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Valuation of a locational spread option: the case of tomatoes in Nairobi and Mombasa Counties in Kenya

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Valuation of a Locational Spread Option: The case of Tomatoes in Nairobi and Mombasa counties in Kenya

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Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

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Abstract

Locational Spread Options are financial instruments that can be used by traders wishing to purchase but not physically acquire produce; to hedge their risks, and / or to take speculative positions, based on their knowledge of market dynamics. In this study, we analyze historical tomato price data in Nairobi & Mombasa counties in Kenya; and observe that the Ornstein Uhlenbeck process best captures the price dynamics due to the mean reverting characteristics noted in the deseasonalized price data. We then derive pricing equations and estimate the model parameters via the use of Maximum Likelihood Estimation. Finally, we use these parameter estimations to perform Monte Carlo simulations, using the antithetic variate variance reduction technique to obtain the option price.

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Dedication

This thesis is dedicated to my daughter, Makena; and to my wife, Karimi; to whom I am thankful for her immense love, support and encouragement.

1 Introduction

1.1 Background

Agriculture plays a crucial role in Kenya's economy; as it contributes 26% of the country's GDP and another 27% indirectly through linages with other economic sectors [1]. Fresh agricultural produce in Kenya is traded on both a wholesale and retail basis, with wholesale markets located throughout major towns; and the retail markets that are located all over estates and villages. This produce is supplied through a supply chain that consists of farmers, brokers, wholesalers, distributors, retail traders and consumers. Tomatoes are extensively grown for local consumption in Kenya; and can grow in different agro-ecological zones, either under irrigation or rain-fed conditions [2]. Major areas of tomato production include Kirinyaga, Meru Central (Mitunguu & Isiolo), Nyeri, Nakuru (Bahati & Kabazi) and Taita Taveta.

In Kenya, commodities are mainly traded via the spot market. Future and forward contracts on commodities are yet to be developed for majority of agricultural commodities; but are at an exploratory stage - with futures based on tea being at the forefront of the conversation [3]. The commodity market in Kenya presently consists of the Mombasa Tea Auction and the Nairobi Coffee Exchange as indicated by the Capital Markets Authority [4]. However, the price information does not reach farmers due to opaque marketing systems; to remedy this, the Capital Markets Master Plan states the need for more transparent commodities markets. The plan therefore made a recommendation (amongst others) that a commodity derivatives exchange should be developed for the commodities traded in the spot commodities market.

A Spread Option is defined as a financial instrument based upon the difference of the prices of 2 underlying assets. A Locational Spread Option, therefore, is a financial instrument based upon price differences between the same commodity in a different location. In the commodity markets, spread options are based on the differences between the prices of the same commodity at 2 different locations (location spreads) or between the prices of the same commodity at 2 different points in time (calendar spreads) or between the prices of inputs to, and outputs from, a production process (processing spreads) as well as between the prices of different grades of the same commodity (quality spreads).

In the energy markets, spreads are used to quantify the cost of production of refined products from the raw material used to produce them. A *Crack Spread* is a simultaneous purchase/sale of crude against the purchase/sale of refined petroleum products. A *Spark Spread* provides insight into the efficiency of a power plant. It can be defined as the cost of converting a specific fuel (e.g. natural gas) into electricity. In the Agricultural markets, a good example of a spread is the Soybean crush spread traded on the Chicago Board of Trade (CBOT) which comprises of futures contracts of Soybean, Soybean Oil and Soybean meal, which gives an indication of the average gross processing margin, and is used by processors to hedge cash positions, or for pure speculation.

The application of commodity spread options in the agricultural markets in Kenya will open up the agro-commodities market to investors seeking a return, as well as traders who may wish to purchase but not physically acquire produce. Therefore, there is a case to be made for the development of financial instruments to be used in the Horticultural sector. This thesis aims to contribute to this development via the pricing of a spread option.

1.2 Problem Statement

Derivative contracts on commodities are yet to be developed locally for majority of agricultural produce. The Capital Markets Authority has recommended the development of a commodity derivatives exchange, which includes the modelling and pricing of financial instruments to be used by market participants; which are yet to be developed for use in the Kenyan context.

1.3 Objectives of the study

1.3.1 Main Objective

1. To design and implement a locational spread option investigate the price differences that exist for tomatoes in Nairobi and Mombasa counties.

1.3.2 Specific Objectives

1. To analyze spot price data of tomatoes in Nairobi and Mombasa, to identify stylized facts such as mean reversion,
2. To ascertain which methodology would suit the development of the model best,
3. To implement the chosen spread option methodology, to develop the locational spread option pricing model, via parameter estimation and application of numerical methods,
4. To compute prices of at the money, in the money and out of the money spread options.

1.4 Significance of the study

The application of commodity spread options in the agricultural markets in Kenya will open up the agro-commodities market to investors seeking a return, as well as traders who may wish to purchase but not physically acquire produce. This study therefore aims to contribute to this via modelling and computing the price of a locational spread option. The thesis is arranged as follows: In Chapter 2 we review existing literature on closed form and non-closed form solutions for spread options, in Chapter 3 we discuss the spread option pricing model, which is based on the Ornstein - Uhlenbeck process; as well as the numerical method used (Monte Carlo simulation) to obtain the option price; in Chapter 4, we discuss the parameter estimation results, as well as our numerical pricing results. Finally in Chapter 5, we conclude our study.

2 Literature Review

Options can either be solved via closed form or non-closed form solutions. Closed form solutions are convenient and allow quick computation of option prices; but may do so at the expense of accuracy and robustness. Non-closed form solutions do not suffer from these disadvantages; but such methods are often computationally involving and time consuming.

Bjerksund & Stensland [5] derive a formula for the spread call value, conditional on following the Kirk's approximation [6], which they show to be a feasible but non-optimal exercise strategy. They then perform numerical investigations, comparing their simulations on three models (their model, Kirk's model [6] and Carmona-Durrleman procedure 7]), using the Monte Carlo simulations as a benchmark; and establish that their formula is more precise than the Kirk's approximation, and marginally more accurate than Carmona-Durrleman's model.

Carmona and Durrleman [7] survey the theoretical and computational problems associated with spread option pricing. They present common features of all the spread options by discussing their roles as speculation devices and risk management tools and review the mathematical framework and numerical algorithms used to price and hedge them. They reviewed a wide scope of existing literature relating to spread option pricing, including closed form pricing models developed by Kirk [6], Bachelier, Samuelson, Carmona & Durrleman [7] amongst others. They outline how pricing and hedging dynamics can be implemented in models for both the spot price and forward curve dynamics. In the Bachelier model, the underlying indexes are modelled by means of lognormal distributions as prescribed by Samuelson, which is motivated by the desire to reproduce the inherent positivity of the indexes. The positivity restriction does not apply to the spreads themselves.

Carmona & Durrleman [7] noted that most pricing algorithms do not address the aspect of hedging, of evaluating the Greeks (partial derivatives of the price). They therefore show that hedging strategies can be computed and implemented in an efficient manner. The closed form formula derived in their paper can be used to compute the Greeks. A comparison of the results of Bachelier's model with those obtained by the closed form formula derived in [7], and the Kirk's approximation show that Carmona and Durrleman's model is superior to the others as it allows for easy computations of the Greeks. They note that the Geometric Brownian Motion models proposed by Samuelson fail to capture Mean Reversion. This feature is included in historical models by assuming that the dynamics of the underlying indexes $S_i(t)$ are given by geometric Ornstein-Uhlenbeck processes instead of Geometric Brownian Motions, which is then represented as a closed form solution in [7].

Commonly used numerical methods to price & hedge financial instruments in the absence of explicit formulae in closed form are as follows: PDE Solvers, Trinomial trees, Monte Carlo computations, and Fourier Transform. Monte Carlo methods give good price approximations but do not address the different sensitivities of the prices [7], which are important as they assist in risk management & hedging. Conversely, Trinomial tree methods allow to compute the partial derivatives along with the price. However, their use results in slow computing times, as well as the fact that they are feasible with 2 assets but may not succeed for higher dimensions.

Hurd & Zhou [8] introduce a new formula for general spread option pricing based on Fourier analysis of the payoff function and found it to be stable & applicable in a wide variety of asset pricing models. They point out that where the stock price follows a GBM process and the strike price is greater than 0, there exists a gap in regard to explicit pricing formulae; with several approximation methods available; e.g. numerical integration methods & analytical methods. They provide a numerical integration method for computing spread options in more than 2 dimensions using the FFT. Their method is based on square integrable integral methods for the payoff function, and is applicable to a variety of spread option payoffs in any model for which the characteristic function of the joint return process is given analytically. They showed that FFT provides an accurate and efficient implementation of the pricing formula in low dimensions - but for higher dimensional problems, the issue of dimensionality sets in.

Cane and Olivares [9] examine spread option pricing under models with jumps driven by Compound Poission Processess (CPPs) and stochastic volatilities in the form of Cox

Ingersoll Ross (CIR) processes. They derive the Characteristic Functions for 2 market models and use Fast Fourier Transforms (FFTs) to accurately compute spread option prices across a variety of strikes and initial price vectors. They noted that Black & Scholes [10] fail to capture critical empirical features of financial markets. They extended Bates' model [11] to 2 market multivariate models with jumps and stochastic volatility and derived the characteristic function under each model. Using FFT and Hurd & Zhou's [8] implementation, they produced results which closely matched those produced by Monte Carlo methods in a shorter time. The prices produced were sensitive to jump and correlation parameters.

Egorova & Jodas [12] discuss numerical analysis and computing of a spread option pricing problem described by a two spatial variable PDE. They develop an explicit difference scheme that retains the benefits of the one-dimensional finite difference method. They then compare these results with those of the Numerical Integration method (NIM) the Fast Fourier Transform (FFT) method & Monte Carlo methods; and establish that the Finite Difference method have the least absolute errors, the computational speed is second only to that of the NIM, and hence they recommend the use of Finite Difference methods for spread option pricing.

Dempster & Hong [13] extended the Fast Fourier Transform technique introduced by Carr & Madan [18] to a multifactor setting for pricing of spread options. They compare the Analytic, Monte Carlo & Fast Fourier Transform methods, and establish that the FFT yields an advantage over the Monte Carlo and Partial Difference methods; mainly due to the reduced computational time of the FFT.

Qi Ai [14] checked the exactness of the Kirk [6] and the Bjerksund - Stensland [15] closed form formulae. He found that they presented larger absolute errors when negative strike prices were given; and also proved inadequate in pricing trivariate spread options. He concluded that Numerical Methods such as Monte Carlo methods would provide better results.

3 Research Methodology

This chapter introduces the model that will be used to model the commodity price, and the pricing technique that will be used to obtain the specific option prices. The determination of the modelling approach, as well as the option pricing technique to be employed is determined from the characteristics of the historical commodity price data. Upon an analysis of our data, we determine that the Ornstein Uhlenbeck process is best suited to model our spread option. We also conclude that Numerical methods, specifically Monte Carlo simulation will be applied to obtain the option price.

3.1 The Ornstein - Uhlenbeck model

In most market applications, the underlying indexes are modelled by means of lognormal distributions to reproduce the positivity of the indexes. Therefore, a series of papers proposed to use Arithmetic Brownian Motion (ABM) for the dynamics of spreads [7]. In this way, prices of options can be derived by computing Gaussian integrals leading to simple closed form formulae. This has been found to be inaccurate, as the marginal distribution of the underlying indexes are Gaussian, and can therefore be negative with positive probability, which the ABM cannot accommodate.

Therefore, one can assume that the dynamics are given by Geometric Brownian Motion (GBM), which is a convenient basis for the formulation of closed form formulae. The GBM however fails to capture one of the main characteristics of commodity price data - that of mean reversion. An analysis of the residuals derived from historical daily tomato prices in Kenya shows that the prices revert to a long-term mean; which means the GBM is unsuitable in our case. We therefore assume that the dynamics of the underlying indexes in our model follow an Ornstein Uhlenbeck process, which incorporates mean reversion.

Alexandridis & Zapranis [16] studied temperature time series data from 7 European cities where Weather Derivatives are traded. They observed that the deseasonalized temperature data residuals follow a mean reverting process, and proceed to model them via an Ornstein Uhlenbeck process, via a model developed by Benth and Saltyte-Benth [17], which follows:

$$Z(t) = \mu(t) + \epsilon(t). \tag{1}$$

where Z(t) is the daily temperature, $\mu(t)$ represents the mean process and $\epsilon(t)$ is the residual process.

The mean process is further given as

$$\mu(t) = S(t) + \sum_{i=1}^{p} \alpha_i (Z(t-i) - S(t-i)). \tag{2}$$

where S(t) is deterministic and α_i, i=1,2,...,p are parameters of the AR(p) process. S(t) plays the role of the long-term average of the temperature, towards which the temperature reverts to. S(t) can therefore be described as the seasonal mean function of temperature.

Substituting (2) into (1) yields:

$$Z(t) - S(t) = \sum_{i=1}^{p} \alpha_i (Z(t-i) - S(t-i)) + \epsilon(t). \tag{3}$$

As long as the residual process has the mean zero, we observe that the expected temperature $\tilde{Z}_t = E[Z(t)]$ follows:

$$\tilde{Z}(t) - S(t) = \sum_{i=1}^{p} \alpha_i (\tilde{Z}(t-i) - S(t-i)) + \epsilon(t). \tag{4}$$

The seasonal mean function S(t) is assumed to have the form:

$$S(t) = a_0 + a_1 t + \sum_{j=1}^{J} b_{1j} \cos(2\pi j (t - b_{2j})/365). \tag{5}$$

The dynamics of the deseasonalized temperature are therefore assumed to follow an O-U process as follows:

$$d\tilde{Z}(t) = -\alpha \tilde{Z}(t) dt + \sigma(t) dB(t). \tag{6}$$

where the temperature is defined as $Z(t) = S(t) + \tilde{Z}(t)$, $\alpha \geq 0$ is a positive constant measuring the speed of mean reversion, and B is a Brownian Motion defined on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$.

Upon analyzing our tomato price data, we noted that it exhibited reversion characteristics as well. We went further to analyze temperature data in Kirinyaga, a major tomato growing region in Kenya [2] and noted that the temperature data reverts to the mean as well. Due to this shared property of mean reversion, we adopt the model developed by [16]& [17] in our case. The price dynamics are given by a Gaussian mean-reverting OU process defined as follows:

$$dT(t) = dS(t) - \kappa(T(t) - S(t))dt + \sigma dW(t). \tag{7}$$

where T(t) is the daily price of Tomatoes, κ is the speed of mean reversion, S(t) is a deterministic function modelling trend and seasonality, σ is the volatility of price variations, and W(t) is a Brownian motion. We model S(t) using a Truncated Fourier series to obtain the deterministic function. Its representation is as below:

$$S(t) = a_0 + a_1(t) + a_2 cos(2\pi(t - a_3)/365) + a_4(1 + sin(2\pi(t - a_5)/365)sin(2\pi t/365). \tag{8}$$

The commodities can then be represented as:

$$dT_1(t) = dS_1(t) - \kappa_1(T_1(t) - S_1(t))dt + \sigma dW^1(t), \tag{9}$$

$$dT_2(t) = dS_2(t) - \kappa_2(T_2(t) - S_2(t))dt + \sigma dW^2(t). \tag{10}$$

where T_1 refers to Tomato prices in Nairobi, and T_2 refers to Tomato prices in Mombasa. S_1 and S_2 are deterministic functions, and W^1 and W^2 are two correlated Brownian Motions with $E[dW^1(t)dW^2(t)] = \rho_{ij}$.

The prices of Tomatoes in Nairobi and Mombasa are correlated, shown in the matrix below:

$$\sum = \begin{bmatrix} 1 & 0.6060 \\ 0.6060 & 1 \end{bmatrix}.$$

We therefore convert the vector of correlated Brownian motion processes to a vector of uncorrelated processes:

$$\begin{bmatrix} dW^1 \\ dW^2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} d\tilde{W}^1 \\ d\tilde{W}^2 \end{bmatrix}.$$

We select the $a'_{ij}s$ suitable to preserve the correlation structures $E(dW^i) = 0$, $E[(dW^i)^2] = dt$, $E[dW^i dW^j] = \rho_{ij} dt \; \forall i \neq j$, which imposes the following conditions:

$$\begin{aligned} a_{11}^2 + a_{12}^2 &= 1 \\ a_{21}^2 + a_{22}^2 &= 1 \\ a_{11}a_{21} + a_{21}a_{12} &= \rho_{12} \end{aligned}$$

we therefore set $a_{11} = 1$, $a_{12} = 0$, $a_{21} = \rho_{12}$, & $a_{22} = \sqrt{1 - \rho_{12}^2}$, from which we obtain:

$$\begin{aligned} dW^1(t) &= d\tilde{W}^1(t) \\ dW^2(t) &= \rho_{12} d\tilde{W}^1(t) + \sqrt{1 - \rho_{12}^2} d\tilde{W}^2(t). \end{aligned}$$

Taking $\tilde{T}(t) =$ T(t)-S(t) in equations (9) & (10), we represent the process as follows:

$$d\tilde{T}_1(t) = -\kappa_1(\tilde{T}_1(t))dt + \sigma_1 dW^1(t), \tag{11}$$

$$d\tilde{T}_2(t) = -\kappa_2(\tilde{T}_1(t))dt + \sigma_2 dW^2(t). \tag{12}$$

where $\tilde{T}(t) =$ T(t)-S(t). $\tilde{T}(t)$ represents the difference between the tomato price and the deterministic component.

Via application of Ito's lemma, the solution of the Stochastic Differential Equation (SDE) above, $\tilde{T}(t)$ is as follows:

$$\tilde{T}_1(t) = x_1 e^{-\kappa_1 t} + \sigma_1 e^{-\kappa_1 t} \int_0^t e^{\kappa_1 s} dW^1(s), \tag{13}$$

$$\tilde{T}_2(t) = x_2 e^{-\kappa_2 t} + \sigma_2 e^{-\kappa_2 t} \int_0^t e^{\kappa_2 s} dW^2(s). \tag{14}$$

where $x_1 = \tilde{T}_1(0)$ and $x_2 = \tilde{T}_2(0)$. $\tilde{T}_1(t)$ and $\tilde{T}_2(t)$ represent the prices of tomatoes for Nairobi and Mombasa at time t respectively.

which further leads to the following pricing equations:

$$\tilde{T}_1(t) = x_1 e^{-\kappa_1 t} + \sigma_1 dW^1(t)(1 + \kappa), \tag{15}$$

$$\tilde{T}_2(t) = x_2 e^{-\kappa_2 t} + \sigma_2 dW^2(t)(1 + \kappa). \tag{16}$$

Proof see Appendix 1.

3.2 Monte Carlo simulation

This section outlines the pricing method that will be used to establish the option price. Closed form solutions for spread option pricing have been developed for different types of spread options; but should be developed on a case by case basis to avoid modelling errors i.e. a closed form solution based on a model whose underlying indexes follow a GBM process cannot be used for underlying indexes whose deseasonalized prices follow a mean reverting process, as is the case in this study. We have therefore opted to obtain the prices via Monte Carlo simulation, which is performed by generating a large number of sample paths to compute the value of the function of the path whose expectation we evaluate, and averaging these values over the sample paths. Its accuracy be improved by increasing the number of simulations, but this leads to an increase in computation time & cost. However, there are methods that can be used to increase efficiency via the use of reduction of variance techniques. Two of the variation techniques in use are the Antithetic variates method and the Control variates method. The antithetic method reduces variance by introducing negative dependence between pairs of replications, whereas control variates take random variables with positive correlation and known expected value under consideration.

The spread option SDEs to be simulated are as follows:

$$dT_1(t) = dS_1(t) - \kappa_1(T_1(t) - S_1(t))dt + \sigma dW^1(t), \tag{17}$$

$$dT_2(t) = dS_2(t) - \kappa_2(T_2(t) - S_2(t))dt + \sigma dW^2(t). \tag{18}$$

where T_1 refers to Tomato prices in Nairobi, and T_2 refers to Tomato prices in Mombasa. S_1 and S_2 are deterministic functions, and W^1 and W^2 are two Brownian Motions. The above spread option SDEs will be used to obtain the spread option price.

In order to simulate equations (17) & (18) we need to discretize them, and we do this using the Euler discretization scheme. The Euler scheme assumes that $0 = t_0 \leq t_1 \leq$

$... \leq t_{n-1}, t_n = t$ and posits the following:

$$dT_1(t) = T_1(t_{i+1}) - T_1(t_i),$$
$$dT_2(t) = T_2(t_{i+1}) - T_2(t_i),$$
$$dS_1(t) = S_1(t_{i+1}) - S_1(t_i),$$
$$dS_2(t) = S_2(t_{i+1}) - S_2(t_i),$$
$$dW^1(t) = \sqrt{t_{i+1} - t_i} Z^1_{i+1},$$
$$dW^2(t) = \sqrt{t_{i+1} - t_i} Z^2_{i+1}.$$

therefore, we substitute in equation (17) and (18) to obtain:

$$T_1(t_{i+1}) - T_1(t_i) = S_1(t_{i+1}) - S_1(t_i) - \kappa_1(T_1(t_i) - S_1(t_i)(t_{i+1} - t_i + \sigma_1\sqrt{t_{i+1} - t_i} Z^1_{i+1}, \quad (19)$$
$$T_2(t_{i+1}) - T_2(t_i) = S_2(t_{i+1}) - S_2(t_i) - \kappa_2(T_2(t_i) - S_2(t_i)(t_{i+1} - t_i + \sigma_2\sqrt{t_{i+1} - t_i} Z^2_{i+1}. \quad (20)$$

Assuming a fixed grid spacing $t_{i+1} - t_i = h$, then $t_i = ih$. Therefore, equation (19) becomes:

$$T_1(i+1) = T_1(i) + S_1(i+1) - S_1(i) - \kappa_1(T_1(i) - S_1(i)) * h + \sigma_1\sqrt{h} Z^1_{i+1}, \quad (21)$$
$$T_2(i+1) = T_2(i) + S_2(i+1) - S_2(i) - \kappa_2(T_2(i) - S_2(i)) * h + \sigma_2\sqrt{h} Z^2_{i+1}. \quad (22)$$

The expectation function for spread option pricing is therefore given as below.

$$f(t) = e^{-rT}\mathbf{E}[(T_2(i+1) - T_1(i+1) - K|\mathcal{F}_t)]^+.$$

The local truncation error (LTE) of the Euler method is defined as the error made in a single step i.e. the difference between the numerical solution after one step, y_1 and the exact solution at time $t_1 = t_0 + h$, given by:

$$y_1 - y_0 = hf(t_0, y_0),$$
$$y_1 = y_0 + hf(t_0, y_0).$$

For the exact solution, we make use of the Taylor expansion of the function y around t_0.

$$y(t_0 + h) = y(t_0) + hy'(t_0) + \frac{1}{2}h^2 y\text{"}(t_0) + O(h^3).$$

The LTE is given by the difference between these equations:

$$y_0(t_0 + h) - y_1 = \frac{1}{2}h^2 y''(t_0) + O(h^3).$$

The algorithm to be used is as follows:

1. Fix the number of monitoring dates & time step $\Delta = T/N$.

2. Starting from $T(0) = x_0$, simulate the spot prices $T_1(i+1)$ and $T_2(i+1)$ as given in equations (21) & (22).

   The increment $W^j(t\Delta) - W^j((t-1)\Delta)$ (where j=1,2) is simulated according to a normal distribution $N(0,\Delta)$:

$$W^{(j)}(t\Delta) - W^{(j)}((t-1)\Delta) = \sqrt{\Delta} * \phi^{-1}(u_i^{(j)}). \tag{23}$$

   where u is a uniform (0,1) random variable and j^{-1} is the inverse cumulative distribution of the standard normal distribution.

3. Update the average according to

$$A^j(i\Delta) = \frac{i-1}{i} x A^j((i-1)\Delta) + \frac{S(i\Delta)}{i}, \tag{24}$$

$$A^j(0) = s_0. \tag{25}$$

   where j=1,2.

4. Compute the discounted option payoff:

$$f(t) = e^{-rT}\mathbf{E}[(T_2(i+1) - T_1(i+1) - K|\mathcal{F}_t)]^+.$$

5. Repeat step 2 to 4 & discount to obtain the option price and average across simulations:

$$\tilde{p} = \frac{1}{N}\sum p. \tag{26}$$

6. Evaluate accuracy by computing the standard error:

$$se = \sqrt{\frac{\sigma^2}{N}}. \tag{27}$$

7. The confidence interval is given by:

$$\tilde{p} + Z_{1-\alpha/2} * se. \tag{28}$$

The antithetic variates method was used. This was done via the following steps:

(a) Computing a vector U of Normal random variables,

(b) Computing a vector of past prices S_1 under the O-U process,

(c) Creating a new vector V given by V=-U,

(d) Computing a new vector of asset prices S_2 under the O-U process based on vector V,

(e) Taking an average of all the values consisting both vectors S_1 and S_2 that we call A_1 and A_2,

(f) Discounting the average values obtained to obtain the option price.

The Greeks measure sensitivity of the prices and are used by practitioners to manage risk. There are 4 major Greeks [20]:

- Delta, which measures the rate of change of the option price with respect to the price of the underlying asset,
- Gamma, which is the rate of change of the Delta with respect to the price of the underlying asset,
- Theta, which measures the rate of change of the option value with respect to the passage of time, and
- Vega, which is the rate of change of the value of the portfolio with respect to the volatility of the underlying asset.

In order to numerically compute the Delta via Monte Carlo simulations, we proceed to compute the limit as $\xi \to 0$ of the following expression [7]:

$$\Delta_1(\xi) \approx \frac{1}{\xi}(P(x_1 + \xi) - P(x_1)).$$

$\Delta_1(\xi)$ is computed for each ξ in a sequence going to zero. The simulation is conducted for x_1 and again for $x_1 + \xi$ [7].

4 Presentation of Research Findings

In this chapter we analyze past tomato price and local temperature data to review data attributes that would inform our model choice. We establish that the price & temperature data exhibit mean reverting properties; therefore confirming the appropriateness of the Ornstein Uhlenbeck model we adopted. Additionally, we obtain parameters from our historical price data; and proceed to use them to price the option via Monte Carlo simulations. We conclude by discussing pricing results for call spread options for At the Money (ATM), In the Money (ITM) and Out of the Money (OTM) options.

4.1 Parameter Estimation

Our market model is based on the dynamics of daily spot commodity prices, and is therefore reliant on historical price data to inform our modelling approach and parameter estimation.[1] Commodity price data for the period February 2014 to February 2018 for a crate (64kg) of tomatoes in Nairobi and Mombasa was obtained and used for purposes of our study. The price data comprises of weekday market prices, and hence weekends are not included in our study. Data outliers were removed using the 3 sigma method, which proved more effective than the interquartile range method (IQR), which when used did not remove all outliers. Moreover, the data contained some missing values, which were filled in using linear interpolation.

Additionally, daily mean temperature readings[2] for Kirinyaga county (a major tomato growing region in Kenya) for the period February 2014 to February 2018 have been obtained and analyzed. The analysis of this particular data set was done only to inform the choice of model; since temperature and price data share the stylized fact of mean reversion [16]. Therefore, the parameters from this data have not been used in the computation of our option price. The raw price data for tomato prices in Nairobi & Mombasa, as well as that for temperature is shown in Figure 1. A cursory look at the data shows that in

[1]The data used in this study is derived from local daily commodity prices compiled by the Ministry of Agriculture, published in the Business Daily newspaper on each weekday.

[2]This data was obtained from the Climate Forecast system, which is availed by the National Centres for Environmental Protection, based in the USA.

all cases, the data is characterized by cycles that exhibit annual seasonality cycles; with some peaks and troughs specifically in the price data.

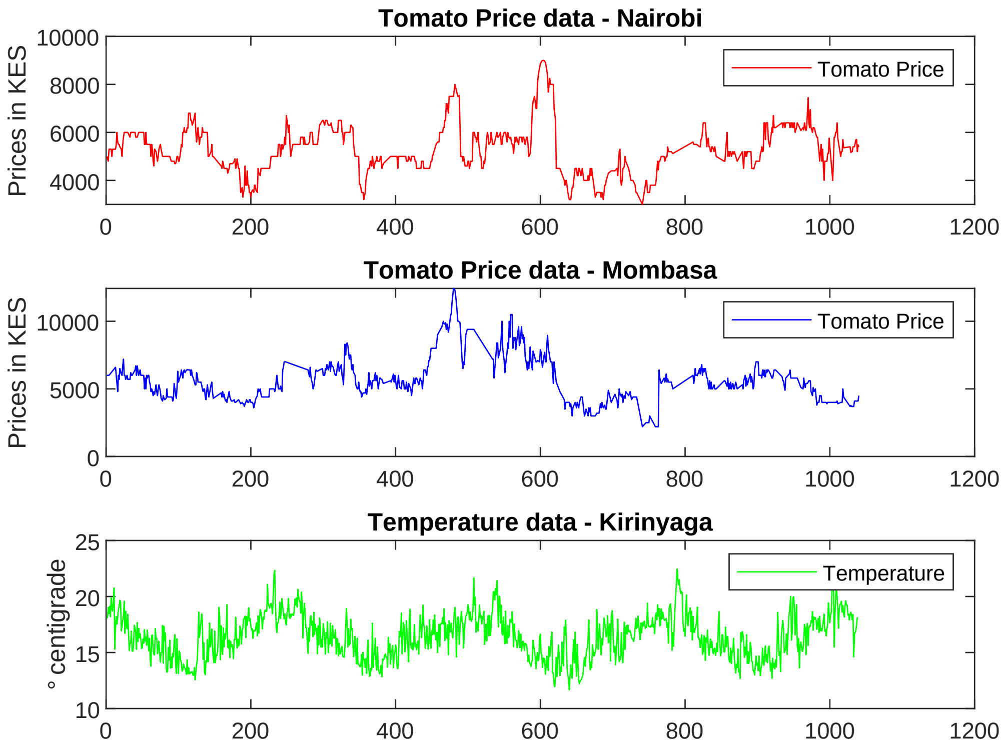

Figure 4.1: *Price data in Nairobi, Mombasa & Temperature data in Kirinyaga*

Recall the O-U model for the price dynamics given in equation (7), which includes a deterministic component, S(t), which models trend & seasonality. We begin by modelling this component for both the price and temperature data, using the Truncated Fourier series given in equation (8). The graphical representation is shown in Figure 2, with the parameter values of S(t) given in Table 1. We then determine the residuals by differencing the deterministic component from the raw data in each case (i.e. for price and temperature data). A multiple linear regression was then applied to the residuals to obtain the residual regression plots as represented in Figure 3. In all cases, the data is seen to revert to a long-term mean. Having confirmed that the price residuals are mean reverting, we proceed to estimate the speed of mean reversion, κ, as well as the standard deviation, σ. We proceed to obtain these parameters via discretization of the O-U process as follows:

$$\Delta T_1(t) = \Delta S_1(t) + \kappa_1(T_1(t-1) - S_1(t-1)) + \sigma_1\sqrt{\Delta t}Z^1(t), \tag{29}$$

$$\Delta T_2(t) = \Delta S_2(t) + \kappa_2(T_2(t-1) - S_2(t-1)) + \sigma_2\sqrt{\Delta t}Z^2(t). \tag{30}$$

Expanding the above equation with $\Delta t = 1$, we have

$$T(t) - T(t-1) = S(t) - S(t-1) + \kappa(T(t-1) - S(t-1)) + \sigma\epsilon(t) \tag{31}$$

this arises from the fact that

$$\begin{aligned} \Delta T(t) &= T(t) - T(t-1), \\ \Delta S(t) &= S(t) - S(t-1), \\ \epsilon(t) &= \sqrt{\Delta t}Z(t) \end{aligned}$$

By rearranging, we have that

$$T(t) - S(t) = T(t-1) - S(t-1) + \kappa(T(t-1) - S(t-1)) + \sigma\epsilon(t). \tag{32}$$

We then set $\tilde{T}(t) = T(t) - s(t)$ to give:

$$\tilde{T}(t) = \tilde{T}(t-1) + \kappa\tilde{T}(t-1) + \sigma\epsilon(t). \tag{33}$$

Equivalently,

$$\tilde{T}(t) = (1+\kappa)\tilde{T}(t-1) + \sigma\epsilon(t). \tag{34}$$

Substituting with $\alpha = 1 + \kappa$, our model is reduced to:

$$\tilde{T}(t) = \alpha\tilde{T}(t-1) + \sigma\epsilon(t). \tag{35}$$

Equation (33) is a simple AR(1) model, hence the parameters κ and σ are estimated via the Maximum Likelihood Estimation method (MLE).

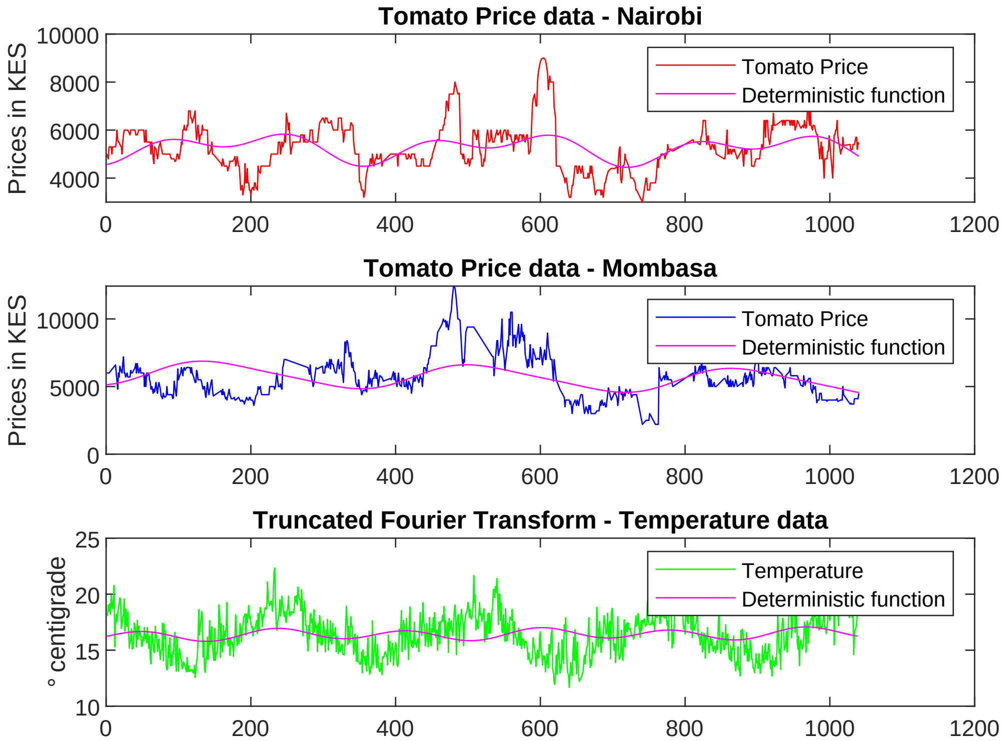

Figure 4.2: *Truncated fourier transform - Price & Temperature data.* This shows the deterministic (deseasonalized) component, S(t) of our model.

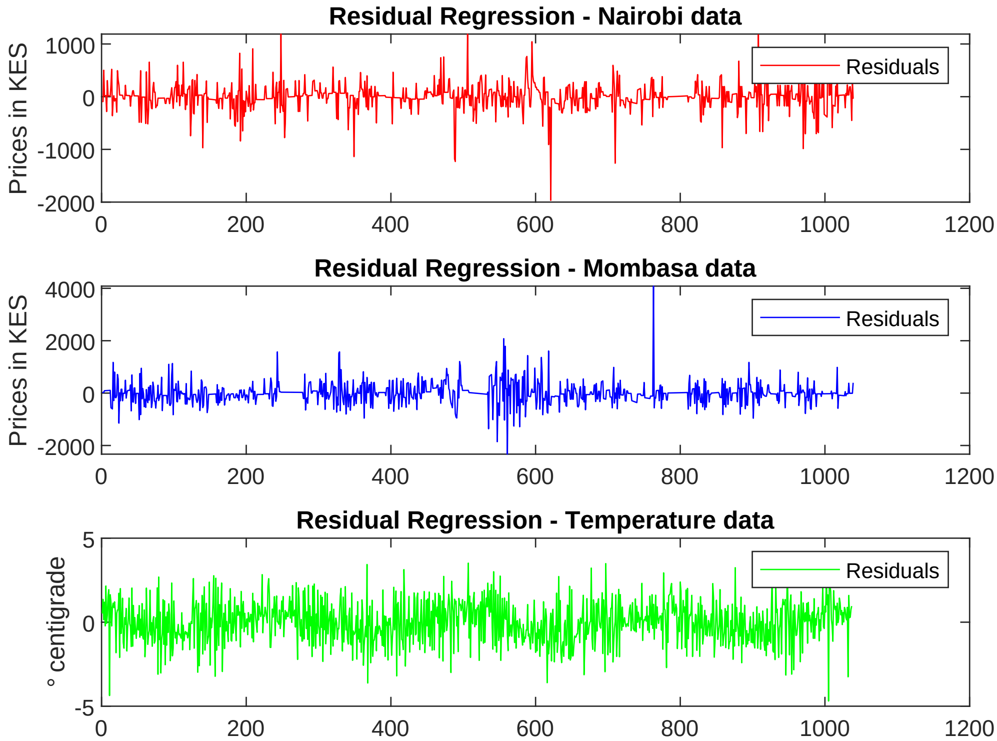

Figure 4.3: *Residual regression - Price & Temperature data.* These residuals were obtained from differencing the deterministic component from the raw data. We observe that the price and temperature data reverts to the mean.

We obtained the following model parameters for Nairobi & Mombasa:

Table 4.1: **Parameter Values**

Parameter	Nairobi	Mombasa
a_0	4.9743	5.9010
a_1	-0.0001	-0.0007
a_2	-0.8866	0.7864
a_3	-2.4918	-0.1943
a_4	0.7568	0.2999
a_5	-0.0273	0.0297
σ	254.1715	401.4495
κ	0.9610	0.9639

The parameters $a_0, a_1, a_2, a_3, a_4 \& a_5$ were estimated from the deseasonalized component

S(t) represented in equation (8) via the Truncated Fourier Transform. The parameters σ and κ were estimated via Maximum Likelihood Estimation on the residual data, following the discretization of equation (11) & (12) into an AR(1) process represented by equation (35). The values of σ for Nairobi and Mombasa are noted to be quite high at 254.1715 and 401.4495 respectively, and the speed of mean reversion κ for both the residuals for Nairobi and Mombasa price data are very close to 1, showing that the residuals revert to the mean at a high rate. These parameters were used to perform the Monte Carlo simulation described in the following section.

4.2 Monte Carlo simulation

The Monte Carlo simulation will enable us to price our spread option. In order to address the efficiency & computing time problem, we used the Antithetic variate variance reduction technique. We simulated call prices for at the money (ATM), in the money (ITM) and out of the money (OTM) options under different number of simulations, results of which are displayed below. We performed these simulations using an Intel (R) Core (TM) I5-6267U CPU, with a clock size of 2.90 GHz.

No. of Simulations	**Option Price**	**Computation time (seconds)**
10	0	0.046
100	12.1750	0.034
1,000	9.3936	0.071
10,000	10.1334	0.378
100,000	9.4405	3.789
1,000,000	9.4792	47.140
10,000,000	9.5861	729.720

Table 4.2: ATM Spread call option prices computed with Monte Carlo simulations. Parameter values are $S_1(t) = 6000$, $S_2(t) = 5000$, $\sigma_1 = 254.1715$, $\sigma_2 = 401.4495$, Strike Price: K=1000, Time to maturity: T=1, Risk free rate: r=0.07.

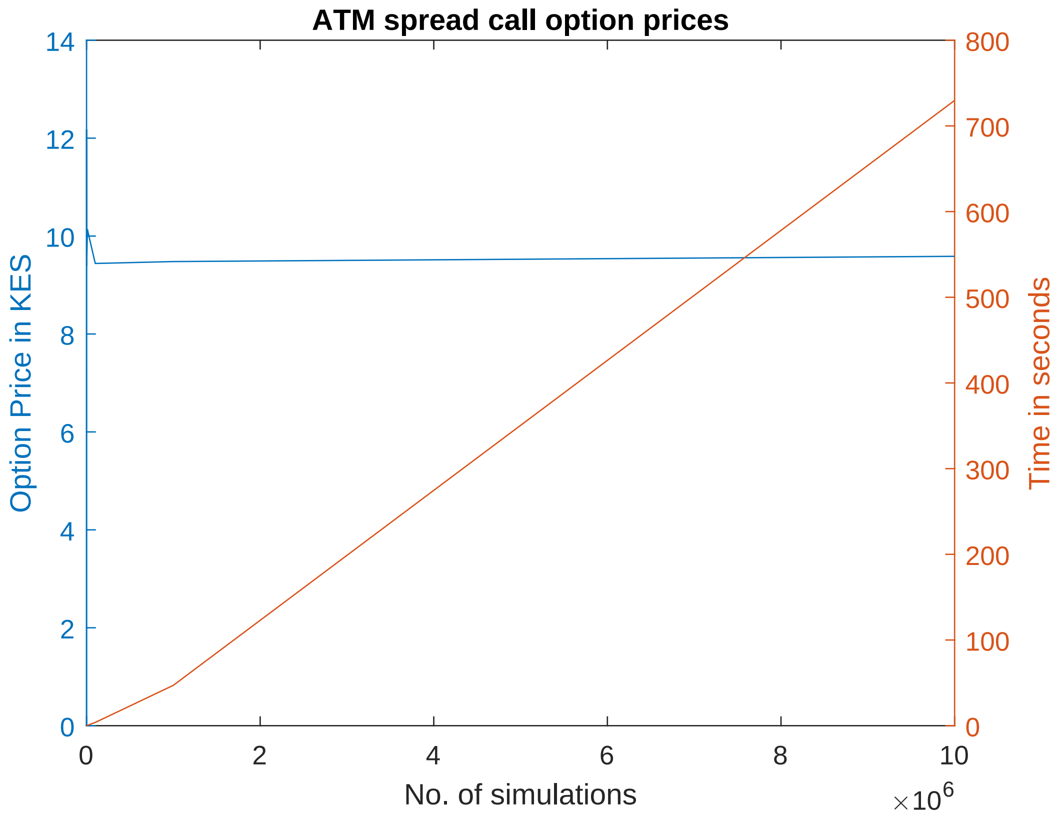

Figure 4.4: *At the money spread call option prices.* We observe that the option prices converge after approximately 100,000 simulations.

No. of Simulations	**Option Price**	**Computation time (seconds)**
10	94.8737	0.037
100	75.9642	0.041
1,000	69.7833	0.051
10,000	61.8587	0.677
100,000	62.1036	8.339
1,000,000	62.1976	91.516
10,000,000	62.7888	720.458

Table 4.3: ITM Spread call option prices computed with Monte Carlo simulations. Parameter values are $S_1(t) = 6000$, $S_2(t) = 5000$, $\sigma_1 = 254.1715$, $\sigma_2 = 401.4495$, Strike Price: K=500, Time to maturity: T=1, Risk free rate: r=0.07.

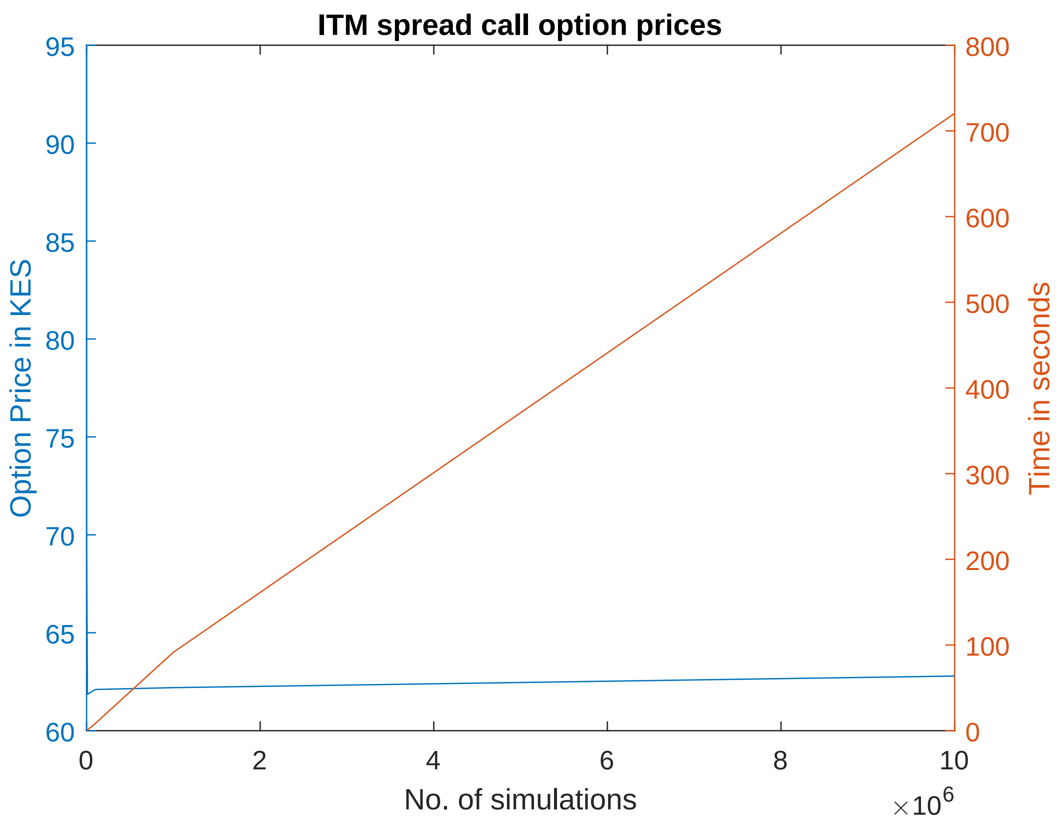

Figure 4.5: *In the money spread call option prices.* We observe that the option prices converge after approximately 100,000 simulations.

No. of Simulations	Option Price	Computation time (seconds)
10	0	0.040
100	0	0.036
1,000	0	0.0047
10,000	0.0472	0.720
100,000	0.0362	8.907
1,000,000	0.0316	101.538
10,000,000	0.0316	1127.878

Table 4.4: OTM Spread call option prices computed with Monte Carlo simulations. Parameter values are: $S_1(t) = 6000$, $S_2(t) = 5000$, $\sigma_1 = 254.1715$, $\sigma_2 = 401.4495$, Strike Price: K=2000, Time to maturity: T=1, Risk free rate: r=0.07.

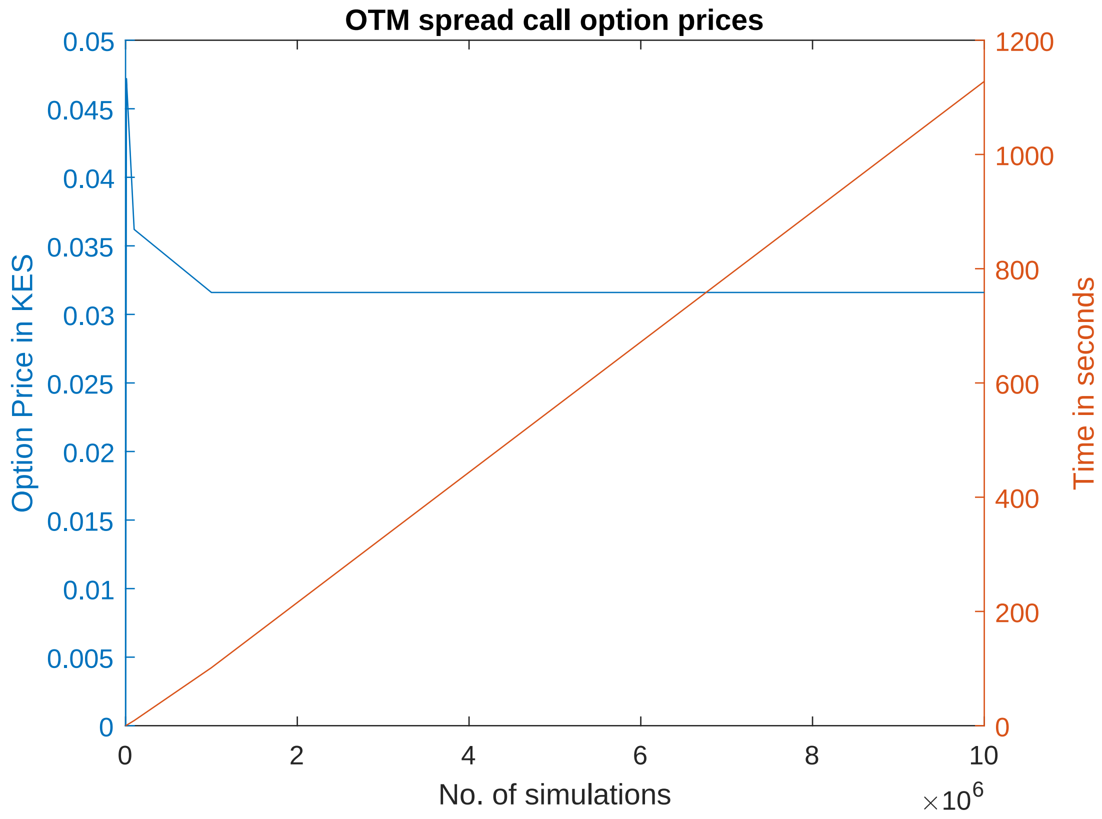

Figure 4.6: *Out of the money spread call option prices.* We observe that the option prices converge after approximately 100,000 simulations.

No. of Simulations	Standard Error - Nairobi	Standard Error - Mombasa
10	80.3761	126.9495
100	25.4172	40.1450
1,000	8.0376	12.6949
10,000	2.5417	4.0145
100,000	0.8038	1.2695
1,000,000	0.2542	0.4014
10,000,000	0.0804	0.1269

Table 4.5: Standard Errors computed with Monte Carlo simulations.

In Table 2, 3 & 4 we show the results from a Monte Carlo simulation performed for spread call options at the money, in the money and out of the money. The option prices for in the money options are of course much higher as the writer requires a higher premium. The converse is true for out of the money options. Figures 4, 5 & 6 graphically represent the speed of convergence and computation times. We established that spread option prices in all cases (ATM, ITM and OTM) converge after approximately 100,000 simulations. Additionally, Table 5 shows that the standard error at this convergence point is also seen to be low in comparison to those seen for fewer simulations. We therefore conclude that the appropriate number of simulations to estimate the spread option price is 100,000. We observe that the computation time in this case is relatively low (3.789 sec for ATM spread call options, 8.339 sec for ITM spread call options and 8.907 for OTM spread call options); as compared to 729.720 sec, 720.458 sec and 1127.878 sec for 10,000,000 simulations for ATM, ITM and OTM spread call options respectively) which therefore presents a good balance between accuracy and computation time.

Additionally, we simulated the value of the Greeks i.e. Delta via Monte Carlo simulation, as shown below:

ξ	$\Delta_1(\xi)$
10	0.01562
9	0.028177778
8	0.0064625
7	-0.002728571
6	0.0126
5	-0.00946
4	0.0066
3	0.0576
2	0.08005
1	0.1971

Table 4.6: Delta computed with Monte Carlo simulations, by re-computing the option price with a slight change in the underlying price, $S_1 = 6000$. The average delta in this case comes to 0.03920, which means that when the price of tomatoes changes by a small amount, the price of tomatoes changes by about 3.92% of this amount.

5 Discussion, Conclusion & Recommendations

We set out to value a locational spread option on Tomatoes in Nairobi and Mombasa counties in Kenya; and did so by determining an appropriate model based on the characteristics of historical data, estimated the parameters of the model, and proceeded to apply numerical methods to determine the price of the spread option. We have demonstrated that an Ornstein Uhlenbeck process is suitable for modelling the locational spread option; as it incorporates mean reversion characteristics of the historical prices. Moreover, we obtained the option price via the use of Monte Carlo simulations, in particular via use of the antithetic variates method. In our approach, we analyzed historical price data to determine whether the GBM, widely used in previous literature, could be used to model the spread in our case. We found that the prices were mean reverting, immediately disqualifying the GBM. We showed that an O-U model was better placed to model the option. Our price dynamics hence follow a Gaussian mean - reverting process as given in equations (9 & 10). We then proceed to model the deterministic function, S(t), which is given in equation (8) and features in equations (9 & 10); and thereafter applied Ito's lemma to obtain the pricing equations for Tomato prices in Nairobi and Mombasa.

We then estimated the parameters via MLE, using the pricing equations given in equation (13 & 14), and subsequently performed Monte Carlo simulations; making use of the antithetic variate technique to hasten the convergence of the prices. We obtained option prices for a varied number of simulations for at the money, in the money and out of the money spread options, and summarized these results in Table 2, 3 & 4. We showed that generally all these options converged after 100,000 Monte Carlo simulations; as represented graphically in figure 4, 5 & 6. We therefore conclude that 100,000 Monte Carlo simulations are appropriate to compute the spread option price, as they provide a good compromise in terms of speed of convergence and pricing accuracy.

In future research, the model can be modified to compute the speed of mean reversion not as a constant, but as a function of time (i.e. $\kappa(t)$ as opposed to κ), as it can be argued that the speed of mean reversion is not constant for the entire time period in the data set, as shown in Alexandridis & Zapranis [16]. Moreover, further research can model volatility as a seasonal function, in order cater for the seasonality observed in the price residuals as observed in Benth & Saltyte-Benth [19]. Future research can be conducted to perform a comparison of numerical pricing methods for this spread option, similar to the work done by Egorova & Jodas [12]; to compare the results from our Monte Carlo simulation to other numerical methods such as Finite Difference, Fast Fourier Transform & Numerical Integration method, to establish which of these methods gives a better computation of the spread option price, from an accuracy, robustness and computational efficiency point of view. More research can be done in modelling the deterministic component of the spread option pricing model; as it can also be modelled using Wavelet Analysis as done by Alexandridis & Zapranis [16], instead of the Truncated Fourier transform used in our discussion. Finally, further research could be conducted on this spread option pricing problem, to develop a closed form solution gives us the price of the option, as well as provides computations for the Greeks, which have proven useful in other markets for hedging purposes.

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Appendix

Appendix 1

Solution of SDE of $\tilde{T}$ via Ito's lemma

$$dT(t) = dS(t) - \kappa(T(t) - S(t))dt + \sigma dW(t). \tag{36}$$

Via Ito's lemma,

$$\begin{aligned}
dy &= \left[\frac{\partial y}{\partial t} + \mu(xt)\frac{\partial y}{\partial x} + \frac{1}{2}\sigma^2(xt)\frac{\partial^2 y}{\partial x^2}\right] dt + \sigma(xt)\frac{\partial y}{\partial x}dW, \\
dy &= \left[\kappa\tilde{T}e^{\kappa t} - \kappa\tilde{T}e^{\kappa t} + \frac{1}{2}\sigma(0)\right] dt + \sigma e^{\kappa t}dW, \\
\int_0^t d(e^{\kappa s}\tilde{T}) &= \int_0^t \sigma e^{\kappa s}dW, \\
e^{\kappa t}\tilde{T}_t - e^{\kappa t}\tilde{T}_0 &= \int_0^t \sigma e^{\kappa s}dW_s, \\
\tilde{T}_t &= \tilde{T}_0 e^{-\kappa t} + \sigma e^{-\kappa t}\int_0^t e^{\kappa s}dW_s.
\end{aligned}$$

Through integration by parts, i.e.

$$\int u dv = uv - \int v du.$$

We evaluate the integral in the above equation as follows:

$$\begin{aligned}
\int_0^t e^{\kappa s}dW_s &= e^{\kappa s}W_s|_0^t - W_s\kappa e^{\kappa s}|_0^t, \\
&= [e^{\kappa t}W_t - e^{\kappa 0}W_0] - [W_t k e^{\kappa t} - W_0\kappa e^{k0}], \\
&= e^{\kappa t}z_t - W_0 - W_t\kappa e^{\kappa t} + W_0\kappa, \\
&= W_t e^{\kappa t}(1+\kappa).
\end{aligned}$$

Therefore

$$\tilde{T}_t = \tilde{T}_0 e^{-\kappa t} + \sigma e^{-kt}[W_t e^{kt}(1+\kappa)], \tag{37}$$

$$\tilde{T}_t = \tilde{T}_0 e^{-\kappa t} + \sigma[W_t(1+\kappa)]. \tag{38}$$

The price dynamics for Tomato prices in Nairobi and Mombasa are as below.

$$\tilde{T}_1(t) = \tilde{T}_1(0)e^{-\kappa_1 t} + \sigma_1[W_1(t)(1+\kappa_1)], \tag{39}$$

$$\tilde{T}_2(t) = \tilde{T}_2(0)e^{-\kappa_2 t} + \sigma_2[W_2(t)(1+\kappa_2)]. \tag{40}$$

Appendix 2

MATLAB code

```
-----Locational Spread Option Pricing MATLAB Code-----
--------------------------Data read--------------------------

 clear; clc; close all;

 B = xlsread('Tomato_det_int.xlsx', 'Sheet1');


 Date = B(:,1);
 Nairobi = B(:,2);
 Mombasa = B(:,3);

 MATLABDate = x2mdate(Date,0,'datetime')

 save('MATLABDate.mat')
 save('Nairobi.mat')
 save('Mombasa.mat')
--------------------------TruncatedF--------------------------

function Gamma = TruncatedF(a0,a1,a2,a3,a4,a5,t)
% Seasonal function with trend, the truncated Fourier transform
%Gamma = TruncatedF(x(1), x(2), x(3), x(4), x(5), x(6), t);
Gamma = a0 + a1*t + a2*cos(2*pi*(t-a3)/365)+ a4*(1+sin(2*pi*(t - a5)/365)).*sin(2*pi*t/365);
end

--------------------------TruncatedFmsa--------------------------

function Beta = TruncatedF(a0,a1,a2,a3,a4,a5,t)
% Seasonal function with trend, the truncated Fourier transform
%Gamma = TruncatedF(x(1), x(2), x(3), x(4), x(5), x(6), t);
Beta = a0 + a1*t + a2*cos(2*pi*(t-a3)/365)+ a4*(1+sin(2*pi*(t - a5)/365)).*sin(2*pi*t/365);
end

--------------------------Est_Sim_Msa--------------------------

%clear; clc; close all;

load('Mombasa.mat')
y = length(Mombasa);
t = (1:1:y)';

init = [20, 1, 1, 10, 1, 10];
options = optimoptions(@lsqnonlin,'Algorithm','trust-region-reflective');
modelfun = @(b) b(1) + b(2)*t + b(3)*cos(2*pi*(t - b(4))/365)...
    + b(5)*(1 + sin(2*pi*(t - b(6))/365)).*sin(2*pi*t/365) - Mombasa;
y = lsqnonlin(modelfun,init,[],[],options)
Beta = TruncatedFmsa(y(1), y(2), y(3), y(4), y(5), y(6), t);

M=Mombasa-Beta

M

[B,BINT,W2] = regress(M(2:end),M(1:end-1))

```

```
-----Est_Sim_Nbi-----

%clear; clc; close all;

load('Nairobi.mat')
x = length(Nairobi);
t = (1:1:x)';

init = [20, 1, 1, 10, 1, 10];
options = optimoptions(@lsqnonlin,'Algorithm','trust-region-reflective');
modelfun = @(b) b(1) + b(2)*t + b(3)*cos(2*pi*(t - b(4))/365)...
    + b(5)*(1 + sin(2*pi*(t - b(6))/365)).*sin(2*pi*t/365) - Nairobi;
x = lsqnonlin(modelfun,init,[],[],options)
Gamma = TruncatedF(x(1), x(2), x(3), x(4), x(5), x(6), t);

J=Nairobi-Gamma

J

[B,BINT,W1] = regress(J(2:end),J(1:end-1))


--------------------------KappaNairobi--------------------------
% Prices at t, X(t)
Pt = J(2:end);

% Prices at t-1, X(t-1)
Pt_1 = J(1:end-1);

% Discretization for daily prices
dt = 1/365;

% PDF for discretized model
mrjpdf = @(Pt, phi, sigmaSq) ...
    exp((-(Pt-phi.*Pt_1).^2)/(2.*sigmaSq)).* ...
    (1/sqrt(2.*pi.*sigmaSq));

% Constraints:
lb = [-Inf 0];
ub = [1 Inf];

% Initial values
x0 = [0 var(J)];

% Maximum likelihood estimation
params_Nbi = mle(Pt,'pdf',mrjpdf,'start',x0,'lowerbound',lb,'upperbound',ub,...
    'optimfun','fmincon');

% Calibrated parameters

kappa_Nai = params_Nbi(1)

sigma_Nai = sqrt(params_Nbi(2))

-----KappaMombasa-----

% Prices at t, X(t)
Pt = M(2:end);
```

```
% Prices at t-1, X(t-1)
Pt_1 = M(1:end-1);

% Discretization for daily prices
dt = 1/365;

% PDF for discretized model
mrjpdf = @(Pt, phi, sigmaSq) ...
    exp((-(Pt-phi.*Pt_1).^2)/(2.*sigmaSq)).* ...
    (1/sqrt(2.*pi.*sigmaSq));

% Constraints:
lb = [-Inf 0];
ub = [1 Inf];

% Initial values
x0 = [0 var(M)];

% Maximum likelihood estimation
params_Msa = mle(Pt,'pdf',mrjpdf,'start',x0,'lowerbound',lb,'upperbound',ub,...
    'optimfun','fmincon');

% Calibrated parameters

kappa_Msa = params_Msa(1)

sigma_Msa = sqrt(params_Msa(2))


--------------------------raintemp--------------------------

%clear; clc; close all;

 Z = xlsread('kiambu_rain_temp_data.xlsx', 'Sheet1');

 DateTime = Z(:,1);
 Rain = Z(:,2);
 Temp = Z(:,3);

 MATLABDates = x2mdate(DateTime,0,'datetime')

 save('MATLABDates.mat')
 save('Rain.mat')
 save('Temp.mat')


--------------------------kirinyaga_temp--------------------------

%clear; clc; close all;

load('Temp.mat')
x = length(Temp);
t = (1:1:x)';

init = [20, 1, 1, 10, 1, 10];
options = optimoptions(@lsqnonlin,'Algorithm','trust-region-reflective');
modelfun = @(b) b(1) + b(2)*t + b(3)*cos(2*pi*(t - b(4))/365)...
```

```
    + b(5)*(1 + sin(2*pi*(t - b(6))/365)).*sin(2*pi*t/365) - Temp;
x = lsqnonlin(modelfun,init,[],[],options)
Alpha = TruncatedF(x(1), x(2), x(3), x(4), x(5), x(6), t);

E=Temp-Alpha

E

[B,BINT,V2] = regress(E(2:end),E(1:end-1))


--------------------------TruncatedFTemp--------------------------

function Alpha = TruncatedF(a0,a1,a2,a3,a4,a5,t)
% Seasonal function with trend, the truncated Fourier transform
%Gamma = TruncatedF(x(1), x(2), x(3), x(4), x(5), x(6), t);
Alpha = a0 + a1*t + a2*cos(2*pi*(t-a3)/365)+ a4*(1+sin(2*pi*(t - a5)/365)).*sin(2*pi*t/365);
end


--------------------------antitheticMC--------------------------

function [call, put ] = antitheticMCtrial (K, TiN, TiM, S, kappa_Nai, kappa_Msa,
sigma_Nai, sigma_Msa, h, T, rho)
K = 500;
N = 10000000;
r=0.07
h=1/N;
T=1;
rho=0.6060

TiN=6000;
TiM=5000;

a0 = 4.9743;
a1 = -0.0001;
a2 = -0.8866;
a3 = -2.4918;
a4 = 0.7568;
a5 = -0.0273;
%n=50;

for i=1:N
    S(i,:)=a0 + a1*i + a2*cos(2*pi*(i-a3)/365)+ a4*(1+sin(2*pi*(i - a5)/365)).*sin(2*pi*i/365);
    %a0+a1*(i,:)+a2*cos*(2*pi S(i,:).*(1+r*h+sigma*dW(i,:));
end

% We generate two standard Gaussian vectors
U = randn(N,1); V = randn(N,1);
% We generate the antithetic random vectors
nU = -U ; nV = -V;



% We generate final prices for both assets from both random vectors
pTi_1 = TiN+ S(i,:)-S(i)-kappa_Nai*(TiN-S(i))*T+sigma_Nai.*sqrt(T).*U
nTi_1 = TiN+ S(i,:)-S(i)-kappa_Nai*(TiN-S(i))*T+sigma_Nai.*sqrt(T).*nU
pTi_2 = TiM+ S(i,:)-S(i)-kappa_Msa*(TiM-S(i))*T+sigma_Msa.*
```

```
(sqrt(T).*V.*rho+sqrt(1-rho).*sqrt(T).*V)
nTi_2 = TiM+ S(i,:)-S(i)-kappa_Msa*(TiM-S(i))*T+sigma_Msa.*
(sqrt(T).*nV.*rho+sqrt(1-rho).*sqrt(T).*nV)

% We compute the payoff vector for the call for both random vectors
resCp = max((pTi_1 - pTi_2) - K, 0);
resCn = max((nTi_1 - nTi_2) - K, 0);
% We compute the payoff vector for the put for both random vectors
resPp = max(K - (pTi_1 - pTi_2), 0);
resPn = max(K - (nTi_1 - nTi_2), 0);

% We compute the average between normal and antithetic payoffs
resC = 0.5*(resCp + resCn);
resP = 0.5*(resPp + resPn);

% We finally discount the average of the payoff
call = exp(-r*T) * mean(resC);
put = exp(-r*T) * mean(resP);
end


--------------------------stderrorMC--------------------------

sqrt((sigma_Nai)^2/10)
sqrt((sigma_Nai)^2/100)
sqrt((sigma_Nai)^2/1000)
sqrt((sigma_Nai)^2/10000)
sqrt((sigma_Nai)^2/100000)
sqrt((sigma_Nai)^2/1000000)
sqrt((sigma_Nai)^2/10000000)

sqrt((sigma_Msa^2)/10)
sqrt((sigma_Msa^2)/100)
sqrt((sigma_Msa^2)/1000)
sqrt((sigma_Msa^2)/10000)
sqrt((sigma_Msa^2)/100000)
sqrt((sigma_Msa^2)/1000000)
sqrt((sigma_Msa^2)/10000000)
```