Introducing a New Coffee Futures Pricing Model for the Nairobi Securities Exchange

Githinji Rosebell[1] and Muthoni Lucy[2]

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Abstract

This study intends to apply a different pricing model for pricing coffee futures at the Nairobi Securities Exchange and suggest an improvement of the existing pricing model currently used at the NSE. This will be done by using Schwartz's model 1 to generate coffee Futures prices, and compare the prices generated by the model with the model currently used to price coffee futures at the NSE. The difference between the series of prices generated by the two models will be tested for significance, and thus guide on whether introducing a new pricing model for the coffee Exchange will have an impact on the current pricing model.

Keywords: Coffee Futures, Futures Pricing Models, Nairobi Securities Exchange

[1] BBS Actuarial Science Student, Strathmore Institute of Mathematical Sciences, Strathmore University
P. O. Box: 215 – 00200, Nairobi
Email: rosebell.githinji@strathmore.edu; githinjirose@gmail.com
[2] Assistant Lecturer, Strathmore Institute of Mathematical Sciences (SIMS), Strathmore University
P. O. Box: 4877 – 00506, Nairobi
Email: LMuthoni@strathmore.edu; lucymuthoni2003@gmail.com

INTRODUCTION

1.1. Background Information

An agreement, which is standardized, between a seller and a buyer to exchange an underlying item for a particular price at a pre-specified date is called a future contract. The underlying item may be a metal, agricultural commodity, mineral or an energy commodity, financial product or a foreign currency. The Futures contract is a derivative security as its value is wholly based on the value of the underlying asset. Unlike forward contracts, futures contracts are exchange traded and regulated, standardized and marked to market.

In the last decade, the size of Futures Exchanges have grown significantly in Africa, Asia and Latin America based on the ground that there is need in particular countries for a way of dealing with price volatility and providing price discovery. However, according to Capital Market Authority (2013) report, 2 out of 3 contracts fail as they have not been designed properly despite having government and donor agencies' support.

Use of futures markets for price risk management purposes also known as hedging as opposed to speculation, it is a tool to realize better prices, but a way to obtain more certainty about the prices one can expect to realize. Greater predictability, in turn, makes it possible to make better decisions and to obtain credit at better terms, indirect benefits, rather that better price realizations, are what allows those who use price risk management. However according to Lamon and Frida (2007), use of organized futures and options markets can be cumbersome for the developing countries producer considering the steps involved in coming up with an exchange and a clearing house. An exchange is basically like a "club" whose members are the futures brokers and it is where they execute the trades. The clearing house is responsible for the settlement and guarantee of all trades on the exchange.

1.2. Coffee in Kenya

Coffee is one of Kenya's top foreign exchange earners coming fifth behind tea, tourism, horticulture and remittances from the diaspora. According to www.staging.nationmedia.com (22and August 2016), coffee farmers are the last class of economic slaves in Kenya. They have no negotiating power, knowledge and responsibility over their crop.

The government of Kenya announced that the steps towards developing institutional and legal frameworks to introduce a Commodities and Futures exchange are underway. A Futures Exchange will offer farmers stable prices and a ready market for their produce as has been proven by Ethiopian and Ugandan Commodities Exchanges, which has resulted in improved output in those countries especially in Ethiopia. This study intends to look at the impact the Commodities and Futures Exchange will have on the coffee prices. This will be done by using a futures pricing model to price the coffee Futures, and comparing the prices generated by the model currently used to generate coffee prices.

CHAPTER 2: LITERATURE REVIEW

Many models consider the relations between prices of futures contracts and corresponding spot prices for instance Anderson (1983). In Schwartz (1998) article he developed a one-factor model for the stochastic behavior of commodity prices that retains most of the characteristics of a more complex two-factor stochastic convenience yield model in terms of its ability to price the term structures of futures prices and volatilities. The model is based on the pricing and volatility results of the two-factor model. When applied to value long-term commodity projects, it gives practically the same results as the more complex model. The inputs to the model are the current prices of all existing futures contracts (and their maturities) and the estimated parameters of the two-factor model. It only requires, however, the numerical solution corresponding to a simple one-factor model.

Many models consider relations between prices of futures contracts and corresponding spot prices, e.g. Anderson (1983), Hirschileifer (1989) and (1990). We also see a textbook by Duffie (1989) trying to explain the relationship between the prices of futures contracts and corresponding prices, but applying the concept on pricing of sugar. Schwartz (1997) compared three models of stochastic behavior of commodity prices: a one factor model and three-factor models. Schwartz (1998) developed a one-factor model that preserves the main characteristics of two-factor models. In this paper, we define the cost as in Black Scholes Merton (1987). For an introduction to the basic concepts for the pricing of derivative assets and real options under the uncertainty and incomplete information, we refer to Bellalah (1995) and (1999b). We use an extension of the analysis in the Schwartz (1997) and (1998) to account for the effects of incomplete information as it appears in the models of Merton (1987) and Bellalah (2001). This paper uses the aforementioned extension to describe the stochastic behavior of commodity prices in the presence of mean reversion and shadow costs of incomplete information.

CHAPTER 3: METHODOLOGY

3.1. Data

The data that has been used in this study was obtained from Ethiopian Coffee Exchange. This is because Ethiopia has already already started trading in the coffee futures. Data and information used in the analyses of the problem was gathered from secondary sources.

The reason why we used Ethiopian data is mainly because NSE is yet to introduce a futures trading. Other reasons include: The two markets are highly correlated as they are both located in the Eastern part of Africa,they are based on agriculture which face the same challenges in both countries e.g frequent draught and finally, the two countries are third world countries.

3.2. Futures Pricing Model

In this model, Schwartz (1997) assumed that the commodity spot price follows the stochastic process:

$$dS = \kappa(\mu - \ln S)Sdt + \sigma S dz \quad (1)$$

Where dz is an increment to a standard Brownian motion and κ refers to the speed of adjustment. Describing $X = \ln S$, and applying lto's lemma to characterize the log price by an Ornstein-Uhlenbeck stochastic process, equation (1) becomes:

$$dX = \kappa(\alpha - X)dt + \sigma dz \quad (2)$$

$$\text{With } \alpha = \mu - \frac{\sigma^2}{2\kappa} \quad (3)$$

Where κ measures the degree of mean reversion to the long run mean log price α. Under standard assumptions, Schwartz (1997) gives the following dynamics of the Ornstein-Uhlenbeck stochastic process under the equivalent martingale measure:

$$dX = \kappa(\alpha^* - X)dt + \sigma dz^* \quad (4)$$

Where $\alpha^* = \alpha - \lambda$ and λ is the market price of risk. λ Can be interpreted as market volatility. We are going to calculate volatility using stochastic methods as illustrated in section 3.4. From equation (4), the conditional distribution of X at time T under the equaivalent martingale measure is normal. The mean and variance of X is:

$$E_0[X(T)] = e^{-\kappa T} X(0) + (1 - e^{\kappa T})\alpha^*$$

$$Var[X(T)] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa T}) \quad (5)$$

When the interest rate is constant, the futures or the forward price of commodity corresponds to the expected price of the commodity for the maturity T. Using the properties of the log-normal distribution, the futures or the forward price given by:

$$F(S,T) = E[S(T)] = exp\left(E_0[X(T)] + \frac{1}{2}Var_0[X(T)]\right) \quad (6)$$

And

$$F(S,T) = exp\left(e^{-\kappa T}\, ln\, S + (1 - e^{-\kappa T})\alpha^* + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T})\right) \quad (7)$$

This equation can be written in a log form as

$$ln\, F(S,T) = e^{-\kappa T}\, ln\, S + (1 - e^{-\kappa T})\, \alpha^* + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T}) \quad (8)$$

Equation (7) is solution to the partial differential equation:

$$.\frac{1}{2}\sigma^2 S^2 F_{SS} + (\kappa(\mu - \lambda) - ln\, S + 1 + (1 - e^{\kappa} T)\alpha * + \frac{\sigma^2}{4k}(1 - e^{-2\kappa T}) \quad (9)$$

Under the terminal boundary condition $F(S,0) = S$

Using this model, we are able to price the Forward contract or a future contract since we can calculate all the parameters in the equation.

3.3. Volatility Estimation Method

It is widely acknowledged that volatility varies with time and is predictable within statistical limits (Hull and White (1987)). Merton (1973) dropped the assumption that volatility in the Black-Scholes model is constant and extended the model to cover the situation in which the volatility is treated as a function of time, that is, volatility follows its own stochastic process. Researchers have proposed several stochastic processes for volatility.

General stochastic volatility models

These models assume that the asset prices evolve according to the geometric Brownian motion:

$$\frac{dS}{S} = \mu dt + \sigma dZ_1 \tag{10}$$

In which the volatility of the underlying asset evolves according to the Itô process given as:

$$d\sigma = p(S,\sigma,t)dt + q(S,\sigma,t)dZ_2 \tag{11}$$

Where increments dZ_1 and dZ_2 are unit Wiener processes

$$\sigma_1 = \sigma_2 = 1, Z_1 \sim N\left(0,\sqrt{t}\right), Z_1 \sim N\left(0,\sqrt{t}\right)$$

The correlation of these processes remains an unknown a parameter ρ, implicit in the theory, to be fitted to data in practice.

Let the value of the option with stochastic volatility be given as $V(S,\sigma,t)$, i.e. V is a function of three variables. It should be noted that although volatility is not a traded asset, one can hedge an option with two other contracts, one being the underlying traded asset, and the other the volatility risk. To illustrate this, we consider a portfolio which contains one option of value $C(S,\sigma,t)$, a quantity- Δ of the underlying asset and a quantity-Δ_1 of another shadow option whose value is denoted as $C_1(S,\sigma,t)$. Here Δ is taken as a coefficient and not as a finite difference operator. The hedge portfolio has value

$$\pi = C(S,\sigma,t) - S\Delta - \Delta_1 C_1(S,\sigma,t) \tag{12}$$

The change of value of the portfolio from time t to $t + dt$ is given as

$$d\pi = dC(S,\sigma,t) - dS\Delta - \Delta_1 dC_1(S,\sigma,t) \tag{13}$$

Thus by Itô's lemma we have

$$d\pi = \left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C}{\partial \sigma^2}\right) dt - \Delta_1 \left(\frac{\partial C_1}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C_1}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C_1}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C_1}{\partial \sigma^2}\right) dt + \left(\frac{\partial C}{\partial S} - \Delta_1 \frac{\partial C_1}{\partial S} - \Delta\right) dS + \left(\frac{\partial C}{\partial \sigma} - \Delta_1 \frac{\partial C_1}{\partial \sigma}\right) d\sigma \tag{14}$$

In order to eliminate randomness in equation (14), we choose

$$\left(\frac{\partial C}{\partial S} - \Delta_1 \frac{\partial C_1}{\partial S} - \Delta\right) = 0 \text{ and } \left(\frac{\partial C}{\partial \sigma} - \Delta_1 \frac{\partial C_1}{\partial \sigma}\right) = 0 \tag{15}$$

The result is then given as:

$$d\pi = \left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C}{\partial \sigma^2}\right) dt - \Delta_1 \left(\frac{\partial C_1}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C_1}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C_1}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C_1}{\partial \sigma^2}\right) dt \tag{16}$$

By arbitrage argument, we have the returns of the portfolio equal to the risk-free rate, i.e.

$$d\pi = r\pi dt = r(C - S\Delta - \Delta_1 C_1) \tag{17}$$

That is

$$\left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C}{\partial \sigma^2}\right) dt - \Delta_1 \left(\frac{\partial C_1}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C_1}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C_1}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C_1}{\partial \sigma^2}\right) dt = r(C - S\Delta - \Delta_1 C_1) dt \tag{18}$$

To separate variables, we collect all the C terms on one side and all the C_1 terms on the other

$$\frac{\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C}{\partial \sigma^2} + rS\frac{\partial C}{\partial S} - rC}{\frac{\partial C}{\partial \sigma}} = \frac{\frac{\partial C_1}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C_1}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C_1}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C_1}{\partial \sigma^2} rS\frac{\partial C_1}{\partial S} - rC_1}{\frac{\partial C_1}{\partial \sigma}} \tag{19}$$

The left-hand side of equation (19) is in terms of Conly and can possibly be expressed as a function of independent variables S, σ, t (Wilmott (1998), 300-301). Thus we have

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \rho\sigma q S \frac{\partial^2 C}{\partial S \partial \sigma} + \frac{1}{2} q^2 \frac{\partial^2 C}{\partial \sigma^2} + rS\frac{\partial C}{\partial S} + (p - \lambda q)\frac{\partial C}{\partial \sigma} - rC = 0 \quad (20)$$

Where the separation constant $\lambda(S, \sigma, t)$ is known as the market price of (volatility) risk. In particular, for an underlying asset, if μ is the growth rate of the tradable asset, then $(\mu - r)/\sigma$ is the excess rate of return (above the risk-free rate) per unit risk- thus it is known as market price of risk and is also referred to as Shapiro ratio (see Lyuu (2002), 220; Hull (2000), 498; Wilmott (1998), 301). Under the simplifying assumption that Wiener process $Z_1$and Z_2 are not correlated then equation (20) becomes:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \frac{1}{2} q^2 \frac{\partial^2 C}{\partial \sigma^2} + rS\frac{\partial C}{\partial S} + (p - \lambda q)\frac{\partial C}{\partial \sigma} - rC = 0 \quad (21)$$

This is a partial differential equation that is analogous to the Black-Scholes PDE, but accounts through λ for the shadow option price C_1.

From the perspective of pattern recognition for processes the PDEs are candidates for fitting a real price history in which the volatility risk through λ has exerted an influence on market prices, and has to be estimated by appropriate methods.

3.4. Test Statistics

The difference between the prices will be tested for significance using two statistical methods which are and principal component analysis and coefficient of determination.

3.4.1. Principal Component Analysis

According to www.en.wikipedia.org (accessed on 23rd October 2016) principal Component Analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or equal to the number of original variables.

Suppose we have a random vector $\underline{X}$ defined as $\underline{X} = (X_1, X_2, \ldots, X_n)$ with a population variance-covariance matrix defined as $var(\underline{X}) = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_p^2 \end{pmatrix}$. Consider the linear combinations defined below:

$$Y_1 = e_{11}X_1 + e_{12}X_2 + \cdots + e_{1p}X_p \tag{22}$$

$$Y_2 = e_{21}X_1 + e_{22}X_2 + \cdots + e_{2p}X_p$$

$$\vdots$$

$$Y_p = e_{p1}X_1 + e_{p2}X_2 + \cdots + e_{pp}X_p$$

Where $\boldsymbol{e_i} = e_{i1}, e_{i2}, \ldots, e_{ip}$ are regression coefficients. Y_i is random with a population variance of

$$var(Y_i) = \sum_{k=1}^{p}\sum_{l=1}^{p} e_{ik}e_{il}\sigma_{kl} = e_i'\Sigma e_i \tag{23}$$

And Y_i and Y_j will have a population covariance of

$$cov(Y_i, Y_j) = \sum_{k=1}^{p}\sum_{l=1}^{p} e_{ik}e_{jl}\sigma_{kl} = e_i'\Sigma e_j \tag{24}$$

Therefore, the first principal component (PC1) Y_1 is the linear combination of x-variables that has maximum variance among all linear combinations, so it accounts for as much variation in the data as possible.

$$var(Y_1) = \sum_{k=1}^{p}\sum_{l=1}^{p} e_{1k}e_{1l}\sigma_{kl} = e_1'\Sigma e_1 = \sum_{j=1}^{p} e_{ij}^2 = 1 \tag{25}$$

The second principal component (PC2) Y_2 is the linear combination of x-variables that accounts for as much of the remaining variation as possible, with the constraint that the correlation between the first and the second component is 0. Thus the variance of PC2 is defined as

$$var(Y_2) = \sum_{k=1}^{p}\sum_{l=1}^{p} e_{2k}e_{2l}\sigma_{kl} = e_2'\Sigma e_2 = \sum_{j=1}^{p} e_{2j}^2 = 1 \tag{26}$$

The first and the second principal component will be uncorrelated with one another in the sense that

$$cov(Y_1, Y_2) = \sum_{k=1}^{p} \sum_{l=1}^{p} e_{1k} e_{2l} \sigma_{kl} = e_1' \Sigma e_2 = 0 \qquad (27)$$

3.4.2. Coefficient of Determination (R^2)

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(P_i - \overline{B_i})^2/(n-k)}{\sum_{i=1}^{n}(P_i - \overline{P_i})^2/(n-1)} \qquad (28)$$

Where, $\bar{P}$ is the average price of all observed futures prices, $\bar{B}$ is the model price of a futures i, n is the number of futures traded and k is the number of parameters needed to be estimated.

It measures the success of the regression in predicting the values of the dependent variable within the sample. R^2 ranges from 0 to 1 (www.mta.org accessed on 23rd October 2016). A very high value of R^2 is therefore associated with a good fit of the observed prices while a small value is associated with a poor fit.

Chapter 4: Data analysis and finding

4.1. Empirical Results

The aim of the study was to improve on the existing coffee futures pricing model developed for Kenyan markets.

4.2. Calibration results

To calibrate parameter values in the above model, I used one factor Hull-White Model calibration method with a constant mean reversion.

Number of observations	36
μ	0.3265
κ	1.1562
α	0.2483
σ_1	0.2740
σ_2	0.2802

σ_3	0.2814
ρ_1	0.8183
λ	0.2565
ρ_2	0.0621

These calibrated results had already been estimated. From the estimated parameters I used κ and λ. Since $\alpha = \mu - \frac{\sigma^2}{2\kappa}$ and $\alpha^* = \alpha - \lambda$, then calculating the future prices is possible using:

$$F(S,T) = \exp\left\{e^{-\kappa T} lnS + (1 - e^{-\kappa T})\alpha^* + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T})\right\} \quad (29)$$

Where S is the closing price, T is calculated at $T = t/365$ since this is daily data and μ is the rate of return calculated as follows:

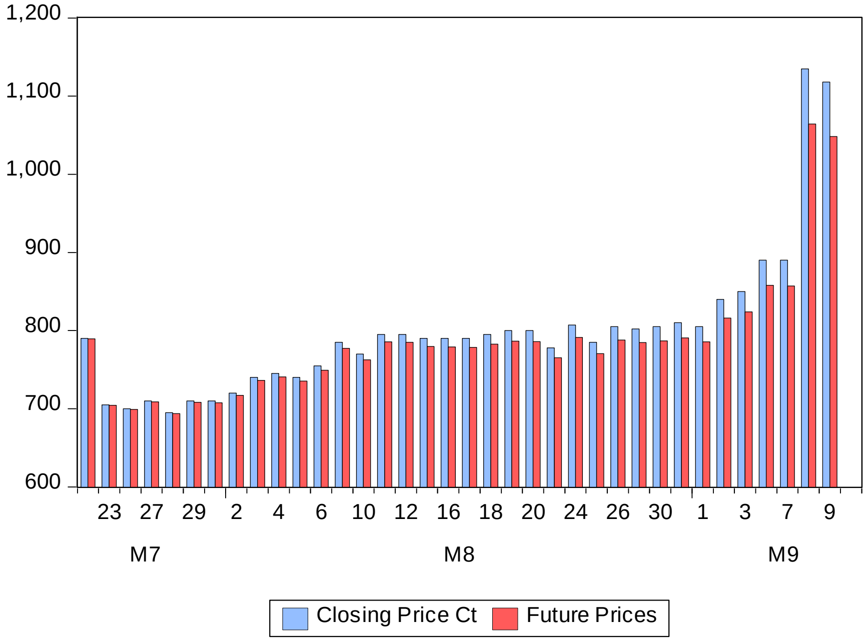
$$\mu = \frac{1}{N}\sum_i^N (o_t + c_t)_i \quad (30)$$

Where $o_t + c_t = \log C_t + \log O_t$

4.3. Graphs

Fig 4.4.1: Line graph comparing the model's prices and observed prices

Fig 4.4.1: Bar graph comparing the model's prices and observed prices



As observed in figures 4.4.1 and 4.4.2, the futures prices generated by the model are lower than the actual prices. This is because the model assumes that the markets are imperfect and therefore penalizes for the cost of getting information.

4.4. Principal component analysis

Table 4.5.1: Principal Component Analysis between the project's model and observed prices

Principal Components Analysis
Date: 11/01/16 Time: 12:05
Sample (adjusted): 7/22/2010 9/09/2010
Included observations: 36 after adjustments
Balanced sample (listwise missing value deletion)
Computed using: Ordinary correlations
Extracting 2 of 2 possible components

Eigenvalues: (Sum = 2, Average = 1)

Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	1.999156	1.998312	0.9996	1.999156	0.9996
2	0.000844	---	0.0004	2.000000	1.0000

Eigenvectors (loadings):

Variable	PC 1	PC 2
CLOSING_PRICE...	0.707107	-0.707107
FUTURE_PRICES	0.707107	0.707107

Ordinary correlations:

	CLOSING_PRICE_CT	FUTURE_P...
CLOSING_PRICE...	1.000000	
FUTURE_PRICES	0.999156	1.000000

4.5. Regression Estimates

In this section, we use definitions from www.halweb.cu3m.es (accessed on 23rd October 2016) to define several parameters found in the generated result-table of regression estimates. These terms are: regression coefficients, standard errors, T-statistic, probability, r-squared, S.E. of regression, sum of squared residuals, Durbin-Watson statistic, Akaike information criterion and Schwartz criterion.

After carrying out regression analysis of our data, we were able to generate the following results using eViews, an inbuilt software found in the Microsoft Office package, specifically within the Excel package. Using the results in the table, we will explain what each term means and proceed to interpret the results depicted.

Table 4.6.1: Regression Analysis between the project's model and observed prices

Dependent Variable: CLOSING_PRICE_CT
Method: Least Squares
Date: 11/01/16 Time: 12:16
Sample (adjusted): 7/22/2010 9/09/2010
Included observations: 36 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
FUTURE_PRICES	1.020416	0.003094	329.8107	0.0000

R-squared	0.975786	Mean dependent var	798.6111
Adjusted R-squared	0.975786	S.D. dependent var	93.97820
S.E. of regression	14.62366	Akaike info criterion	8.230543
Sum squared resid	7484.797	Schwarz criterion	8.274529
Log likelihood	-147.1498	Hannan-Quinn criter.	8.245895
Durbin-Watson stat	0.159330		

4.5.1. Regression Coefficients

Each coefficient multiplies the corresponding variable in forming the best prediction of the dependent variable. The coefficient measures the contribution of its independent variable to the prediction. The coefficient of the series called C is the constant or intercept in the regression. It is the base level of the prediction when all of the other independent variables are zero. The other coefficients are interpreted as the slope of the relation between the corresponding independent variable and the dependent variable.

4.5.2. Standard Errors

These measure the statistical reliability of the regression coefficients. The larger the standard error, the more statistical noise infects the coefficient. The standard error of the coefficient is 0.003094 which is small enough to make any impact.

4.5.3. Probability

This is the probability of drawing a t-statistic of the magnitude of the one just to the left from a t-distribution. Since the probability that the true value C is zero is 0.0000 is less than our level of significance $\alpha = 0.05$ we will fail to reject the null hypothesis that the true coefficient is zero.

4.5.4. R-squared

This measures the success of how a variability of one factor can be caused by the relationship with another factor. R^2 ranges between 0 and 1, 1 if the regression fits perfectly, and zero if it fits no better than the simple mean of the dependent variable R^2 is the fraction of the variance of the dependent variable explained by the independent variables. It can be negative if the regression does not have an intercept or constant or if two-stage least squares is used. It is calculated using the formulae:

$$\frac{Estimated\ variation}{Total\ variation}$$

Which can also be written as: $1 - \frac{SSE}{TSS}$

From our estimates R^2 is 0.975786 and hence it perfectly fits.

4.5.5. S.E. of regression

This is a summary measure of the size of the prediction errors. It has the same units as the dependent variable. The Standard Error in our estimate is given as 14.62366.

4.5.6. Durbin-Watson Statistic

This is a test statistic for autocorrelation. The value ranges from 0-4. A value of two implies there is autocorrelation and values that tend to 4 show negative autocorrelation and values tending to 0 implies positive autocorrelation. Since our Durbin-Watson test statistic (0.159330) less than 2, there is evidence of positive serial correlation.

4.5.7. Akaike Information Criterion (AIC)

It is a measure of how well a model fits a dataset adjusting the ability of the model to fit any dataset whether related or not. It is based on the sum of squared residuals but places a penalty on extra coefficients. Under certain conditions, you can choose the length of a lag distribution, e.g. by choosing the specification with the lowest value of AIC. Our value of AIC is 8.2305.

4.5.8. Schwarz Criterion

This is an alternative to the AIC and also known as Bayesian information criterion with basically the same interpretation but a larger penalty for extra coefficients. The smaller the Schwartz Criterion, the better the fit of the model. The value for this model is 8.27.

4.6. Discussion and Conclusion

The purpose of this study was to compare our project's model with the model developed for Nairobi Stock Exchange by Muthoni et al (2016), and suggest possible improvements in the model.

4.6.1. Choice of Models

Two different models were used. For this study, we used a model that assumes that the commodity spot price follows the stochastic process:

$$dS = \kappa(\mu - \ln S)Sdt + \sigma S dz \tag{31}$$

And thus the forward prices were given by:

$$F(S,T) = exp\left(e^{-\kappa T} \ln S + (1 - e^{-\kappa T})\alpha^* + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T})\right) \tag{32}$$

Muthoni et al (2016) used a three factor model; the three factors being the spot price of the commodity, instantaneous convenience yield and the instantaneous interest rate. To futures prices were calculated using the formulae:

$$F(s,\delta,r,T) = S\ exp[\frac{-\delta(1 - e^{-kT})}{k} + \frac{(r + \lambda s)(1 - e^{-\alpha T})}{\alpha} + C(T)]$$

4.6.2. Models' Prices vis-à-vis Observed Prices

Below is a comparison of the observed futures prices indicated by closing price, and the model's futures prices developed by Muthoni et al (2016).

Fig 4.7.1: Muthoni et al (2016) Model vs. Observed Prices

Source: Lucy Muthoni, P.O. Box. 4877-00506, Nairobi, Kenya – one of the authors of Muthoni et al (2016).

As the figure suggests, the model's prices are much higher compared to the observed prices. This does not reflect the presence of cost of information, which, when treated as a discount factor, leads to lower model prices compared to observed prices.

Comparing the figure 4.7.1 above with the figure 4.4.1 discussed earlier, we see that this project's model indeed produces prices which are lower than the observed market prices, putting into consideration the atmosphere of markets with incomplete information.

4.6.3. Regression Estimates Results

Under Muthoni et al (2016) , R^2 calculated was 0.9693. In this study, the calculated R^2 is 0.9758. That means that this thesis' model has a higher capability of predicting futures value compared to Muthoni et al (2016)'s model.

4.7. Suggested Improvements to Muthoni et al (2016)'s Model

Muthoni et al (2016) used a three factor model; the three factors being the spot price of the commodity, instantaneous convenience yield and the instantaneous interest rate. To futures prices were calculated using the formulae:

$$F(s,\delta,r,T) = S\ exp[\frac{-\delta\left(1-e^{-kT}\right)}{k} + \frac{(r+\lambda s)\left(1-e^{-\alpha T}\right)}{\alpha} + C(T)] \qquad (33)$$

We noticed that under this model, the second and the third terms of the exponents are treated as cumulative function instead of discount functions. From deep analysis of this model, we would like to suggest the following edition:

$$F(s,\delta,r,T) = S\ e^{-\left(\frac{\delta\left(1-e^{-kT}\right)}{k}+\frac{(r+\lambda s)\left(1-e^{-\alpha T}\right)}{\alpha}+C(T)\right)} \qquad (34)$$

Equation (34) shows that every term in the exponent is treated as discount factor, therefore reflecting the cost of information which is $-C(T)$.

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