretical Review

(Artzner, Freddy, & Heath, 1999) studied both market risks and nonmarket risks, without complete markets assumption, and discussed methods of measurement of these risks. They presented and justified a set of four desirable properties for measures of risk, and called the measures satisfying these properties “coherent.” They examined the measures of risk provided and the related actions required by the SEC⁴ and FINRA⁵ rules, and by quantile-based methods. They demonstrated the universality of scenario-based methods for providing coherent measures. They offered suggestions concerning the SEC method. They also suggested a method to repair the failure of subadditivity of quantile-based methods such as VaR.

(Hua & Wilmott, 1996) presented a new model for pricing and hedging a portfolio of derivatives taking into account the effect of an extreme movement in the underlying. They made no assumptions about the timing of this ‘crash’, the probability distribution or size. They however put an upper bound on the latter.

⁴ Securities and Exchange Commission (SEC) protects investors and maintain the integrity of the securities markets, both exchange and OTC (Over-the-Counter) markets.

⁵ FINRA (Financial Industry Regulatory Authority) (formerly NASD (National Association of Securities Dealers) is the largest self-regulatory organization (SRO) in the securities industry in the United States. An SRO is a membership-based organization that creates and enforces rules for members based on the federal securities laws. FINRA is overseen by the SEC.

http://www.investopedia.com/ask/answers/112.asp#ixzz4lbRDrW00
The pricing and hedging followed from the assumption that the worst scenario actually happens i.e. the size and time of the crash are such as to give the option its worst value. The optimal static hedge followed from the desire to make the best of this worst value. There are many applications for this crash modelling, but they focused on using the model to evaluate the Value at Risk for a portfolio of options.

(Wilmott, 2006) presented the CrashMetrics methodology for one stock and the multi index model. The methodology is based on VaR and is specifically designed for the analysis of and protection against market crashes. If a portfolio consists of a single underlying equity and its derivatives, then the change in its value during a crash is represented by the option-pricing formulae for all of the derivatives and equity in the portfolio i.e. the change in portfolio value is given by summation of all the individual contracts in the portfolio. An assumption is that we are not too near to the expiries and strikes of the options which enables one to approximate the portfolio change in value by the Taylor series in the change in the underlying asset. In practice however, this will not be a good enough approximation. For a multi asset model, CrashMetrics handles underlyings using an index or benchmark. One can measure the performance of a portfolio of assets and options on these assets by relating the magnitude of extreme movements in any one asset to one or more benchmarks such as the S&P 500. The relative magnitude of these movements is measured by the Crash Coefficient for each asset relative to the benchmark. Unlike the RiskMetrics\textsuperscript{6} and CreditMetrics\textsuperscript{7} datasets, the CrashMetrics dataset does not have to be updated frequently because of the rarity

\textsuperscript{6} RiskMetrics methodology for calculating the VaR assumes that a portfolio or investment's returns follow a normal distribution. The method uses the Variance-Covariance method to calculate VAR.

\textsuperscript{7} CreditMetrics is a tool for assessing portfolio risk due to changes in debt value caused by changes in obligor credit quality.
of extreme market movements. CrashMetrics is very robust because it does not use unstable parameters such as volatilities or correlations.

Moreover, it does not rely on probabilities, but instead considers worst cases. CrashMetrics is a good risk tool because it is very simple and fast to implement and can be used to optimize portfolio insurance against market crashes.

2.2.1. Criticisms against CrashMetrics methodology

(Dowd, 2002) states that CrashMetrics is open to criticism on the grounds that it relies heavily on the Greek approximations in circumstances where those approximations are not likely to be good. Nonetheless, the basic idea of identifying worst-case outcomes and then evaluating their liquidity consequences is a good one and can be implemented in other ways as well. For example, he suggested that one might identify the worst-case outcome as the expected outcome at a chosen confidence level using extreme value methods. Yet, still however, he suggests the method is still rather simplistic, and with complicated risk factors such as often arise with credit-related risks, one might want a more sophisticated model that was able to take account of the complications involved.

2.3. Empirical Review

(Gencay & Selcuk, 2004) investigated the relative performance of Value-at-Risk (VaR) models with the daily stock market returns of nine different emerging markets. In addition to well-known modeling approaches, such as Variance-covariance method and historical simulation, they studied the extreme value theory (EVT) to generate VaR estimates and provide the tail forecasts of daily returns at the 0.999 percentile along with 95% confidence intervals for stress testing purposes. The results indicated that EVT based VaR estimates are more accurate at higher quantiles.
(Gomez, Mendoza, & Nancy, 2012) using daily data, found that CrashMetrics complements more traditional stress testing techniques, providing not only a stringent loss scenario, but one that is cemented on an observed market shock and on the estimated sensitivities of the change in portfolio value during periods of financial turmoil. Given that correlations between assets are stronger during a market crash, their findings indicated that financial institutions seem relatively more exposed to market risk under this methodology than using other market risk measures. Thus, results indicate that CrashMetrics provides vital information from a prudential perspective, alerting policymakers of significant individual or sector-specific exposures to market risk and thus, allowing preemptive action to be undertaken in a timely and efficient manner.

(Zarpellon & Banca, 2014) applied Wilmott’s idea of Crash Coefficients in order to improve standard parametric VaR estimations. The idea was basically to leave out of their analysis all the ordinary market movements and focus on extreme movements. This could seem unnatural because it may appear as they were wasting information, but were just aiming the observations that really matter in risk management. They provided some fundamental statistics about index and stocks, which they used in their considerations later on. They afterwards substituted Betas, in standard parametric VaR, with Crash Coefficients, in order to obtain an easier and more reliable VaR assessments.

2.4. Summary of Literature Review

Various literature on the use of CrashMetrics suggest that it indeed does provide stringent loss scenarios in the event that a crash occurs. It is therefore a more robust method in estimating portfolio changes under extreme market events.
However, (Dowd, 2002) states that CrashMetrics is still relatively too simple for complicated risk factors such as often arise with credit-related risks.
3. Research Methodology

3.1. Introduction

This chapter describes the procedures and methodologies that will be used in conducting the study to arrive at conclusions regarding the effectiveness of CrashMetrics in estimating changes in portfolio values during extreme market conditions.

3.2. Research Design

The research design to be used is a comparative research design. This study will evaluate and compare results from VaR and CrashMetrics which both examine the changes of portfolio value given changes in the portfolio components. The two will be calculated using Betas for VaR and Crash Coefficients for CrashMetrics.

3.3. Data Collection

In carrying out our exercises, we will use daily data of stock and index returns for the period beginning 2nd January 2015 to 31st December 2017, for a total of 566 observations. The equity portfolio will be constructed using Markowitz Portfolio Theory as presented by (Markowitz, 1952). Any method can be chosen since the highlight is not the allocation of weights in the portfolio but rather the change in value of the portfolio given weights. We will generate the weights of the portfolio in each of the invested stocks. The NASI in our case will be the benchmark. The will be used as the benchmark in calculating Beta VaR for our equity portfolio since it contains all stocks on the NSE. We will apply the CrashMetrics methodology for the generated portfolio.
3.4. Data Analysis

The methodology presented below is as used by (Wilmott, 2006). It is the only methodology available on CrashMetrics. However, it will be adjusted to accommodate a portfolio of stocks. We seek to calculate the variation in the value of a portfolio given a significant change in assets invested in \((\text{denoted by } U_i \forall i = 1, ..., N)\). We know that there exists a relationship between the change in the value of the portfolio and the \(U_i's\) which can formally be expressed as:

\[
\Delta \pi = F(\Delta U_1, \Delta U_2, ..., \Delta U_N)
\]

Where \(\Delta \pi\) is the absolute change in the value of the portfolio and the \(U_i's\) are the absolute changes in the underlying assets. The function \(F(.)\) generically represents the sum of all the formulae for each of the contracts in the portfolio. Defining \(U_i's\) given that each entity's portfolio will have many assets is of great relevance.

3.4.1.1. The Multi-Asset/Single Index CrashMetrics Model

In this model by (Wilmott, 2006) we can link all extreme movements in any one asset in a portfolio to a single index or a benchmark. The magnitude of such movements relative to the benchmark are captured by the Crash Coefficient of each asset denoted by \(k_i\). Thus if the benchmark moves by \(x\%\) then the \(i^{th}\) asset will move by \(k_i * x\%\) where \(x\%\) is the percentage change in the benchmark and \(k_i\) denotes the Crash Coefficient.

In estimating such a coefficient, we are only interested in relating extreme movements in the series and so in calculating \(k_i\) we only utilize the largest rises and falls in the benchmark index.

CrashMetrics hence does not require constant updating, since it only utilizes extreme market movements in the data which are by nature rare events.
The index is not required to be constructed from the underlying assets in the portfolio. It can be any representative quantity in the market.

Once our crash co-efficients are estimated, we need to define $F(\Delta U_1, \Delta U_2, \ldots, \Delta U_N)$ explicitly so as to relate the returns in the benchmark, denoted by $X$, with the change in the value of the portfolio. We therefore need a function such that:

$$F(\Delta U_i) = F(\Delta i kX)$$  \hspace{1cm} (2)

And so the change in value of the portfolio can be expressed as:

$$\Delta \pi = F(k1.X, k2.X, kN .X)$$  \hspace{1cm} (3)

For characterizing the functional form of $F(.)$, we relate the returns of the benchmark with the returns on the assets through the Crash Coefficient which is estimated by running an OLS regression where the returns on the $i^{th}$ asset are the dependent variable and the returns on the index are the independent variable. The OLS regression methodology is discussed below.

3.4.1.2. Ordinary Least Squares Regression

In a linear regression framework, we assume some output variable $y$ is a linear combination of some independent input variables $X$ plus some independent noise $\epsilon$. The way the independent variables are combined is defined by a parameter vector $\beta$:

$$y = X\beta + \epsilon$$  \hspace{1cm} (4)

We also assume that the noise term $\epsilon$ is drawn from a standard Normal distribution:

$$\epsilon \sim N(0, 1)$$
For some estimate of the model parameters \( \hat{\beta} \), the model’s prediction errors/residuals \( \varepsilon \) are the difference between the model prediction and the observed output values:

\[
\varepsilon = y - X\hat{\beta}
\]  

(5)

The Ordinary Least Squares (OLS) solution to the problem (i.e. determining an optimal solution for \( \hat{\beta} \)), involves minimizing the sum of the squared errors with respect to the model parameters, \( \hat{\beta} \). The sum of squared errors is equal to the inner product of the residuals vector with itself \( \sum \varepsilon_i^2 = \varepsilon^T \varepsilon \):

\[
\varepsilon^T \varepsilon = (y - X\hat{\beta})^T (y - X\hat{\beta})
\]  

(6)

\[
\varepsilon^T \varepsilon = y^T y - y^T (X\hat{\beta}) - (X\hat{\beta})^T y + (X\hat{\beta})^T (X\hat{\beta})
\]  

(7)

\[
\varepsilon^T \varepsilon = y^T y - (X\hat{\beta})^T y - (X\hat{\beta})^T y + (X\hat{\beta})^T (X\hat{\beta})
\]  

(8)

\[
\varepsilon^T \varepsilon = y^T y - 2(X\hat{\beta})^T y + (X\hat{\beta})^T (X\hat{\beta})
\]  

(9)

\[
\varepsilon^T \varepsilon = y^T y - 2(X\hat{\beta})^T y + \beta^T X^T X \hat{\beta}
\]  

(10)

To determine the parameters, \( \hat{\beta} \), we minimize the sum of squared residuals with respect to the parameters:

\[
\frac{\partial}{\partial \beta} [\varepsilon^T \varepsilon] = 0
\]  

(11)

\[
0 = -2X^T Y + 2X^T X \hat{\beta}
\]  

(12)

\[
X^T Y = X^T X \hat{\beta}
\]  

(13)

Solving for \( \hat{\beta} \) gives the analytical solution to the Ordinary Least Squares problem.
3.4.1.3. Beta Value-at-Risk

To arrive at the way Crash Coefficients are use as substitutes for Betas in VaR, we need to look at how Beta VaR methodology is structured.

Parametric VaR is defined as

\[ \text{Var}_i = \text{VM}_i \cdot |z_\alpha| \cdot \sigma_i \]  \hspace{1cm} (15)

Where:

\( \text{VM}_i \) = Market value of the position in asset i, and then we suppose it is equal to 1 to obtain VaR in percentage;

\( \sigma_i \) = estimated volatility for the assets i

\( z_\alpha \) = scalar factor; the scalar factor defines which kind of parametric VaR we have, so with normal distribution of returns, we have 1.96 at level of significance, \( \alpha = 5\% \), to obtain VaR at 95%

Applying the idea of Beta, as discussed in Section 3.1.4.1. We move from Parametric VaR to Beta (Parametric) VaR:

\[ \text{Var}_i = \text{VM}_i \cdot |z_\alpha| \cdot \beta_{ij} \cdot \sigma_j \]  \hspace{1cm} (16)

Where:

\( \beta_{ij} \) = Beta Coefficients estimated by OLS regression in Section 3.1.4.1.
3.4.1.4. CrashMetrics

Crash Coefficients are estimated by using OLS regression. The Crash Coefficients are obtained by regressing the returns for asset \( i \) on the returns of the index. However, one does not use full sample data. Instead, one sorts the data on returns for the assets \( i \) for all with respect to returns on the index from largest to smallest or smallest to largest. One then proceeds to run a regression on the \( n^{th} \) highest and lowest quantiles of the data depending on the level of significance.

The coefficients obtained from the regression are Crash Coefficients and are denoted by \( k_{ij} \).

Substituting Betas in the Beta VaR, gives us our Crash Coefficient VaR as shown below

\[
CCVaR_i = VM_t |z_a| k_{ij} \sigma_j
\]

Where:

\( k_{ij} = \text{estimated Crash Coefficients} \)

\( CCVaR_i = \text{Crash Coefficient VaR} \)
4. Data Analysis and Presentation of Results

4.1. Introduction

This chapter presents the empirical results of this study. The sections will be divided based on the tests carried out with the subsequent results obtained in the study in a bid to answer the specific research objective.

4.2. Preliminary VaR Analysis

4.2.1. Descriptive Statistics for NASI and select listed banks

In this section, some descriptive statistics regarding the NASI as well as the select listed banks are reported. It is going to be a short description as nothing about those statistics is particularly interesting, but those numbers are the basis for the calculation of the VaR, so they are necessary. NASI is Kenya’s All Share Index and is used as the benchmark for the calculation of betas as used in Beta VaR as well as calculation of Crash Coefficients as used in CrashMetrics.

The NASI has an average daily return of -0.04% and a daily standard deviation of 0.03% (volatility). The volatility is our main goal in parametric VaR computation. To compute a volatility longer than a day we multiply one day volatility by the square root of time. The returns of the NASI still have a quite symmetric distribution as well as a strong leptokurtosis, which means the distribution has a fat tail.

Of the select listed banks on the NSE, Barclay’s Bank stocks have the lowest daily mean return of -0.12% whereas Diamond Trust Bank stocks has the highest mean daily return of -0.05%. It is worth noting that NASI has a higher daily mean return than any of the listed bank’s stocks.
CFC Bank has the highest daily standard deviation of 2.34% whereas Kenya Commercial Bank has the lowest standard deviation of 1.72%. NASI returns have a lower daily standard deviation compared to the returns of any bank’s stock.

The stock returns also exhibit strong leptokurtosis with the returns of Cooperative stocks exhibiting the highest excess kurtosis\(^8\) of 20.98. The results for this section are shown in Table 1 below.

Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Assets</th>
<th>Mean Error</th>
<th>Standard Error</th>
<th>Sample Variance</th>
<th>Daily Standard Deviation</th>
<th>Kurtosis</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASI</td>
<td>-0.4%</td>
<td>0.03%</td>
<td>0.01%</td>
<td>0.76%</td>
<td>8.24</td>
<td>-0.78</td>
</tr>
<tr>
<td>BCBL</td>
<td>-0.12%</td>
<td>0.07%</td>
<td>0.02%</td>
<td>1.57%</td>
<td>7.78</td>
<td>-0.69</td>
</tr>
<tr>
<td>KN</td>
<td>-0.09%</td>
<td>0.10%</td>
<td>0.05%</td>
<td>2.34%</td>
<td>3.13</td>
<td>0.01</td>
</tr>
<tr>
<td>CFCB</td>
<td>-0.08%</td>
<td>0.07%</td>
<td>0.03%</td>
<td>1.77%</td>
<td>23.98</td>
<td>-1.84</td>
</tr>
<tr>
<td>KN</td>
<td>-0.05%</td>
<td>0.07%</td>
<td>0.03%</td>
<td>1.73%</td>
<td>7.18</td>
<td>-0.30</td>
</tr>
<tr>
<td>DTL KN</td>
<td>-0.06%</td>
<td>0.08%</td>
<td>0.04%</td>
<td>1.93%</td>
<td>5.94</td>
<td>-0.11</td>
</tr>
<tr>
<td>KN</td>
<td>-0.07%</td>
<td>0.07%</td>
<td>0.03%</td>
<td>1.72%</td>
<td>7.85</td>
<td>0.13</td>
</tr>
<tr>
<td>SCBL</td>
<td>-0.06%</td>
<td>0.08%</td>
<td>0.03%</td>
<td>1.86%</td>
<td>4.60</td>
<td>-0.53</td>
</tr>
</tbody>
</table>

\(^8\) Kurtosis is a measure of the tail of a probability distribution of a random variable. The normal distribution has a kurtosis of 3 and all other distributions are expressed as having x excess kurtosis above 3.
4.2.2. Cross Asset Correlations

In periods of very high or very low returns on stocks, assets generally exhibit very high correlations with each other according to CrashMetrics. In this study however, returns of the eight bank stocks exhibit very low correlations with other banks stocks. These are stocks for CFC, Standard Chartered and Diamond Trust Banks. Dropping the returns for the mentioned stocks, one can observe very high correlation averaging at 0.63. The results are shown in Table 2 below. The full cross asset correlations are shown in Table 3 below.

Table 2: Cross Asset Correlation for All Variables

<table>
<thead>
<tr>
<th>Assets</th>
<th>NSEASI</th>
<th>BCBL</th>
<th>COOP</th>
<th>EQBNK</th>
<th>KNCB</th>
</tr>
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<td></td>
<td>Index</td>
<td>KN</td>
<td>KN</td>
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<td>NASI</td>
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<td>0.56</td>
<td>0.57</td>
<td>0.63</td>
<td>0.68</td>
</tr>
<tr>
<td>BCBL KN</td>
<td>0.56</td>
<td>1.00</td>
<td>0.64</td>
<td>0.64</td>
<td>0.68</td>
</tr>
<tr>
<td>COOP</td>
<td>0.57</td>
<td>0.64</td>
<td>1.00</td>
<td>0.69</td>
<td>0.73</td>
</tr>
<tr>
<td>KN</td>
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</tr>
<tr>
<td>EQBNK KN</td>
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<td>0.64</td>
<td>0.69</td>
<td>1.00</td>
<td>0.79</td>
</tr>
<tr>
<td>KNCB KN</td>
<td>0.68</td>
<td>0.68</td>
<td>0.73</td>
<td>0.79</td>
<td>1.00</td>
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Table 3: Cross Asset Correlation for All Variables

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<th>Assets</th>
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<th>NASI KN</th>
<th>BCBL Index</th>
<th>BCBL KN</th>
<th>CFCB Index</th>
<th>CFCB KN</th>
<th>COOP Index</th>
<th>COOP KN</th>
<th>DTL Index</th>
<th>DTL KN</th>
<th>EQBNK Index</th>
<th>EQBNK KN</th>
<th>KNCB Index</th>
<th>KNCB KN</th>
<th>SCBL Index</th>
<th>SCBL KN</th>
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<tbody>
<tr>
<td>NASI</td>
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<td>-0.11</td>
<td>0.57</td>
<td>0.17</td>
<td>0.63</td>
<td>0.68</td>
<td>0.36</td>
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</tr>
<tr>
<td>BCBL</td>
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<td>0.68</td>
<td>0.24</td>
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<td>-0.07</td>
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<tr>
<td>COOP</td>
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<td>0.73</td>
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<td>-0.02</td>
<td>0.01</td>
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<td>0.00</td>
<td>0.69</td>
<td>-</td>
<td>1.00</td>
<td>0.79</td>
<td>0.23</td>
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<tr>
<td>KNCB</td>
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<td>0.68</td>
<td>0.08</td>
<td>0.73</td>
<td>0.01</td>
<td>0.79</td>
<td>1.00</td>
<td>0.21</td>
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<tr>
<td>SCBL</td>
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<td>0.24</td>
<td>-0.07</td>
<td>0.23</td>
<td>0.28</td>
<td>0.23</td>
<td>0.21</td>
<td>1.00</td>
<td></td>
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</tr>
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</table>

4.3. Beta Value-at-Risk

The parametric VaR is based on the concepts of volatility and a multiplying factor which comes from base the distribution of returns. The Betas represent the behavior of an equity in normal market conditions respect to benchmark index. They are estimated using equation 14 from section 3.4.1.2. We can appreciate how the Betas are statistically reliable just looking at R² coefficient.
Aggressive equities like Equity Bank, KCB and Co-operative Bank exhibit very high Betas, while defensive equities such as CFC and Diamond Trust Bank exhibit very low Betas with CFC having a negative beta. It is interesting to note that for CFC and Diamond Trust, the Betas are statistically insignificant and hence one cannot use them to accurately explain the relationship between NASI returns and their returns. For the rest of the equities, the Betas are significant and hence can be used to explain the relationship between NASI’s returns and those of the stock returns.

The Beta Parametric (95%) VaR was estimated using equation 16 from section 3.4.1.3. Equity Bank has a Beta VaR of 0.50%, which is the highest VaR for all the stocks in the portfolio. In addition to this, Equity Bank has the highest Beta which means its price changes are most sensitive to changes in prices of NASI. The Beta Parametric VaR for the portfolio is 1.62%. The results for the Betas are shown in Table 4 below whereas the results for the Beta VaR are shown in Table 5.

Table 4: Comparison between Beta and Crash Coefficients using R² coefficients

<table>
<thead>
<tr>
<th></th>
<th>BCBL</th>
<th>CFCB</th>
<th>COOP</th>
<th>DTL</th>
<th>EQBNK</th>
<th>KNCB</th>
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<tr>
<td>Crash</td>
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<tr>
<td>Coefficient</td>
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<tr>
<td>R - Square</td>
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<tr>
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<tr>
<td>R - Square</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>T-Statistic</td>
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<tr>
<td>P-Value</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>BCBL</th>
<th>CFCB</th>
<th>COOP</th>
<th>DTL</th>
<th>EQBNK</th>
<th>KNCB</th>
<th>SCBL</th>
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<tbody>
<tr>
<td>Crash</td>
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<td>-0.1434</td>
<td>0.8746</td>
<td>0.1468</td>
<td>1.1328</td>
<td>0.9683</td>
<td>0.3467</td>
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<tr>
<td>Coefficient</td>
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<td></td>
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<tr>
<td>R - Square</td>
<td>0.3139</td>
<td>0.0126</td>
<td>0.3223</td>
<td>0.0290</td>
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<td>T-Statistic</td>
<td>6.6727</td>
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<td>0.5149</td>
<td>12.3356</td>
<td>11.3250</td>
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<td>Beta</td>
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<td>0.5572</td>
<td>0.5572</td>
<td>0.5572</td>
<td>0.5572</td>
<td>0.5572</td>
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<td>R - Square</td>
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<td>T-Statistic</td>
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<td>6.2007</td>
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<td>P-Value</td>
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<td>0.1970</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0049</td>
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Table 5: Comparison between Beta VaR and CrashMetrics

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<tr>
<th></th>
<th>Daily Beta VaR</th>
<th>Daily CC VaR</th>
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<td>BCBL KN</td>
<td>0.1944%</td>
<td>0.1988%</td>
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<tr>
<td>CFCB KN</td>
<td>-0.0610%</td>
<td>-0.0864%</td>
</tr>
<tr>
<td>COOP KN</td>
<td>0.3690%</td>
<td>0.3766%</td>
</tr>
<tr>
<td>DTL KN</td>
<td>0.0396%</td>
<td>0.1176%</td>
</tr>
<tr>
<td>EQBNK KN</td>
<td>0.5038%</td>
<td>0.4871%</td>
</tr>
<tr>
<td>KNCB KN</td>
<td>0.3473%</td>
<td>0.3444%</td>
</tr>
<tr>
<td>SCBL KN</td>
<td>0.2299%</td>
<td>0.2231%</td>
</tr>
</tbody>
</table>

4.4. CrashMetrics

The Crash Coefficients have were estimated using equation 14 from section 3.4.1.2. They are more statistically reliable than Betas when we compare the R² coefficient. Aggressive equities like Equity Bank, KCB and Co-operative Bank exhibit very high Crash Coefficients, while defensive equities such as CFC and Diamond Trust Bank exhibit very low Crash Coefficients with CFC having a negative Crash Coefficient.

These results are consistent with those of Betas from Section 4.3. The Crash Coefficients for CFC and Diamond Trust are insignificant as well similar to their Beta counterparts from Section 4.3. Hence, one cannot use them to accurately explain the relationship between NASI returns and their returns. For the rest of the equities, the Crash Coefficients are significant and hence can be used to explain the relationship between NASI’s returns and those of the stock returns.
The CrashMetrics value was calculated using equation 17 from section 3.4.1.4. Equity Bank has a CrashMetrics value of 0.49% which is the highest value for all the stocks in the portfolio. In addition to this, Equity Bank has the highest Crash Coefficient which means its price changes are most sensitive to changes in prices of NASI in the tail of the distribution of returns. The CrashMetrics value for the portfolio is 1.66%. The results are shown for the Crash Coefficients are shown in Table 4 above whereas the results for the Crash Coefficient VaR are shown in Table 5 above.

4.5. Comparisons between CrashMetrics and Beta VaR

The Crash Coefficients estimated are higher than Betas estimated. The Crash Coefficients are more statistically than Betas significant as evidenced by the higher R^2 coefficient for Crash Coefficient models. Moreover, the estimated portfolio CrashMetrics value is higher at 1.66% than that of Beta VaR at 1.62%. The results are shown for the Betas are shown in Table 3 above whereas the results for the Beta VaR are shown in Table 4 above.

In employing CrashMetrics, (Wilmott, 2006) uses 40 extreme moves in Daimler-Benz's stocks versus returns on the DAX index. It is his finding that the crash coefficient is more stable than the beta. Moreover, for large moves in the index, the stock and the index are far more closely correlated than under normal market conditions. In other words, when there is a crash all stocks move together.

Results from (Zarpellon & Banca, 2014) show that Beta VaR for Unicredit shares, lead to an underestimation of VaR position compared to the CrashMetrics VaR. CrashMetrics also produces a lower estimation error compared to Beta VaR.

Hence, the CrashMetrics methodology succeeds in providing a higher value of potential portfolio change during a period of extreme price movements than Beta Parametric VaR.
5. Conclusion

5.1. Summary and Conclusions

This paper studies the effectiveness of CrashMetrics, a methodology by (Wilmott, 2006), in estimating portfolio changes compared to Beta Parametric VaR, a methodology of calculating VaR proposed by (Sharpe, 1964). The study estimated the Beta value-at-risk and CrashMetrics value for a hypothetical portfolio. The period considered was January 2015 and December 2016 and the stocks considered were those of listed banks in the NSE. This enabled us to capture the volatility in bank stocks’ returns due to the implementation of the interest rate cap on 14th September 2016 as well as the foreclosure of two prominent banks namely Chase Bank on 7th April 2016 and Imperial Bank on 14th October 2015.

In this study returns of some banks Stocks exhibit very low correlations with other banks stocks. These are stocks for CFC, Standard Chartered and Diamond Trust Banks. Dropping the returns for the mentioned Stocks, one can observe very high correlation averaging at 0.63. This is in line with the assumptions made by (Wilmott, 2006) that the cross asset correlations are perfectly correlated with a value of 1.

The Betas and Crash Coefficients were estimated using an OLS regression model by regressing stock returns on NASI returns. In estimating the Betas, the regression was run on the full sample data whereas in estimating crash coefficients, the regression was run on the highest and lowest 5th percentiles of the sample data. The Betas were then used to estimate the Beta Parametric VaR whereas the Crash Coefficients were used to estimate the CrashMetrics value.

Using the P - Value approach to check whether or not the coefficients were significant reveals that for certain stocks, namely CFC and Diamond Trust Bank, both the Crash Coefficients and Betas were insignificant in explaining the effect of NASI’s price changes on stock price changes.
For the other stocks, both the Crash Coefficients and Betas succeeded in explaining the effect of NASI's price changes on stock price changes. It is also worth noting that Crash Coefficients were more statistically significant than Betas as evidenced by higher $R^2$ coefficients for Crash Coefficients than Betas.

A comparison of the Beta VaR and CrashMetrics value reveals that the estimated portfolio CrashMetrics value is higher at 1.6612% than that of Beta VaR at 1.6229%. The results show that the CrashMetrics methodology succeeds in providing a higher value of potential portfolio change during a period of extreme price movements than Beta Parametric VaR. Beta VaR therefore underestimates the possible loss a portfolio stands to lose during a crash which is essentially a period of extreme price fluctuations.

5.2. Recommendations

CrashMetrics provides higher values for losses that a portfolio may incur compared to Beta VaR during a crisis. Hence, CrashMetrics methodology can be advocated for when VaR needs to be calculated during a period of a market crisis. This methodology would however not be recommended for complex portfolios due to the simplistic assumptions made while modeling for VaR. It is recommended for a bank if it calculates VaR for each of its numerous retail customers' portfolios.

5.3. Limitations and areas of further studies

There was difficulty in obtaining data for some listed banks such as I&M Bank and NIC Bank. Moreover, there was a bias towards stocks for listed banks while constructing the hypothetical portfolio. This was done because the crisis that was chosen for study was during January 2015 to December 2016 where there were foreclosures of two banks, namely Chase and Imperial Banks and implementation of the interest rate cap which took effect on 14th September 2016.
The scope of this study can be improved by including Conditional Value-at-Risk and Expected Shortfall, which are considered as better alternatives to the Standard VaR. Moreover, a portfolio with more asset classes can be considered in order to show how CrashMetrics behaves when employed across a much more diversified portfolio.
References


Zarpellon, M., & Banca, V. (2014). Crash Coefficients: a faster and more reliable way to compute parametric VaR.