Strathmore Institute of Mathematical Sciences (SIMS)
End of Semester Examination for the Degree of Bachelor of Business Science in Financial Economics/Financial Engineering/Actuarial Science

BSA 3108: Theory of Finance

DATE: 14th **September, 2021** **Time: 2 Hours**

Instructions

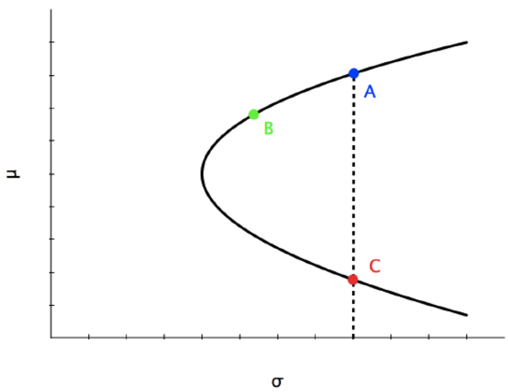
- **This examination consists of FIVE questions.**
- **Answer Question ONE (COMPULSORY) and any other TWO questions.**

1. (a) Graphically demonstrate the Fisher separation theorem for the case where an individual ends up lending in financial markets **(6 Marks)**

   (b) Given the exponential utility function $U(W) = -e^{-aW}$. Does the function have constant relative risk aversion? **(4 Marks)**

   (c) Briefly explain all three forms of market efficiency **(6 Marks)**

   (d) The graph below traces out the minimum variance frontier from modern portfolio Theory (MPT).



In answering each part of this question, assume, as Harry Markowitz did when he invented Modern portfolio Theory, that all investors have mean-variance preferences, that is, utility functions that are increasing in their portfolio's expected return and decreasing the variance or standard deviation of their portfolio's random return. Assume, as well, that there is no risk-free asset, so that all investors must choose portfolios on or inside the minimum variance frontier.

(i) Would any investor ever choose to hold portfolio C? Briefly explain **(3 Marks)**

(ii) Suppose you observed one investor - call him or her "investor 1" - holding portdfolio A and another investor - call him or her "investor 2" - holding portfolioB. Which investor is risk averse: investor 1 or investor 2? Briefly explain. **(3 Marks)**

(e) In the context of state preference framework, describe the following concepts:

(i) Arrow-Debreu securities **(2 Marks)**

(ii) Complete capital market **(2 Marks)**

(f) Distinguish between statistical and economic models. What are the advantages and limitations of either? **(4 Marks)**

2. (a) Consider a consumer with income of \$100 who faces a 50 percent probability of suffering a loss that reduces his or her income to \$25 dollars. Suppose that this consumer can buy an insurance policy for x dollars that protects him or her fully by paying off \$75 if the loss occurs. Finally, assume that the consumer's preferences can be described by a Von Neumann-Morgenstern expected utility function with Bernoulli utility function of the constant relative risk aversion form

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1-\gamma}$$

with $\gamma = 2$.

(i) Write down an expression for the consumer's expected utility if he or she decides to buy insurance. **(2 Marks)**

(ii) Write down an expression for the consumer's expected utility if he or she decides not to buy the insurance. **(4 Marks)**

(iii) Find the value x^* of the premium that makes the investor exactly indifferent between buying and not buying the insurance. **(4 Marks)**

(b) Consider an investor with preferences over the mean μ_p and variance σ_p^2 of the return on his or her portfolio that are described by the utility function

$$U(\mu_p, \sigma_p^2) = \mu_p - (\frac{A}{2})\sigma_p^2,$$

where a higher value of the parameter A corresponds to a greater aversion to risk. Suppose that this investor allocates the fraction w_1 of his or her initial wealth to risky asset 1, with expected return $\mu_1 = 2$ and standard deviation of its random return equal to $\sigma_1 = 2$, the fraction w_2 of initial wealth to risky asset two, with expected return $\mu_2 = 5$ and the standard deviation of its random return equal to $\sigma_2 = 2$, and the remaining fraction $1 - w_1 - w_2$ to a risk-free asset with return $r_f = 1$

Assuming that the correlation between the two risk asset returns is zero, the investor's portfolio will have return

$$\mu_p = (1 - w_1 - w_2)r_f + w_1\mu_1 + w_2\mu_2 = 1 - w_1 - w_2 + 2w_1 + 5w_2 = 1 + w_1 + 4w_2$$

and variance

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 = 4w_1^2 + 4w_2^2$$

Therefore, the investor chooses w_1 and w_2 to maximize

$$\mu_p - (\frac{A}{\sigma_p^2}) = 1 + w_1 + 4w_2 - 2A(w_1^2 + w_2^2)$$

(i) Write down the expressions for the optimal portfolio weights, w_1^* and w_2^* **(4 Marks)**

(ii) Provide an intuitive link between the parameter A and the optimal portfolio choices implied by your solution to part (i) **(2 Marks)**

(c) Briefly explain the concept of **separation principle** in the context of modern portfolio theory **(4 Marks)**

3. (a) Consider the following information.

State	Asset 1	Asset 2	Asset 3	Asset 4	State Probability
1	5%	2%	5%	5%	0.3
2	7%	6%	4%	5%	0.2
3	3%	9%	7%	5%	0.5
Asset value	20,000	10,000	10,000	n.a	

(i) Determine the market price of risk assuming CAPM holds. Define all the terms used. **(6 Marks)**

(ii) Describe any three limitations of the CAPM model? **(6 Marks)**

(b) Ms. Bethel, manager of the Humongous mutual fund, knows that her fund currently is well diversified and that it has a CAPM beta of 1.0. The risk free rate is 8% and the CAPM risk premium, $[E(R_m) - R_f]$, is 6.2%. She has been learning about measures of risk and knows that there are (atleast) two factors: changes in industrial production index, $\overline{\delta_1}$, and unexpected inflation, $\overline{\delta_2}$. The APT equation is

$$E(R_i) - R_f = [\overline{\delta_1} - R_f]b_{i1} + [\overline{\delta_2} - R_f]b_{i2}$$
$$E(R_i) = 0.08 + [0.05]b_{i1} + [0.11]b_{i2}$$

(i) If her portfolio currently has a sensitivity to the first factor of $b_{i1} = -0.5$ what is its sensitivity to unexpected inflation? **(4 Marks)**

(ii) If she rebalances her portfolio to keep the same expected return but reduce her exposure to inflation to zero $b_{i2} = 0$, what will its sensitivity to the first factor become? **(4 Marks)**

4. (a) You are given the following information:

Security	State 1	State 2	Security Prices
j	\$12	\$20	$p_j = \$22$
k	24	10	$p_k = 20$

(i) Compute the prices of pure security state prices. **(6 Marks)**

(ii) What is the initial price of a third security i, for which the payoff in state 1 is \$6 and the payoff in state 2 is \$10? **(4 Marks)**

(b) Explain the Grossman-Stiglitz Paradox. In line with what the paradox posits, why is market inefficiency important? **(4 Marks)**

(c) Explain the implications of the Efficient market hypothesis in

(i) Corporate finance **(3 Marks)**

(ii) Portfolio management **(3 Marks)**

5. (a) Describe any three characteristics of a perfect capital market **(6 Marks)**

(b) Define the following measures of risk (in words and mathematically). List their advantages and limitations:

(i) Value at Risk (VaR) **(3 Marks)**

(ii) Variance of returns **(3 Marks)**

(c) Let S be the price of stock t, and suppose that at $t + \Delta t$ the stock can only change into two values: up $Su > S$ or down $Sd < S$ (here u and d represent the relative increase or decrease of the stock price). What are the probabilities of the up and down moves of the stock price, if the market is assumed to have no arbtrage opportunities? **(4 Marks)**

(d) Assuming one step binomial model, consider stock with spot price $S = \$80$, price after up move $Su = \$100$ and after down move $Sd = \$60$. Assuming riskless rate of return $r = 0$, price European put option with the strike price $K = \$90$ and expiring in $T = 1$ year. **(4 Marks)**

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