



Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BBS ACTUARIAL SCIENCE, BBS FINANCIAL ENGINEERING, BBS FINANCIAL
ECONOMICS
END OF SEMESTER EXAMINATION BBSA
2204: FINANCIAL MATHEMATICS II

DATE: 19th January 2022

Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

Question One

Part 1

- i. Describe what is meant by the “no arbitrage” assumption in financial mathematics.

[2]

A 9-month forward contract is issued on 1 April 2015 on a stock with a price of £6 per share on that date. Dividends are assumed to be received continuously and the dividend yield is 3.5% per annum.

- ii. Calculate the theoretical forward price per share of the contract, assuming no arbitrage and a risk-free force of interest of 9% per annum.

[3]

The actual forward price per share of the contract is £6.30 and the risk-free force of interest is as in part (ii).

- iii. Outline how an investor could make an arbitrage profit.

[2]

[Total 7]

Part 2

An insurance company has sold a pension product to an individual. Under the arrangement, the individual is to receive an immediate annuity of £500 per year annually in arrear for 12 years. The insurance company has invested the premium it has received in a fixed-interest bond that pays coupons annually in arrear at the rate of 5% per annum and which is redeemable at par in exactly eight years.

- i. Calculate the duration of the annuity at an interest rate of 4% per annum effective. [2]
 - ii. Calculate the duration of the bond at an interest rate of 4% per annum effective. [3]
 - iii. State with reasons whether the insurance company will make a profit or a loss if there is a small increase in interest rates at all terms. [2]
- [Total 7]

Part 3

In January 2014, the government of a country issued an index-linked bond with a term of two years. Coupons were payable half-yearly in arrear, and the annual nominal coupon rate was 6%. The redemption value, before indexing, was £100 per £100 nominal. Interest and capital payments were indexed by reference to the value of an inflation index with a time lag of six months.

A tax-exempt investor purchased £100,000 nominal at issue and held it to redemption. The issue price was £97 per £100 nominal.

The inflation index was as follows:

<i>Date</i>	<i>Inflation Index</i>
July 2013	120.0
January 2014	122.3
July 2014	124.9
January 2015	127.2
July 2015	129.1
January 2016	131.8

- i. Set out a schedule of the investor's cashflows, showing the amount and month of each cashflow. [3]
 - ii. Determine the annual effective real yield obtained by the investor to the nearest 0.1% per annum. [5]
- [Total 8]

Part 4

The annual rates of return from an asset are independently and identically distributed. The expected accumulation after 20 years of £1 invested in this asset is £2 and the standard deviation of the accumulation is £0.60.

i. Calculate the expected effective rate of return per annum from the asset, showing all the steps in your working. [4]

ii. Calculate the variance of the effective rate of return per annum. [4]

[Total 8]

[Total Marks 30]

Question Two

Part 1

- i. State the three conditions that are necessary for a fund to be immunized from small, uniform changes in the rate of interest. [3]
- ii. A pension fund has liabilities of £10m to meet at the end of each of the next ten years. It is able to invest in two zero-coupon bonds with a term to redemption of three years and 12 years respectively. The rate of interest is 4% per annum effective.

Calculate:

- a) The present value of the liabilities of the pension fund
 - b) The duration of the liabilities of the pension fund
 - c) The nominal amount that should be invested in the zero-coupon bonds to ensure that the present values and durations of the assets and liabilities is the same [7]
- iii. One year later, just before the pension payment then due, the rate of interest is 5% per annum effective
 - a) Determine whether the duration of the assets and the liabilities are still equal.
 - b) Comment on the practical usefulness of the theory of immunization in the context of the above result.

[6]

[Total 16]

Part 2

State the characteristics of index-linked government bonds.

[Total 4]

[Total 20 Marks]

Question Three

Part 1

A company is undertaking a new project. The project requires an investment of £5m at the outset, followed by £3m three months later.

It is expected that the investment will provide income over a 15-year period starting from the beginning of the third year. Net income from the project will be received continuously at a rate of £1.7m per annum. At the end of this 15-year period there will be no further income from the investment.

Calculate at an effective rate of interest of 10% per annum:

- i. The net present value of the project [3]
- ii. The discounted payback period [4]

A bank has offered to loan the funds required to the company at an effective rate of interest of 10% per annum. Funds will be drawn from the bank when required and the loan can be repaid at any time. Once the loan is paid off, the company can earn interest on funds from the venture at an effective rate of interest of 7% per annum.

- iii. Calculate the accumulated profit at the end of the 17 years. [4]
- [Total 11]

Part 2

A one-year forward contract on a stock is entered into on 1 January 2011 when the stock price is £68, and the risk-free force of interest is 14% per annum. The stock is expected to pay an annual dividend of £2.50 with the next dividend due in eight months' time. On 1 April 2011, the price of the stock is £71, and the risk-free force of interest is 12% per annum. The dividend expectation is unchanged.

Calculate the value of the contract to the holder of the long forward position on 1 April 2011.

[Total 9]

[Total 20 Marks]

Question Four

Part 1

The value of the assets held by a pension fund on 1 January 2010 was £10 million. On 30 April 2010, the value of the assets had fallen to £8.5 million. On 1 May 2010, the fund received a contribution payment of £7.5 million and paid out £2 million in benefits. On 31 December 2010, the value of the fund was £17.1 million.

- i. Calculate the annual effective money-weighted rate of return (MWRR) for 2010. [3]
- ii. Calculate the annual effective time-weighted rate of return (TWRR) for 2010. [3]
- iii. Explain why the MWRR is higher than the TWRR for 2010. [2]

The fund manager's bonus for 2010 is based on the return achieved by the fund over the year.

- iv. State, with reasons, which of the two rates of return calculated above would be more appropriate for this purpose. [2]
- [Total 10]

Part 2

The annual yields from a fund are independent and identically distributed. Each year, the distribution of $1 + i$ is log-normal with parameters $\mu = 0.05$ and $\sigma^2 = 0.004$, where i denotes the annual yield on the fund.

- i. Calculate the expected accumulation in 20 years' time of an annual investment in the fund of £5,000 at the beginning of each of the next 20 years. [5]
 - ii. Calculate the probability that the accumulation of a single investment of £1 made now will be greater than its expected value in 20 years' time. [5]
- [Total 10]

[Total 20 Marks]

Question Five

Part 1

The annual rates of return from a particular investment, Investment A, are independently and identically distributed. Each year, the distribution of $(1 + i_t)$, where i_t is the rate of interest earned in year t , is log-normal with parameters μ and σ^2 . The mean and standard deviation of i_t are 0.06 and 0.03 respectively.

- i. Calculate μ and σ^2 .

[5]

An insurance company has liabilities of £15m to meet in one year's time. It currently has assets of £14m. Assets can either be invested in Investment A, described above, or in Investment B which has a guaranteed return of 4% per annum effective.

- ii. Calculate, to two decimal places, the probability that the insurance company will be unable to meet its liabilities if:
 - a. All assets are invested in Investment B.
 - b. 75% of assets are invested in Investment A and 25% of assets are invested in Investment B.

[6]

- iii. Calculate the variance of return from each of the portfolios in (ii)(a) and (ii)(b).

[3]

[Total 14]

Part 2

A ten-month forward contract was issued on 1 September 2012 for a share with a price of £10 at that date. Dividends of £1 per share are expected on 1 December 2012, 1 March 2013 and 1 June 2013.

- i. Calculate the forward price assuming a risk-free rate of interest of 8% per annum convertible half-yearly and no arbitrage.

[4]

- ii. Explain why it is not necessary to use the expected price of the share at the time the forward matures in the calculation of the forward price.

[2]

[Total 6]

[Total 20 Marks]