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Modeling SME credit ratings using non-homogenous backward semi-Markovian approach

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Declaration of Authorship

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

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“*Education Is Not the Filling of a Pail, But the Lighting of a Fire*”

W. B. Yeats

Abstract

Considering the growth in SME lending in Kenya and the obvious risks it posses to the banking sector, we establish a credit risk model that is responsive to the jumps in the economy. This is based on simulation of implied values of credit worthiness over a period of 12 months for 1000 SMEs, in which case we establish a case for the discrete time non-homogeneous semi-Markov approach as a proxy for internal rating model for a portfolio of SME loans. While viewing credit risk as a reliability issue, the model provides a credit indicator which gives a prospective measure of credit risk for an SME portfolio. Banks seeking to comply with the new IFRS9 guidelines can espouse this model to adequately measure impairment of financial instruments.

Key Words: Credit Risk Modeling, IFRS9, Backward Semi-Markov Models

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List of Abbreviations

CBK	Central Bank (of) Kenya
SME	Small (and) Medium Enterprises
SMEBS	Small (and) Medium Enterprise Businesses
IRB	Internal Risk Based
IRM	Internal Rating Mmodels
IFRS	International Financial Reporting Standard
CRA	Credit Rating Agency
CRB	Credit Reference Bureau
AIA	American International Assurance
PD	Probability of Default
LGD	Loss Given Default
NPL	Non Performing Loan
ILRFM	Individual Level Reduced Form Model
PRFM	Portfolio Reduced Form Model
CTMP	Contious Time Markov Process
DTMP	Discrete Time Markov Process
DTNHMP	Discrete Time Non Homogeneous Markov Process
DTNHSMP	Discrete Time Non Homogeneous Semi(-)Markov Process
CAMELS	Capital Assets Manamegent Earnings Liquidity (and) Sensitivity

Dedicated to my daughter, Christiana Waridi

Chapter 1

Introduction

1.1 Background to the study

Credit creation is the main income generating activity for commercial banks, despite its downside being inherent credit risk exposure. Credit risk is the potential of a bank borrower or counterparty defaulting in its obligations in accordance with agreed terms. "Default" is the failure of the obligor to make the defined payments, and hence all instruments may be defined as being "in default" or " not in default" which is a primitive rating system.

In Kenya, financial reforms allowed the formation of many financial institutions to meet the high credit demand. Furthermore, there has been a notable boom experienced in the volume and value of loans issued in the Kenyan banking sector in the last few years, and especially in the SME sector. However, the cost of credit for SMEs remains high due to a number of factors as reported by WorldBank (2015), one of them being the limited use and sharing of positive information about borrowers. The same report recommends positive information sharing from all credit providers including payment service providers and utility companies to improve on data quality. According to CBK (2013a), another contributing factor to the cost of credit is the number of non-performing loans which is a major indicator of credit risk.

In its 2013 report, the Central Bank of Kenya confirms that credit risk has the largest influence on the soundness of financial institutions which is a view echoed by Kargi (2011). Credit risk management underpins credit risk modeling, and the efficacy of such models will either introduce systemic and model risks or ensure a robust risk management system. CBK (2013b) issued new risk guidelines in which the Central Bank of Kenya acknowledges internal rating models (IRM) for banks as effective tools for credit risk management.

In accordance with the guidelines offered by *Basel II* and *Basel III*, banks are allowed to integrate their own estimates of different risk exposures adopting the internal-risk-based (IRB) approach (Basel et al., 1999). In these guidelines, banks are required to calculate a one-year Value at Risk using a 99% confidence level $\alpha = 0.01$ from the notion of "incremental risk charge", for losses associated with credit products. According to IASB (2015) , banks are required to monitor the changes in credit quality to enable them recognise and measure the Expected Credit Losses of a given portfolio in adopting the IFRS 9.

In relation to monitoring potential loan default, ratings migration indicate a change in credit quality of an entire loan portfolio (Altman & Saunders, 1998). These transition matrices reveal the distribution of the rated entities based on their initial ratings at the end of a given time interval; say a year. It has been recognized that the transitional analysis approach would offer support for banks to concisely model credit rating transition probabilities as an important part of the management of credit risk (Tsaig, Levy, & Wang, 2011). A focal issue in the Basel directive emphasizes that the level of capital a bank holds should correspond to the riskiness of its underlying assets portfolio.

One of the major indicators of this riskiness is credit risk whose impact can be recognised in two dimensions. First, the direct impact of non-performing loans on the banking sector has resulted in erosion of bank's profitability, stagnation of economic resources and cautious behaviour of corporations and consumers due to decline in the confidence of the financial system (Gitahi, 2013). In her work, she confirms that bad borrowers have exploited information asymmetry to create multiple bad debts in the banking industry in Kenya leading to poor performance by individual commercial banks and instability in the banking sector.

The second dimension of the impact of credit risk incurred by banks is the decline in the quality and the increase in the cost of credit. An increase in the number of defaulting loans in a bank's loan book would lead to a tightening of conditions to enable the bank manage the losses being incurred. These conditions may include an increase in the interest rates charged for a particular portfolio, a demand for collateral where none was required, or a reduction for the term, of the facility being offered. All this facets have a direct impact on the cost and quality of loans being offered and how accessible the loans are to a customer.

In Kenya, non-performing loans have led to the collapse of 41 banks as at end of December 2016. While commercial banks have faced a myriad of challenges over the years, the main cause of default persists, and is directly related to not understanding the credit standards of borrowers and poor credit risk management (CBK, 2012). Ideally, credit decisions should only be made after a thorough evaluation of the risk profile of the credit facility and the characteristics of the borrower. This information is availed through credit reports from the licensed CRBs and CRAs.

From this study, we establish the environment in which this credit ratings′ transition takes place. Once this is achieved, a reliability model is developed to assist financial institutions to have a 360° view of the client being rated and hence increase the reliability of credit ratings. The findings will enable the said institutions to adequately carry out portfolio choice and risk assessment, asset pricing, and in the evaluation of regulatory capital as stated in the New Basel Accord.

1.2 Small and Medium Enterprise credit rating in Kenya

In Kenya, the term SME stands for "small and medium enterprises". They are majorly distinguished by the number of employees and annual turnover. Micro Enterprises have up to 10 employees with a turnover of less than Ksh.0.5 million, Small Enterprises have between 10 to 50 employees with a turnover of between Ksh.0.5 million and Ksh.5 million; while Medium Enterprises have between 50 to 100 employees with a turnover that is greater than 5 million Kenya Shillings.

Some unique characteristics of SMEs which are broadly identified are: low capitalization, limited recognizable assets, short business lifespan, poor access to capital markets, very large cash intensity in transactions, absence of dependable credit information/history, poor financial disclosure, and high credit risk perceptions coupled with high borrowing cost.

SMEs in Kenya have the significant contribution to the economy namely: they represent 90% of enterprises, provide employment to 60% of the total employed population and contribute 20% to the total national GDP. IN fact, SME as a sector has had a particular allure for banks since it has shown to provide higher returns, higher growth prospects and a myriad of cross selling opportunities. However, with this many opportunities, the lenders are not short of risks too. For instance, while the general non-performing loans (NPLs) rate stands at 6.8% on average for the whole bank lending book, SME lending presents NPLs of 9% on average (CBK, 2015).

For this reason, credit evaluation for SMEs ofttimes, has presented a challenge for financial institutions. It is often not cost effective to apply the intensive, bespoke credit analysis used for large and global corporate customers. Yet these SMEs and commercial customers have generally been quite susceptible to purely statistical approaches of credit rating, as is regularly applied to consumer loans. However, in order to keep lending costs down and also improve the predictability, consistency and objectivity of credit decisions, the tactical application of a standardized model does offer considerable advantage.

Due to lack of robust internal credit risk modeling tools, lenders have turned towards Credit Rating Agencies (CRAs) are financial firms that have been incorporated to analyze financial statuses, and provide opinions on default risk in form of ratings that extend the " not in default" category of the primitive rating system as defined by QuantPerspective (2012). These ratings cover economic and industrial analysis, customer profiling, business risk review and financial statements analysis. Firms may take on one of nine credit ratings as well as the default (D) rating and Not Rated (NR) state. They are therefore represented chronologically from the best credit quality to the worst as: AAA, AA, A, BBB, BB, B, CCC,CC,C, D, NR (Not Rated).

Credit Ratings have not been exploited to their potential in Kenya, because credit information sharing is a new concept. It was passed into law in 2008 and implemented in 2013 according to the laws of Kenya (CBK, 2013b). In essence, availability of quality and sufficient data has been the biggest hindrance to this kind of analysis. Also, until 2014, there were no licensed CRAs in Kenya, and hence, no existing SME ratings prior to that.

1.3 Research problem

The Basel Accord and International Financial Reporting Standards (IFRS9) provide guidelines for measuring and managing credit risk. One of the main components of these measures, is the application of probabilities of default to determine the Expected Losses over a predefined time period. These credit risk measures are the basis on which capital reserves for banks are made since they are prospective.

Though helpful in quantifying the requirements of The Basel Accord and IFRS 9, Credit Rating Agencies fail to look at credit risk as a reliability issue. The ratings they provide do not give the migration patterns over a given period of time. Failure to take into account rating transitions is equal to believing that a rated entity shall remain in the same rating over its lifetime; a statement that is untrue. Changes in the credit quality of a portfolio over time, in fact, has a direct correlation to the capital allocation made for a given portfolio (IASB, 2015).

To the best of my knowledge, the only study seeking to understand the credit risk models in Kenya is the research conducted by Wagacha and Othieno (2016). The researchers sought to determine if consumer credit risk in Kenya is modelled as a semi-Markov process. In their conclusion, they site the challenge they encountered with acquiring actual data for validating their model.

This study, therefore, is an extension of the propositions of Wagacha and Othieno (2016) that the semi-Markovian approach can be applied in modelling credit risk in Kenya. Specifically, it seeks to model SME ratings using a Discrete Time Non-Homogeneous Semi-Markov approach while considering the initial and final backward recurrence process to be able to provide a reliability function for the portfolio. The study tests the theoretical model using credit data on 2500 SME accounts obtained from Metropol Credit Reference Bureau for the period of January 2015 to January 2016.

1.4 Research objective

The main objective of the study is to develop a credit rating model for SME credit in Kenya.

The supporting objectives are to:

1. Apply a hybrid Merton approach in modeling credit worthiness of an SME.
2. Apply a non-homogenous backward semi-Markov approach in modeling credit risk in a portfolio of SMEs in Kenya.

1.5 Research hypothesis

The research hypothesis for this study is: The Kenya SME credit ratings are modeled on a non-homogeneous backward semi-Markovian process.

1.6 Value of the study

The knowledge obtained in this study should enable commercial banks to adequately carry out portfolio choice, risk assessment and asset pricing. The reliability model will be a parameterized tool that can easily be input into IRMs for assessment of credit quality.

The study will contribute to literature as an empirical proof of the efficacy of the DTNHSMP in credit transition modeling. In theory, the study demonstrates that a backward semi-Markov approach to credit risk management would yield more accurate outcomes for measuring a financial institutions credit risk exposure.

For the Central bank of Kenya, the model developed may be used to establish a standard for evaluation of risk capital requirements as stated in the New Basel Accord and IFRS 9.

1.7 Scope of the study

The study included all Micro, Small and Medium Enterprises that have a SMEBs rating as offered by Metropol Credit Reference Bureau. The data under consideration is monthly credit ratings from December, 2015 to December, 2016.

1.8 Assumptions

To obtain the transition probabilities, the following assumptions are made:

1. Firms are properly rated
2. Firms with no response have the same migration probability as those with responses which make up *25%* of all the borrowing SMEs.

Chapter 2

Literature review

2.1 Credit rating migration patterns

Also referred to as transition or migration matrices, these are tools that are used for assessing the applicability of Credit Ratings in credit risk modelling. Grzybowska, Karwanski, and Orłowski (2010) define a transition matrix as a method of evaluating intensities and is based on directly observed historical performance.Lando and Skødeberg (2002) refer to transition matrices as a tool for reporting ratings migration.

These ratings change with time and one way of observing their evolution is by means of Markov processes (Jarrow, Lando, & Turnbull, 1997). Built on estimates of rating transition probabilities, transition matrices are the core of credit risk management because they provide the probability of an entity remaining at its current rating or migrating to another rating. These matrices contain important elements that relay the stability of a rating and thus should be included in a credit rating model to ensure maximum efficiency of IRMs used by financial institutions.

Previously, there have been empirical studies applying both discrete time and continuous time Markov Processes to model credit risk spread as components of PD and LGD, (Camacho Valle, 2013). However, the suitability of Markov processes has been challenged with the main issues raised being the dependence on chronological time of the rating evaluation (Nickell, Perraudin, & Varotto, 2000); duration inside a state (Carty & Fons, 1994); and the dependence of the new ratings on the previous ratings and not just the last evaluated (Carty & Fons, 1994); (Nickell et al., 2000). To solve some of the issues raised, different semi-Markov models have been proposed in the recent past by Guglielmo D'Amico, Janssen, and Manca (2004); D'Amico, Janssen, and Manca (2005); Vasileiou and Vassiliou (2006); Guglielmo D'Amico, Janssen, and Manca (2010).

D'Amico, Di Biase, Janssen, and Manca (2010) demonstrates that semi-Markovian processes solve the non-Markovianity problems wholesomely by introducing the backward process, giving one the opportunity to assign different transition probabilities to the function representative of the duration inside the most recently visited state. To be more precise, the introduction of the backward process at the initial and final times allow one to have a complete knowledge of the waiting times at the beginning and the end of the observation period of the model.

We define the following:

- Initial backward takes into account the time in which the system went into the state and also, if the arrival time is before the beginning of the observed time horizon.
- Final backward takes into account the time in which the last transition before the end of the observation time is done.

In the credit risk modelling problem, a complete understanding of the duration inside current state is of basal importance. The use of forward and backward processes allows the possibility of modeling all the waiting time scenarios that could happen in the neighbours of the initial and final observation times as shown by Guglielmo D'Amico et al. (2010). For this study we focus on the initial and final backward semi-Markov process to model migration patterns.

Therefore, the solution to a credit risk problem would also be considered a reliability problem according to Jacques and Raimondo (2007). This said, the ratings carried out by CRAs become a reliability measure of a firm, and the down state is considered a downgrade. Considering a random and discrete time, we will pursue Linda (2004) hypothesis that the next transition depends on the current state and its dependence to previous states, but after a random duration (waiting time), making a Semi-Markov environment a better fit than a Markov environment.

Credit transition matrices characterize changes in credit quality of an owing entity and are cardinal inputs to risk management applications including portfolio risk assessment, pricing of bonds and assessment of risk capital (Schuermann, 2008). This chapter provides an overview of credit risk management tools and credit migration matrix modelling approaches which should enable us to model migration patterns that would ensure effective credit risk management.

2.2 The risk management in Kenya

CBK (2010) points to the application of the CAMELS rating system, an international benchmark used by the central Bank of Kenya in analyzing the soundness of financial institutions. The aspects of a bank's condition that are assessed are: capital adequacy, assets, management capability, earnings, liquidity and sensitivity. Fredrick (2013) recognizes that a number of studies previously conducted have examined the efficacy of the CAMELS ratings and generally, conclude that publicly available data together with regulatory CAMELS ratings can predict problem banks. However, in a case study for the American International Assurance- Vietnam, (AIA), it was established that the CAMELS model overlooks the provision as well as allowance for loan loss ratios.

Heuristic modeling has also been identified as a major component of a majority of Kenyan banks′ credit risk models. This modeling uses judgemental rules and is based on how well the management know an entity; a very shortsighted and subjective approach. In a case study by Kithinji (2010), he alludes to the fact that subjective decision making by a bank's management may lead to credit being extended to business entities that they are affiliated to, or to persons with a reputation for non-financial acumen e.g celebrities, and hence create a risk exposure to the bank.

2.3 Current models

Camacho Valle (2013) identifies three broad approaches to modelling credit risk as reduced form models, structural models and factor models. On the other hand, Linda (2004) identifies two broad methodologies for modelling credit risk: a reduced form approach making use of intensity models to estimate stochastic hazard rates and an option-theoretic structural approach pioneered by Merton (1974).

Both Valle and Linda concur that the reduced form models decompose risky debt prices in order to estimate the random intensity process underlying default while the structural approach models the economic process of default. This points out that reduced form models mainly focus on the accuracy of the probability of default (PD). However, under the Merton (1974) structural model, default occurs after ample warning (Linda, 2004). Consequently, it is postulated that default occurs after gradual decent in the assigned rating for an entity to the point of default. This implies that the PD steadily approaches zero as the time to maturity approaches (Camacho Valle, 2013).

Finally, Linda (2004) suggests that the reduced form models or intensity based models would provide more realistic credit spreads. While structural models view default as the outcome of a gradual deterioration process in an entities value, intensity-based models view default as a sudden and unexpected event, resulting in PD estimates that are more consistent with empirical observations.

In conclusion, Camacho Valle (2013) noted that reduced form modes can be classified as portfolio reduced form model (PRFM) and individual level reduced form model (ILRFM). He further highlights that the ILRFM is based on a credit scoring system while the PRFM assumes an intensity jump process. PRFMs are reported to perform better in capturing the properties of credit risk (Hao & Zhang, 2009). However, inspite of RFM being realistic, they are not as tractable hence, this study takes a PRFM approach while employing a hybrid of the structural form model and reduced form model for credit risk assessment (Merton, 1976).

2.4 A case for the Semi-Markov models

Within the PRFMs, Discrete Time Markov Processes (DTMP) and Continuous Time Markov Processes (CTMP) have been applied in empirical studies to model credit risk spread and as components of PD and LGD (Camacho Valle, 2013). However, many papers highlight the problem of unsuitability of Markov processes in the credit risk environment.

The main problem of non-Markovianity is the duration inside a state (Carty & Fons, 1994); the dependence of the current state where the current state depends on various previous states assigned to an entity (Carty & Fons, 1994); (Nickell et al., 2000); and the underestimation of migration probabilities by DTMPs (Linda, 2004). The three problems have been addressed by a number of authors in some past papers: Guglielmo D'Amico, Janssen, and Manca (2005) using the semi-Markov processes.

Guglielmo D'Amico et al. (2005) presents a generalisation of the semi-Markov processes transition probabilities which challenges many set backs raised above against the Markov processes, however, they fail to address the downward problem. In Guglielmo D'Amico, Janssen, and Manca (2016), the results on the asymptotic behaviour of the discrete time non-homogeneous semi-Markov processes are extended to the semi-Markov transition probabilities with a backward process. This has a practical application of allowing the recovery of the distribution function of the time of default conditioned on the rating and its duration.

2.5 Conclusion

The literature reviewed in this study highlights the milestones made so far in the development of tools used for the management of credit risk. Many tools have been empirically tested to model credit risk spread and come up with components of the probability of default and the loss given default. In more advanced economies, the CRAs also compute and make available transition matrices that provide a key component in credit risk assessment. In Kenya, there exists a very young credit information sharing platform with scarce data.

In developed economies, CRAs publish "rating transition matrices" that show the frequency of the migration of ratings over a given time interval, say 1 year. These changes in the credit rating occur due to industry and domicile effects, and business cycle effects (Nickell et al., 2000). In Kenya, credit risk management guidelines are provided by the CBK which has recommended that banks must receive sufficient information to enable a comprehensive evaluation of the true risk profile of an entity, the bare minimum being the borrower's credit rating (CBK, 2013b).

However, we are cognizant of the fact that these ratings change over time, leading to potential credit risk to the banks. These transitions have not been addressed by the CBK risk management guidelines, and hence, the banks are left without tools for monitoring credit quality which would allow one to, more accurately, determine the overall characteristics of the credit portfolio and adequacy of loan loss reserves.

While many models have been reviewed for appropriateness in developing transition matrices, the discrete time non-homogeneous semi-Markov environment is preferred,(Guglielmo D'Amico et al., 2016), since it is the only model that allows one to factor both provisions and recoveries for a loan portfolio when setting the loan loss provisions. This is made possible by the capability of the backward semi-Markovian approach which allows an entity to switch between rating classes i.e. an entity can move from a state of being in default at time s to a performing state in time $s + 1$; interchangeably which is the current state of the industry in kenya.

In view of the current increase in the activity witnessed for SME lending, there is a need to evaluate the credit rating migration patterns that exist, and develop a reliability model for the same in which case, this study assumes a discrete time non-homogeneous backward semi-Markov approach. In the following chapter we are going to simulate the initial PDs and model the credit migrations based on a backward semi-markov approach and finally test the model with real data for SMEs that have been rated in Kenya.

Chapter 3

Methodology

The study seeks to develop a reliability model for an SME portfolio by observing past credit rating migration patterns of an entity, while adopting to a discrete time non-homogeneous semi-Markov credit risk framework. This chapter contains the research design, target population, data collection and the model framework.

3.1 Research design

The study is an experimental-based research work involves simulation of credit ratings using a jump-diffusion process. The migration probabilities of the said ratings will be determined, and then, the developed model will be tested using actual credit ratings from SMEs that have been rated in Kenya.

3.2 Population and sampling

The statistical analysis utilises a sample containing credit ratings for 2,500 Kenyan SMEs gathered from Metropol Credit Reference Bureau database. The data in focus is the monthly credit ratings, which is used to validate the migration model.

3.3 Data collection methods

First, Monte Carlo simulation is employed to generate the credit ratings of a sample of 1000 SMEs for a period of 12 months. The credit rating data is then used to generate an initial transition matrix.

Secondary data for validating the model is obtained from Metropol Credit Reference Bureau and includes historical monthly credit ratings. The reference period is January 2015 to January 2016 (13 months). The analysis considers 30,000 data points from 2,500 firms, all of which have data points over the whole period.

3.4 The Modeling framework

3.4.1 SME credit ratings

In Kenya, SME Credit Rating is an independent assessment of the overall condition of an SME. The essence of the rating is to measure the financial strength of the firm and the probability of default on a financial obligation. There are seven different classes of ratings, in which, six are up states (working states), while one is a down state (default state). With inadequate rating data available, it was apparent that a standard rating model for different loaners was required.

As in the paper by Wagacha and Othieno (2016) , the study adopted a logistic regression model to determine the initial rating of a customer which was in line with the current practice in the Kenyan Banking industry. If x denotes the number of attributes (being k in number) and *b* being the weights attached to them, the score obtained on scoring instance i is :

$$Score_i = b_1 x_{i1} + b_2 x_{i2} + \cdots + b_k x_{ik} = b' x_i \tag{3.1}$$

where *b* and *x* are vectors.

$$ProbDefault_i = F(Score_i) \tag{3.2}$$

in which case F is defined as the logistic distribution function $\Lambda(z)$ in the following manner

$$\Lambda(z) = \frac{\exp(z)}{1 + \exp(z)} \tag{3.3}$$

Applying this to the first equation results in

$$\text{Prob Default}_i = \Lambda(\text{ Score}_i) = \frac{\exp(b' x_i)}{1 + \exp(b' x_i)} = \frac{1}{1 + \exp(-b' x_i)} \tag{3.4}$$

Adopting the suggestion made by De Andrade and Thomas (2007) that one can espouse the corporate structural models if the consumers behavioral score is used as a surrogate for credit worthiness of the borrower. In this study, the Merton Model, is considered to be the most notable for modeling consumer credit risk (Marrison, 2002). Consequently, the rating is done using this model for simulated values of the behavioural scores.

De Andrade and Thomas (2007) considered credit worthiness to be a continuous time diffusion process with jumps. Thus $S(t)$ satisfies

$$dS_i(t) = \mu_i dt + \sigma_i dW_t + A dY_t \tag{3.5}$$

where μ_i is the drift of the process and corresponds to the natural drift in credit worthiness caused in part by the aging of the loan account, and dW_t is a Brownian motion process describing the natural variation in the behavioral scores while dY_t is a Poisson jump process with the magnitude of a random jump being described by A; which is included to model jumps in behavioral scores due to major events like a change in the economic situation which could lead to the population odds changing dramatically.

3.4.2 Discrete time non-homogeneous semi-markov process

In this section we will have an introduction of the Discrete Time Non-Homogeneous Semi-Markov Process $(DTNHSMP)$ as defined by Guglielmo D'Amico et al. (2016). The authors consider two random variables that are jointly evolving. $J_n : \omega \to I$ with state space $I = \{1, 2, \cdots, m\}$ represents the state at the $n-th$ transition. $T_n : \omega \to N$, represents the time of the $n-th$ transition. in this case, the process (J_n, T_n)is a non-homogeneous Markovian renewal process.

This process is associated with the Kernel $\mathbf{Q} = [Q_{ij}(s,t)]$ for all $i, j \in I, t \geq 0$ and is denoted by the probability

$$Q_{ij}(s,t) = P[J_{n+1} = j, T_{n+1} \leq t \mid J_n = i, T_n = s] \tag{3.6}$$

These transition probabilities can be always represented in the form

$$P[J_{n+1} = j \mid J_n = i, T_n = s] =: F_{ij}(s,t)p_{ij}(s) \tag{3.7}$$

From equation (3.6), we define $[p_{ij}(s)] = \mathbf{P}(s)$ as the transition matrix of the embedded Markov Chain J_n. Also, we define $F_{ij}(s,t) = Q_{ij}(s,t)$ / $p_{ij}(s)$ if $p_{ij}(s) > 0$ while $F_{ij}(s,t)$ can be an arbitrary distribution function such that $F_{ij}(0) = 0$ if $p_{ij} = 0$. The function $F_{ij}(s,t)$ is the cumulative density function (CDF) of some random variable T_{ij} which is the a *holding time* in state i, if the next state will be j.

This function $F_{ij}(s,t)$ is what distinguishes the Discrete Time Non-Homogenous Markov Process from the Discrete Time Non-Homogenous Semi-Markov Process since in the

Markov environment, it has to be a geometric distribution function while in the semi-Markov case, the distribution function may be of any type. By means of $F_{ij}(s,t)$, we can take into account the duration problem since we know that the transition probabilities depend on the time spent in a certain state.

Given that $Q_{ij}(s,t) = p_{ij}(s)\ F_{ij}(s,t)$, then the function

$$H_i(s,t)\colon = P[T_{n+1} \le t \mid J_n = i, T_n = s] = \sum_{j=1}^{m} Q_{i,j}(s,t) \tag{3.8}$$

is the CDF of some random variable which is denoted by T_i, that is called the *waiting time* in state i when a successor state is unknown.

We also have the function $b_{i,j}(s,t)$, $s \ge 0$ shown below that represents the transition probability from the state i to the state j under condition that duration of the state is equal to s.

$$b_{i,j}(s,t) = P[J_{n+1} = j, T_{n+1} = t \mid J_n = i, T_n = s] = \begin{cases} Q_{i,j}(s,s) = 0 & \text{if } t = s \\ Q_{i,j}(s,t) - Q_{i,j}(s,t-1) & \text{if } t > s \end{cases} \tag{3.9}$$

We define the following :

1. The counting process $N(t), t \ge 0$ of the associated kernel Q as

$$N_t = sup\{n \mid T_n \le t\}, \forall t \in N \tag{3.10}$$

2. and the process $Z = (Z(t), t \ge 0)$ by

$$Z(t) = J_{N(t)} \tag{3.11}$$

which is the NHSMP of Kernel Q and represents, for each waiting time, the state occupied by the process at each time.

The transition probabilities can now be defined as

$$\phi_{ij}(s,t) = P[Z(t) = j \mid Z(s) = i, T_{N(s)} = s] \tag{3.12}$$

where $\phi_{i,j}(s,t)$ represents the probability of staying in the state j at time t, given that at time s the system entered the state i. These transition probabilities are obtained when solving the evolution equation below:

$$\phi_{ij}(s,t) = d_{ij}(s,t) + \sum_{\beta \in 1}^{m} \sum_{\theta=s+1}^{t} \phi_{\beta j}(\theta,t) b_{i\beta}(s,\theta), \tag{3.13}$$

3.4.3 DTNHSMP with initial and final backward recurrence process

To appropriately consider the duration problem, we introduce the backward recurrence time process $C(t) = t - T_{N(t)}$ denoting the time since the last transition (Janssen & Manca, 2007). The transition probability can be conditioned to the backward process and hence generating the transition probabilities with initial and final backward equations as shown below. We adopt the notation for Guglielmo D'Amico et al. (2010).

The initial backward equation is given by

$$^{b}\phi_{ij}(l,s;t) = P[Z(t) = j \mid Z(s) = i, C(s) = s - l] \tag{3.14}$$

This equation represents the semi-Markov transition probability with initial backward recurrence time. It is indicative of the system being in the state i at time s and that it entered in this state at time l and $s - l$ represents the initial backward time.

The final backward equation is given by

$$\phi_{ij}^{b}(s;l',t) = P[Z(t) = j, C(s) = t - l' \mid Z(s) = i] \tag{3.15}$$

This equation represents the semi-Markov transition probability with final backward recurrence time space.It is indicative of the system having entered in the state i at time s and we are seeking to find the probability of being in the state j at time t with the entrance in this state at time l' where $t - l$ is the final backward time.

Putting the initial and backward cases together, we end up with

$$^{b}\phi_{ij}^{b}(l,s;l',t) = P[Z(t) = j, C(s) = t - l' \mid Z(s) = i, C(s) = s - l] \tag{3.16}$$

To obtain the evolution equations, we introduce the following notation:

$$d_{ij}(l,s;t) = \begin{cases} \frac{1-H_i(s-l,t)}{1-H_i(s-l,s)} & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases} \tag{3.17}$$

where $d_{ij}(l,s;t)$ gives the probability that the system does not have any transition from state i between times l and t given that no transition occurred from state i between time l and s.

and

$$b_{ij}(l,s;t) = \frac{(b_{ij}(s-l,t))}{(1-H_i(s-l,s))} \tag{3.18}$$

where $b_{ij}(l,s;t)$ represents the probability to make the next transition from state i to j from time l to time t given that the system does not make transition from the state i between times l and s.

Therefore, the respective evolution equations associated with the equations (3.14),(3.15),(3.16) are:

$$^b\phi_{ij}(l,s;t) = d_{ij}(l,s;t) + \sum_{\beta\in 1}^{m}\sum_{\theta=s+1}^{t} \phi_{\beta j}(\theta,t) b_{i\beta}(l,s;\theta), \tag{3.19}$$

which provides the probability that the system is in the state j at time t given that it was in the state i at time s and entered this state at time l.

$$\phi^b_{ij}(s;l',t) = d_{ij}(s,t)1_{l'=s} + \sum_{\beta=1}^{m}\sum_{\theta=s+1}^{l'} \phi^b{}_{\beta j}(\theta;l',t) b_{i\beta}(s,\theta), \tag{3.20}$$

where $1_{l'=s} = 1$ iff $l' = s$ otherwise, it is equal to 0.

which provides the probability that the system will arrive in the state j just at time $l'$and will remain in this state, without any other transition,up to time $t$given that it entered at time s in state i and

$$^b\phi^b_{ij}(l,s;l',t) = d_{ij}(l,s,t)1_{l'=s} + \sum_{\beta=1}^{m}\sum_{\theta=s+1}^{l'} \phi^b{}_{\beta j}(\theta;l',t) b_{i\beta}(l,s;\theta), \tag{3.21}$$

where $1_{l'=s} = 1$ iff $l' = s$ otherwise, it is equal to 0.

finally gives the probability that the system entered in state j at time l' and remained inside this state without any other transition up to the time t given that it entered in the state i at time $l$and it did not move up to s

3.4.4 The backward semi-Markov reliability credit risk model

The credit risk reliability model is presented in this study. Credit ratings as provided by CRAs provides a reliability degree of a firm. In the case of Metropol's (Kenya) historical database, there are ten different classes of ratings where the first nine are performing states, and D is a defaulting state.

$$
\begin{aligned}
I &= \{S.AAA, S.AA, S.A, S.BBB, S.BB, S.B, S.CCC, S.CC, S.C, S.D\} \\
U &= \{S.AAA, S.AA, S.A, S.BBB, S.BB, S.B, S.CCC, S.CC, S.C\} \\
D &= \{S.D\}
\end{aligned}
$$

For purposes of this model, we follow the standards set by the CBK prudential guidelines provisions for loan classification which is based on the number the loan is past its repayment due date, where the states $Normal(N), Watch(W), Sub-Standard(SS)$ are all transient and the state $Doubtful(D) = Loss(L)$ is an absorbing state. We link the state space I with this current loan classification in the Kenyan Banking Industry as follows: $N = \{S.AAA, S.AA, S.A\}, W = \{S.BBB, S.BB, S.B\}, SS = \{S.CCC, S.CC, S.C\}$ and $D = L = \{S.D\}$.

In the empirical analysis, we solve for the system of equations (3.19) to (3.20) above to find the transition probability functions : ${}^b\phi_{ij}(l, s; t)$, which represent the probability of being in the rating j at time t, being in the rating i at time s, given that the arrival was with the last transition at rating i at time $s - l$. This takes into account the permanence of time of the system in a rating while considering the different probabilities of rating change in a function of the different times of evaluation. The different probability values ${}^b\phi_{ij}(l, s; t)$ and ${}^b\phi_{i-j}(l, s; t)$ solve the downward problem.

After solving the evolution equation (3.21) in the empirical analysis, we resolve for the non-homogeneous backward reliability function; which gives the probability of the firm never going into the default state from the time s up to the time t, given that entrance into the state i was at the time $s - l$, and it was the last transition before time s; and is given by

$$
\begin{aligned}
{}^bR_i{}^b(l, s; l', t) &= P[Z(h) \in U : \forall h \in (s, t) \cap N | Z(s) = i, T_{N(s)} = s - l, T_{N(s)+1} > s] \\
&= \sum_{j \neq D} {}^b\phi_{ij}{}^b(l, s; l', t).
\end{aligned}
$$

Chapter 4

Results and discussions

In the previous section, we define the relationships that exist for a backward non-homogeneous semi-Markov reliability model. We estimate the credit ratings and backward semi-Markov transition probabilities by the Monte-Carlo method and test the model with actual SME credit ratings.

4.1 Simulation analysis

Monte-Carlo method is a widely used technique in approximating a complex process by simulating a large number of random realisations of the process. It is often used when computation of an exact result with a deterministic method is not possible. It was introduced in 1940 by John Von Neumann, Stanislaw Ulam and Nicholas Metropolis, while working on a Manhattan project. Since then its application has spun many fields of study such as mathematics, biology, physics and chemistry.

In finance, many problems encountered are complex and would often not have a numerical solution when statistical or deterministic tools are applied. It was introduced by Hertz (1964) to solve problems related to corporate finance and by Boyle (1977) for simulating a derivative instrument prices. In credit risk analysis, Monte Carlo method has been applied mainly to the estimation of the maximum value likely to be lost if a counter party defaults Glasserman and Li (2005) and also in structural models to estimate the PD (Beem, 2010).

4.1.1 Simulating the credit ratings

Following the logistic regression carried our on the characteristics of each entity, a rating cretarion was determined based on the probabilities of default. The dynamics of the ratings over a *12* month period was then determined. Table 4.1 below represents the initial transition matrix based on the simulations of Ratings on 1000 consumers over 12 months.

TABLE 4.1: Initial Transition Matrix ($P(s)$)

$P(s)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
SAAA	0.2157	0.5128	0.4732	0.4040	0.4000	0.4063	0.3436	0.3394	0.4247	0.4195
SAA	0.1516	0.1943	0.2104	0.2686	0.2012	0.1942	0.1484	0.1308	0.1949	0.1657
SA	0.1303	0.0604	0.1387	0.2012	0.1954	0.1651	0.1150	0.1244	0.1929	0.0975
SBBB	0.1152	0.0316	0.0490	0.1293	0.1106	0.1273	0.0980	0.1075	0.1910	0.0919
SBB	0.1143	0.0232	0.0006	0.0662	0.0672	0.1065	0.0685	0.0835	0.1538	0.0410
SB	0.1124	0.0000	0.0000	0.0019	0.0425	0.0541	0.0367	0.0667	0.0907	0.0067
SCCC	0.1101	0.0000	0.0000	0.0000	0.0162	0.0342	0.0241	0.0533	0.0229	0.0000
SCC	0.1098	0.0000	0.0000	0.0000	0.0007	0.0000	0.0020	0.0047	0.0000	0.0000
SC	0.1091	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SD	0.1053	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

This table provides the probability of an entity migrating from one state to the another as seen at the beginning of the observation period. The assumption here is that the state *SD* is not an absorbing state.

4.1.2 Simulating the transition probabilities

For the non-homogeneous case,the following probabilities apply: The table 4.2 represents the probability that in a time interval *t* there was no new rating for the entity having started at state *i* at the initial time.

TABLE 4.2: Probability of not exiting the initial state $H_i(s,t)$

$H_i(1,t)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
T0	0.9439	0.1542	0.3514	0.5400	0.6926	0.8278	0.8967	0.9604	0.9656	0.9668
T1	0.9304	0.1319	0.3338	0.5172	0.6742	0.8194	0.8873	0.9596	0.9625	0.9638
T2	0.9187	0.1296	0.3309	0.5138	0.6742	0.8163	0.8872	0.9554	0.9593	0.9607
T3	0.9123	0.1090	0.3156	0.5098	0.6702	0.8106	0.8839	0.9495	0.9526	0.9542
T4	0.8677	0.1017	0.3154	0.5098	0.6653	0.8022	0.8787	0.9448	0.9489	0.9507
T5	0.8618	0.0971	0.3079	0.4825	0.6453	0.7978	0.8741	0.9447	0.9477	0.9496
T6	0.8040	0.0886	0.3015	0.4810	0.6434	0.7968	0.8705	0.9404	0.9442	0.9461
T7	0.7779	0.0861	0.2961	0.4783	0.6397	0.7940	0.8681	0.9369	0.9414	0.9435
T8	0.1332	0.0680	0.2852	0.4757	0.6337	0.7930	0.8665	0.9363	0.9409	0.9429
T9	0.1089	0.0587	0.2841	0.4604	0.6223	0.7859	0.8662	0.9352	0.9393	0.9415
T10	0.1051	0.0453	0.2785	0.4584	0.6177	0.7856	0.8638	0.9344	0.9385	0.9407
T11	0.1045	0.0284	0.2546	0.4489	0.6124	0.7725	0.8447	0.9166	0.9223	0.9250
T12	0.1026	0.0113	0.1917	0.4095	0.6041	0.7707	0.8368	0.9120	0.9158	0.9188

While considering the backward recurrence time, we are interested in the outcome of the table 4.3 below which represents the probability that an entity had no transition from state *i* with initial backward time $s - u$.

TABLE 4.3: Probability of no transition from state i $D(u, s; t)$

$D(0,1;t)$	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
SAAA	**0.1**	0	0	0	0	0	0	0	0	0
SAA	0	**0.1418**	0	0	0	0	0	0	0	0
SA	0	0	**0.2553**	0	0	0	0	0	0	0
SBBB	0	0	0	**0.1615**	0	0	0	0	0	0
SBB	0	0	0	0	**0.1305**	0	0	0	0	0
SB	0	0	0	0	0	**0.1236**	0	0	0	0
SCCC	0	0	0	0	0	0	**0.1092**	0	0	0
SCC	0	0	0	0	0	0	0	**0.1079**	0	0
SC	0	0	0	0	0	0	0	0	**0.1005**	0
SD	0	0	0	0	0	0	0	0	0	**0.1002**

The transition probabilities with full consideration of the backward recurrence times is shown in the tables 4.4, 4.5 and 4.6 below which represent the transition probabilities obtained by solving the evolution equation for the backward semi-Markov process. The transition probabilities represent the probability that the system entered in state j at time l' and remained inside this state without any other transition up to the time t given that it entered in the state i at time $l$and it did not move up to s.

TABLE 4.4: Probabilities (${}^b\phi_{ij}^b(1,1)$)

${}^b\phi_{ij}^b(1,1)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
SAAA	0.1135	0.3195	0.1145	0.1536	0.1170	0.5934	0.1386	0.1508	0.9038	0.6027
SAA	0.0068	0.0124	0.0352	0.0460	0.0296	0.0153	0.0494	0.0367	0.0133	0.0151
SA	0.0116	0.0032	0.0418	0.0416	0.0345	0.0047	0.0707	0.0219	0.0104	0.0118
SBBB	0.0149	0.0157	0.0648	0.0559	0.0491	0.0207	0.0865	0.0130	0.0146	0.0120
SBB	0.0119	0.0341	0.0864	0.0652	0.0685	0.0381	0.0992	0.0096	0.0162	0.0090
SB	0.0208	0.0679	0.1145	0.0986	0.0890	0.0453	0.0873	0.0355	0.0322	0.0063
SCCC	0.0373	0.0565	0.1454	0.1427	0.1034	0.0501	0.0622	0.0663	0.0262	0.0067
SCC	0.0998	0.0427	0.1846	0.2992	0.0779	0.0104	0.0061	0.1047	0.0101	0.0058
SC	0.1024	0.0596	0.1852	0.3161	0.0723	0.0183	0.0012	0.1077	0.0051	0.0088
SD	0.0998	0.0580	0.1805	0.3081	0.0705	0.0178	0.0011	0.1050	0.0049	0.0086

TABLE 4.5: Probabilities (${}^b\phi_{ij}^b(1,6)$)

${}^b\phi_{ij}^b(1,6)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
SAAA	0.2693	0.9838	0.2808	0.0729	0.3174	0.1534	0.5728	0.8010	0.3594	0.2555
SAA	0.0714	0.0373	0.0049	0.0056	0.0028	0.0384	0.0260	0.0294	0.0071	0.0852
SA	0.0624	0.0348	0.0017	0.0100	0.0022	0.0418	0.0414	0.0320	0.0011	0.0793
SBBB	0.0648	0.0262	0.0260	0.0121	0.0031	0.0451	0.0591	0.0136	0.0013	0.0976
SBB	0.0701	0.0100	0.0547	0.0240	0.0068	0.0495	0.0794	0.0008	0.0005	0.1072
SB	0.0668	0.0039	0.0817	0.0390	0.0142	0.0787	0.1051	0.0258	0.0180	0.0672
SCCC	0.0926	0.0279	0.1159	0.0404	0.0444	0.0758	0.0977	0.0671	0.0677	0.0480
SCC	0.0003	0.0551	0.2075	0.0759	0.1648	0.1072	0.1825	0.1491	0.0807	0.0318
SC	0.0151	0.0543	0.2176	0.0812	0.1809	0.1117	0.1961	0.1565	0.0751	0.0380
SD	0.0148	0.0529	0.2121	0.0792	0.1763	0.1089	0.1911	0.1525	0.0732	0.0370

4.1.3 Simulating the reliability function

Finally, the rating indicator ${}^bR_i{}^b(l, s; l', t)$ gives the probability of the firm never going into the default state from the time s up to the time t, given that entrance into the state i was at the time $s - l$, and it was the last transition before time s and is represented in table 4.7. This would enable a bank to establish the extent of exposure at any time in future.

TABLE 4.6: Probabilities (${}^{b}\phi_{ij}^{b}(1,12)$)

${}^{b}\phi_{ij}^{b}(1,12)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
SAAA	0.1785	0.1692	0.1135	0.9386	0.0674	0.1097	0.5051	0.9100	0.2117	0.0852
SAA	0.0694	0.0549	0.0356	0.0307	0.0005	0.0307	0.0303	0.0305	0.0603	0.0086
SA	0.0962	0.0445	0.0387	0.0293	0.0013	0.0256	0.0348	0.0343	0.0661	0.0117
SBBB	0.1267	0.0430	0.0265	0.0352	0.0006	0.0128	0.0273	0.0332	0.0673	0.0361
SBB	0.1568	0.0399	0.0185	0.0574	0.0054	0.0013	0.0076	0.0267	0.0852	0.0616
SB	0.1679	0.0265	0.0290	0.0920	0.0190	0.0177	0.0746	0.0115	0.1006	0.0905
SCCC	0.2034	0.0211	0.0328	0.1271	0.0372	0.0393	0.1164	0.0516	0.1118	0.1331
SCC	0.1751	0.0835	0.0435	0.2508	0.0259	0.0683	0.2186	0.1366	0.0608	0.2948
SC	0.1631	0.0936	0.0419	0.2656	0.0233	0.0711	0.2292	0.1402	0.0534	0.3129
SD	0.1590	0.0913	0.0408	0.2589	0.0227	0.0693	0.2234	0.1367	0.0520	0.3049

TABLE 4.7: Probability of never going into default ($R_i(s,t)$)

$R_i(1,t)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
T1	0.9906	0.9628	0.9498	0.8414	0.6969	0.5972	0.4810	0.4455	0.4258	0.0000
T2	0.9898	0.9577	0.8542	0.7129	0.5607	0.5475	0.4693	0.4391	0.4085	0.0000
T3	0.9827	0.8766	0.8371	0.6538	0.5376	0.5268	0.4382	0.3902	0.3485	0.0000
T4	0.9450	0.8534	0.7948	0.6143	0.5242	0.4383	0.4150	0.3472	0.3125	0.0000
T5	0.9277	0.8387	0.7382	0.5832	0.4792	0.4381	0.3635	0.3361	0.2952	0.0000
T6	0.8737	0.8250	0.6483	0.5813	0.4486	0.4372	0.3438	0.3205	0.2598	0.0000
T7	0.8606	0.7575	0.5159	0.4088	0.3825	0.3518	0.3408	0.2974	0.1358	0.0000
T8	0.8589	0.6292	0.4793	0.3908	0.3611	0.3517	0.2564	0.2095	0.1192	0.0000
T9	0.8465	0.5004	0.4603	0.3838	0.3343	0.3126	0.2037	0.1689	0.1055	0.0000
T10	0.6776	0.5001	0.4032	0.3689	0.3153	0.3068	0.1408	0.1359	0.1044	0.0000
T11	0.5132	0.4501	0.3976	0.3487	0.3080	0.2696	0.1394	0.1098	0.1040	0.0000
T12	0.5024	0.4478	0.3278	0.2477	0.2336	0.2304	0.1127	0.1072	0.1031	0.0000

4.2 Empirical analysis

The empirical analysis lays a foundation for the adoption of the discrete time non-homogenous backward semi-Markov modeling of credit risk for a portfolio of SME loans and hence provides a conceivable internal credit rating model for the Kenyan banking industry. The table 4.8 below represents the generated the initial transition matrix from the actual ratings received.

TABLE 4.8: Initial Transition Matrix ($P(s)$)

$P(s)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
SAAA	0.8628	0.7414	0.9535	0.9185	0.8969	0.8759	0.8630	0.8640	0.8640	0.7680
SAA	0.0172	0.0387	0.0468	0.0438	0.0555	0.0635	0.0592	0.0813	0.1730	0.1207
SA	0.0072	0.0001	0.0060	0.0341	0.0380	0.0468	0.0546	0.0597	0.0690	0.0320
SBBB	0.0068	0.0000	0.0051	0.0047	0.0152	0.0090	0.0079	0.0517	0.0589	0.0017
SBB	0.0049	0.0000	0.0049	0.0037	0.0030	0.0057	0.0069	0.0354	0.0279	0.0014
SB	0.0017	0.0000	0.0041	0.0020	0.0040	0.0023	0.0029	0.0249	0.0025	0.0003
SCCC	0.0017	0.0000	0.0034	0.0009	0.0039	0.0025	0.0022	0.0032	0.0019	0.0002
SCC	0.0014	0.0000	0.0025	0.0005	0.0025	0.0023	0.0015	0.0021	0.0016	0.0000
SC	0.0009	0.0000	0.0016	0.0005	0.0020	0.0020	0.0008	0.0015	0.0009	0.0000
SD	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0001	0.0000

This table provides the probability of an entity migrating from one state to the another as seen at the beginning of the observation period. The assumption here is that the state *SD* is not an absorbing state.

4.2.1 The transition probabilities

For the non-homogeneous case,the following probabilities were generated when actual scores were applied to the models: The table 4.9 represents the probability that in a time interval t there was no new rating for the entity having started at state i at the initial time.

TABLE 4.9: Probability of not exiting the initial state $H_i(s,t)$

$H_i(1,t)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
T0	0.6040	0.2981	0.1518	0.0681	0.0399	0.0177	0.0091	0.0066	0.0047	0.0003
T1	0.6499	0.3178	0.1533	0.0708	0.0402	0.0182	0.0094	0.0069	0.0048	0.0003
T2	0.6562	0.3268	0.1549	0.0712	0.0402	0.0184	0.0095	0.0069	0.0049	0.0004
T3	0.6563	0.3271	0.1588	0.0720	0.0411	0.0208	0.0096	0.0069	0.0049	0.0004
T4	0.6644	0.3295	0.1629	0.0739	0.0439	0.0211	0.0097	0.0070	0.0049	0.0004
T5	0.6668	0.3353	0.1649	0.0788	0.0468	0.0213	0.0098	0.0071	0.0050	0.0004
T6	0.6672	0.3363	0.1719	0.0796	0.0476	0.0228	0.0100	0.0071	0.0050	0.0004
T7	0.6739	0.3470	0.1721	0.0822	0.0478	0.0231	0.0100	0.0071	0.0050	0.0004
T8	0.6872	0.3529	0.1730	0.0844	0.0496	0.0239	0.0101	0.0072	0.0050	0.0004
T9	0.6916	0.3548	0.1751	0.0871	0.0504	0.0242	0.0101	0.0073	0.0052	0.0004
T10	0.6978	0.3678	0.1776	0.0882	0.0505	0.0252	0.0104	0.0074	0.0052	0.0004
T11	0.7056	0.3688	0.1809	0.0929	0.0547	0.0297	0.0104	0.0075	0.0052	0.0004
T12	0.7104	0.4091	0.1948	0.0964	0.0580	0.0300	0.0110	0.0078	0.0055	0.0006

While considering the backward recurrence time on the actual scores, we are interested in the outcome of the table 4.10 below which represents the probability that an entity had no transition from state i with initial backward time $s-u$.

TABLE 4.10: Probability of no transition from state i $D(u, s; t)$

$D(0,1;t)$	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
SAAA	**0.1000**	0	0	0	0	0	0	0	0	0
SAA	0	**0.1949**	0	0	0	0	0	0	0	0
SA	0	0	**0.1255**	0	0	0	0	0	0	0
SBBB	0	0	0	**0.1103**	0	0	0	0	0	0
SBB	0	0	0	0	**0.1032**	0	0	0	0	0
SB	0	0	0	0	0	**0.1027**	0	0	0	0
SCCC	0	0	0	0	0	0	**0.1013**	0	0	0
SCC	0	0	0	0	0	0	0	**0.1003**	0	0
SC	0	0	0	0	0	0	0	0	**0.1002**	0
SD	0	0	0	0	0	0	0	0	0	**0.1005**

The migration patterns are now given by the actual transition probabilities generated from the actual scores with full consideration of the backward recurrence times and is shown in the tables 4.11, 4.12and 4.13 below which represent the transition probabilities obtained by solving the evolution equation for the backward semi-Markov process. The transition probabilities represent the probability that the system entered in state j at time l' and remained inside this state without any other transition up to the time t given that it entered in the state i at time $l$and it did not move up to s.

TABLE 4.11: Probabilities (${}^b\phi^b_{ij}(1,1)$)

${}^b\phi^b_{ij}(1,1)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
SAAA	0.0500	0.0516	0.3187	0.3823	0.2578	0.0236	0.3375	0.1903	0.1546	0.1100
SAA	0.0049	0.0582	0.0655	0.0630	0.0555	0.0017	0.0716	0.0559	0.0430	0.0192
SA	0.0062	0.0601	0.0733	0.0298	0.0642	0.0609	0.1237	0.0075	0.0293	0.0184
SBBB	0.0415	0.1761	0.1441	0.0250	0.1390	0.0368	0.0910	0.0100	0.0580	0.0158
SBB	0.0466	0.1748	0.1576	0.0285	0.1438	0.0417	0.0592	0.0076	0.0517	0.0190
SB	0.0541	0.2073	0.1649	0.0174	0.1733	0.0558	0.0530	0.0139	0.0627	0.0308
SCCC	0.0009	0.0386	0.0365	0.0295	0.0417	0.0184	0.0538	0.0120	0.0310	0.0128
SCC	0.0093	0.0379	0.0398	0.0307	0.0444	0.0201	0.0414	0.0126	0.0262	0.0103
SC	0.0184	0.0325	0.0341	0.0224	0.0479	0.0277	0.0533	0.0057	0.0186	0.0089
SD	0.0886	0.1446	0.2116	0.1527	0.1667	0.0290	0.0278	0.0568	0.0492	0.0170

TABLE 4.12: Probabilities (${}^b\phi^b_{ij}(1,6)$)

${}^b\phi^b_{ij}(1,6)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
SAAA	0.5311	0.2480	0.0230	0.0268	0.0206	0.3363	0.1775	0.1206	0.1151	0.4669
SAA	0.0696	0.0165	0.0169	0.0032	0.0401	0.0688	0.0620	0.0042	0.0116	0.1052
SA	0.0957	0.0225	0.0170	0.0029	0.0388	0.0433	0.0361	0.0222	0.0049	0.0893
SBBB	0.1197	0.0053	0.0529	0.0450	0.0712	0.0899	0.1177	0.0191	0.0233	0.0708
SBB	0.1565	0.0267	0.0598	0.0315	0.0731	0.0868	0.0910	0.0062	0.0559	0.0660
SB	0.2299	0.0687	0.0322	0.0158	0.0944	0.0468	0.0141	0.0200	0.1499	0.0118
SCCC	0.0800	0.0334	0.0274	0.0079	0.0273	0.0720	0.0227	0.0418	0.0209	0.0112
SCC	0.0864	0.0278	0.0207	0.0101	0.0252	0.0813	0.0305	0.0412	0.0163	0.0266
SC	0.0909	0.0236	0.0228	0.0114	0.0227	0.0638	0.0250	0.0379	0.0179	0.0367
SD	0.1480	0.0597	0.1418	0.0495	0.0065	0.0680	0.1060	0.1112	0.1411	0.0153

4.2.2 The reliability measure

Finally, the rating indicator ${}^bR_i{}^b(l, s; l', t)$ is generated for the actual scores and it provideds the probability that a firm will never go into the default state from the time s up to the time t, given that entrance into the state i was at the time $s - l$, and it was the last transition before time s and is represented in 4.14. This measure is key in enabling a bank to establish the extent of exposure at any time in future.

TABLE 4.13: Probabilities (${}^b\phi_{ij}^b(1,12)$)

${}^b\phi_{ij}^b(1,12)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
SAAA	0.5639	0.3277	0.2683	0.2487	0.0274	0.1581	0.0794	0.1488	0.4547	0.0982
SAA	0.0778	0.0385	0.0185	0.0104	0.0071	0.0235	0.0353	0.0348	0.0513	0.0121
SA	0.1824	0.0271	0.0297	0.0108	0.0018	0.0106	0.0448	0.0522	0.0884	0.0117
SBBB	0.1851	0.0341	0.0321	0.0109	0.0260	0.0036	0.0128	0.0238	0.1214	0.0168
SBB	0.1958	0.0435	0.0319	0.0012	0.0345	0.0166	0.0061	0.0342	0.1223	0.0165
SB	0.2506	0.1031	0.0486	0.0130	0.0784	0.0128	0.0172	0.0856	0.1598	0.0248
SCCC	0.0831	0.0205	0.0667	0.0301	0.0189	0.0266	0.0059	0.0215	0.0933	0.0244
SCC	0.0707	0.0219	0.0615	0.0327	0.0207	0.0211	0.0118	0.0282	0.0964	0.0228
SC	0.0784	0.0138	0.0521	0.0312	0.0271	0.0225	0.0077	0.0341	0.1026	0.0284
SD	0.2419	0.0531	0.0256	0.0956	0.0663	0.0521	0.1168	0.1415	0.1168	0.1324

TABLE 4.14: Probability of never going into default ($R_i(s,t)$)

$R_i(1,t)$	SAAA	SAA	SA	SBBB	SBB	SB	SCCC	SCC	SC	SD
T1	0.9441	0.8332	0.7372	0.7306	0.6321	0.4799	0.4586	0.4456	0.4400	0.0000
T2	0.8471	0.7760	0.7231	0.7180	0.5303	0.4768	0.4208	0.4101	0.4043	0.0000
T3	0.7940	0.7586	0.7208	0.6878	0.4783	0.4569	0.3879	0.3582	0.3447	0.0000
T4	0.7851	0.7034	0.6593	0.6406	0.4735	0.4385	0.3722	0.3529	0.3422	0.0000
T5	0.7655	0.6836	0.6535	0.6149	0.4735	0.3990	0.3661	0.3336	0.3226	0.0000
T6	0.7564	0.6419	0.5983	0.5669	0.4146	0.3910	0.3562	0.3236	0.3199	0.0000
T7	0.7185	0.6016	0.5762	0.5268	0.4035	0.3903	0.3339	0.3092	0.2695	0.0000
T8	0.7047	0.5783	0.5406	0.5103	0.3993	0.3787	0.2845	0.2726	0.2384	0.0000
T9	0.6209	0.5468	0.5348	0.5090	0.3981	0.3728	0.2753	0.2441	0.2375	0.0000
T10	0.5839	0.5027	0.4665	0.4595	0.3980	0.3555	0.2494	0.2439	0.2232	0.0000
T11	0.5823	0.4820	0.4251	0.3457	0.3248	0.3203	0.2440	0.2363	0.2209	0.0000
T12	0.5582	0.3830	0.3746	0.3215	0.3211	0.2744	0.2209	0.2204	0.2015	0.0000

The ${}^b\phi_{ij}^b(s,t)$ computed above for $(0 \leq s \leq t \leq 12)$ correspond to the probability of default for each entity at a given state i. However, the most appealing thing about the backward semi-Markov model is its capacity to forecast over longer periods which is in line with the IFRS9 requirements for institutions to that they provision for a year for less risky entities and for the full life time for riskier entities.

The study results generate default probabilities for twelve months. The variance in the quality of a credit portfolio will advice on reserves to be made for other states that trigger provisions. The non-homogenous backward semi-Markov model is adopted in that provisioning ca now done prior to occurrence of loss event which protects the firm from deliquesce.

The study illustrates the applicability of using logistic regression to model the probabilities of default of randomly selected entities. It further concurs with those who espouse the hybrid Merton model for determining the creditworthiness of an entity.

Of interest to a bank other than the PDs is the probability that in a given time interval, there will be no new rating evaluation for an SME that started at state i at the starting time and is given by table 4.9. This provides the expected reserves if the SMEs creditworthiness doesn't change over a given time interval. Also, the bank would also be interested with the permanence of an entity in a state and its subsequent movement to other states and the impact of this on the provisions required.

Lastly, the new IFRS9 guidelines have pushed to the forefront the importance of watching the credit quality of a portfolio of SMEs given their assumed PDs at a given state. The semi-Markov reliability indicator is used to establish the level of exposure which is given by the probability that an entity has no default in a time t given that it stated at state i at a previous rating time $s - l$. From the empirical analysis, we find that there is less than 6% chance that any SME will default in the first 12 months.

A comparison of the adequacy of reserves provided through the current Kenyan banking practice and the non-homogenous semi-Markov model brings to light the discrepancy between prospective and retrospective provisioning. Appendix A provides the classification of the real scores used in the empirical analysis. The fact that this model is better in forecasting credit risk for a portfolio of SME loans is evident and hence attaining the objective of the study.

Chapter 5

Conclusion

Markov processes are widely used in the modeling of credit risk. Different approaches have been discussed in the past and a semi-Mark environment seems to capture the empirical state of credit risk. In this study we have reviewed extensively different works in favor of non-homogeneous semi-Markov processes. The study provided a general overview of the historical evolution of credit risk models namely; factor models, reduced form models and structural form models. The study also explores the assumptions and difficulties one would face when applying any of the said techniques in credit risk management.

From the study, we establish the applicability of a modified Merton model in espousing the credit worthiness of an entity. The modified Merton took into consideration economic jumps that occur periodically. The modified model was used to generate scores for a period of 12 months given that the initial score was provided by mapping a range of scores to probabilities of default as presented for 1000 randomly selected entities.

With this hindsight, the study hinges on the strengths of using a backward semi-Markov model as compared to Markov models in modeling credit risk by establishing the relationships that exist in a discrete time non-homogeneous semi-Markovian environment and how they build up to a credit risk reliability model are described. The study includes the initial and final backward recurrence processes to the semi-Markov environment to completely mitigate against the duration problem that exists in the Markov environment.

The study establishes a case for the adoption of the backward discrete time non-homogenous semi-Markov credit risk framework in modeling credit risk at portfolio level for SME loans though the modeling of credit rating transitions and how this migration patterns affect the capital adequacy of a bank in Kenya in light of the IFRS9 requirements. However when considering bank capital provisions, the backward DTNHSMP creates a situation where a bank will end up over provisioning. This is because, this model does not take into consideration the recoveries made from collateralized loans. It is therefore prudent for further research to be done to look into the forward DTNHSMP which will take into account the recoveries likely to be made from the losses envisioned.

In the soon to be implemented IFRS9, loans offered are financial instruments that are included within the scope of the impairment requirements. The impairment models are based on whether there has been a significant increase in the credit risk of a financial asset since its initial recognition and then, one is to determine the amount of impairment to be recognised as expected credit losses (ECL). Lifetime expected credit loss is the present value of expected credit losses that arise if a borrower defaults on its obligation at any point throughout the term of the financial asset. To establish the extent of exposure at any time in future, the backward DTNHSMP discussed in this study provides the reliability function that provides the probability that the entity has no default in a given time spun having started at an initial state a short while before the current time.

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Chapter 6

Appendices

CBK Risk classification for an SME portfolio