STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BACHELOR OF BUSINESS SCIENCE (ACTURIAL SCIENCE, FINANCE, FINANCIAL ECONOMICS)
BSM 2111 STATISTICAL INFERENCE

Date: 21st July 2017 Time: 2 Hours

Instruction: Anwer Question 1 and any other 2 Questions

Question One (30 Marks)

(a) Diferentiate between following terms as used in Estimation theory

(i) Point estimation and interval estinmation **(2 Marks)**

(ii) Sufficient estimator and a consistent estimator **(2 Marks)**

(b) Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from a normal population with mean θ and variance σ^2. Show that the statistic defined by

$$s^2 = \tfrac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

is an unbiased estimator of σ^2 where s^2 is the sample variance **(5 Marks)**

(c) Let $x_1, x_2, x_3, \ldots, x_n$ be independently and identically distributed random variables with probability density function

$$f(x) = \begin{cases} P^x(1-P)^{1-x}, x = 0,1 \\ 0, elsewhere \end{cases}$$

Show that T=$\sum_{i=1}^{n} x_i$ is sufficient for P **(7 Marks)**

(d) A random sample of size 100 trees of same age of a species A gave an average height of 50 mm with a standard deviation of 2.8mm while 150 trees of species B gave an average height of 55.5 mm and a standard deviation of 2.9mm. Construct a 95% confidence interval for the difference between the averages of the two tree species. **(5 Marks)**

(e) Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from a uniform distribution from $(\theta - 2, \quad \theta + 2)$

(i) Find the moment estimator for θ **(5 Marks)**

(ii) Given the following observations, 10.6, 12.3, 4.8, 11.0, 12.37, determine the moment estimator for θ, hence find the estimate of its mean **(4 Marks)**

Question Two 20 (Marks)

(a) Suppose $X_1, X_2, X_3,..., X_n$ is a random sample from a distribution defined by

$$f(x,\theta) = \begin{cases} \frac{e^{\lambda}\lambda^{x}}{x!}, 0 < \theta < 1, x = 0,1,2,..., n \\ 0, elsewhere \end{cases}$$

obtain a sufficient estimator for λ **(5 Marks)**

(b) Consider a linear regression equation $Y = \alpha + \beta X + e$, using the method of least squares derive the estimators for α and β **(10 Marks)**

(c) The data below shows the budgetary allocation in thousand Kenya shillings , Y and production level (in tonnes) , X for a sugar manufacturing company. Use matrix notation to fit a least squatres regression line of Y on X straight line

Budget ('000s Kshs), Y	2	3	4	6	1
Production level, X	1	2	3	7	4

(5 Marks)

Question Three (20 Marks)

(a) Let $X_1, X_2, X_3,..., X_n$ be a random sample from a normal population with μ and variance σ^2, show that $S^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)}{n}$ is a consistent estimator of σ^2, the population variance. **(3 Marks)**

(b) Suppose deposit amounts by customers to a savings account are uniformly distributed over the interval (α , β). Given also that the density of the distribution is defined by;

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha}, \alpha \leq x > \beta \\ 0, elsewhere \end{cases}$$

(i) Show that this is probability density function **(3 Marks)**

(ii) Determine the moment generating function of X **(4 Marks)**

Hence find

(iii) Mean of X **(3 Marks)**

(c) An insurance company has established that the number of car accident claims they receive are uniformly distributed on an interval $(0, \beta)$. Determine

(i) the Maximum likelihood estimator for β **(4 Marks)**

(ii) The mean number of claims and the variance. **(3 Marks)**

Question Four (20 Marks)

(a) Given a random sample of size n from the population whose probability density function is defined by

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi\beta}} \exp(\frac{-(X-\alpha)^2}{2\beta}, -\infty \leq x > \infty \\ 0, elsewhere \end{cases}$$

Determine the Maximum likelihood estimator of α and β. **(6 Marks)**

(b) Suppose $x_1, x_2, x_3, ..., x_m$ is a random sample from a normal population with mean μ_1 and known variance σ^2. Also let $y_1, y_2, y_3, ..., y_n$ be another random sample from a normal population with mean μ_2 and known variance σ^2. Assume that these samples are independent of each other. Construct a confidence interval for the difference of the means μ_1 and μ_2. **(9 Marks)**

(c) Several life insurance firms have policies geared to working class citizens. To get more information about this group, a major insurance firm interviewed working class individuals to find out the type of life insurance they preferred, if any. The table below shows the preferences by gender.

Gender	Preferred a term policy	Preferred a whole-life policy	No preference
Adult Male	160	30	10
Adult Female	140	120	40

Is there evidence that the life insurance preference of working class citizens depends on their sex? Use $\alpha = 0.05$ **(5 Marks)**

Question 5 (20 Marks)

(a) Let $X_1, X_2, X_3, ..., X_n$ is a random sample of size n from a normal population whose probability density function is defined by

$$f(X, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp - \frac{1}{2\sigma^2}(X-4)^2, \ -\infty < X < \infty,$$

(i) Determine the M.L.E for σ^2 **(5 Marks)**

(ii) Data gathered on the pre-tax profits (in Million Kenya shillings) from five branches of supermarket stores are obtained as;

5.2, 3.0, 2.5, 4.1, 1.0, 2.1, 3.5, 5.1, 4.0, 2.6.

Find the estimate of the variance from the sampled data. **(3 Marks)**

(b) An industrial psychologist feels that a big factor in job turnover among assembly line workers is the individual employee's self esteem. She believes that workers who change jobs often (population A) have, on average, lower self esteem, as measured by a standardized test, than workers who do not (population B). To determine whether she can support her belief with statistical analysis, she draws a simple random sample of employees from each population and gives a test measuring esteem. The results are summarized in the output below

Descriptive Statistics: population A, population B

Variable	Total Count	Mean	Standard Dev
Pop. A	10	52.30	9.23
pop. B	17	69.47	11.78

The psychologist believes that the relevant populations scores are normally distributed, with equal, although unknown variances. At the 0.01 level of significance

(i) State type I and type II errors for this problem **(2 Marks)**

(ii) test whether her belief is valid **(3 Marks)**

(iii) briefly comment what she should make of the findings **(1 Mark)**

(c) A National Statistics office of a city surveyed 150 households and reported that in 2016, 42% percent of households in the city had internet access.

(i) Estimate the standard error of the proportion of households that had internet access.

(2 Mark)

(ii) Construct the 99% confidence interval for the sample proportion of 50% **(3 Marks)**

(iii) Interpret meaning 99% confidence interval in part b (ii) above **(1 Mark)**