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Abstract

This paper reports a small study of secondary school students problem-solving practices in Uganda. A sample of 225 students (109 male and 116 female), in eight government and private secondary schools were used. Students problem-solving processes and strategies were assessed using three non-routine tasks. Solutions of students from government and private schools and differences by gender were categorised and rated as exceptional, proficient, satisfactory, limited, poor and blank. The solutions mean scores were also examined using t-tests of independent samples. The results of the t-tests for independent samples indicated no statistically significant differences in students scores by gender. The results of the t-tests for independent samples indicated a statistically significant difference in students scores by school type. The results suggest that teachers in both government and private schools need to realise that when concepts and skills they teach make sense to students they learn faster, they remember better, and they are better able to use concepts and skills in subsequent problem-solving situations. This study needs replicating at different school levels and contexts using qualitative data collection techniques. Mathematics educators could benefit from knowledge of students problem-solving practices as students should be involved in solving problems rather than mastering skills and not applying them since "the ability to solve problems is at the heart of mathematics" (Cockcroft, 1982, p. 73).

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Keywords. Heuristics; Non-routine tasks; Polya; Problem-solving process; Uganda.

Introduction

In every country, education is basically intended to develop individuals who are independent critical thinkers, self-confident, motivated and multitalented, and who can comfortably fit into the various needs of adult life and society. In short, the individuals should be competent problem solvers. Problem-solving is often viewed in different ways. For example, one perspective regards problem-solving as a process where previously acquired knowledge is applied in a novel and unknown situation or a process of engaging in a task for which a prior solution method/procedure/algorithm/routine is not known before hand (Burton, 1984; National Council of Teachers of Mathematics [NCTM], 2000; Polya, 1957). Thus, involvement in problem-solving enables learners to exercise and acquire confidence in their intellectual power. Another view is that problem-solving provides a context for practising and applying concepts

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and skills that Charles (2009, p.1) calls "teaching FOR problem-solving" (emphasis in original). Teaching is itself a problem-solving activity. Furthermore, NCTM (2000, p.52) regards "problem-solving [as] an integral part of all mathematics learning." Problem-solving improves mathematical proficiency that entails five strands: "conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition" (National Research Council [NRC], 2001, p. 116) to enhance students building of helpful attitudes and beliefs about mathematics. However, Lesh (1981) has argued and cautioned that "knowing to compute does not ensure that a person will know when to compute, which operation to use in a particular situation, or how to use an answer once it is obtained" (p.235).

I agree that learners need to be confident in their mathematical knowledge and in their ability to seek, acquire and apply new knowledge. They should demonstrate persistence, resolve, flexibility and ingenuity in finding the solutions to problems. They should value and be aware of the importance of communication skills in mathematics and develop a positive attitude toward mathematics. They should appreciate the usefulness of computational competence, mathematical processes and problem-solving skills which are used in the decision making and modelling processes in society. However, the ability to perform basic computations and follow procedures may not be sufficient for learners to be successful in problem-solving. Learners need to develop and apply habits of mind such as thinking critically, thinking creatively and meta-cognition in the different phases of the problem-solving process.

A major purpose of studying mathematics is to learn to solve problems. Cockcroft (1982) argued that "the ability to solve problems is at the heart of mathematics" (p. 73). Some problems can be either routine requiring only the application of a known procedure or algorithm or non-routine requiring the development of a process or the conducting of an investigation to solve them. Several scholars have developed several heuristics and strategies for mathematical problem-solving and frameworks for analysing the problem-solving process (Burton, 1984; Heinze, 2005; Polya, 1957; Silver, Mamona-Downs, Leung & Kennedy, 1996). The most versatile and commonly used scheme of problem-solving is the one formulated by Polya (1957). Polya's four phases of problem-solving in what he calls the LIST entails: understanding the problem, devising a plan, carrying out the plan, and looking back. A similar model was suggested by Burton (1984) comprising of entry, attack, review and extension through organising questions, procedures, and skills for entry, attack, and review-extension as one hunts for a resolution rather than a solution.

Problem-solving ability has been investigated in several ways and several heuristics for problem-solving have been suggested in the literature. For example, Collis, Romberg and Jurdak (1986) used the structure of learned outcomes (SOLO) taxonomy in which the response modes are categorised into five levels, namely: (1) pre-structural, (2) uni-structural, (3) multi-structural, (4) relational, and (5) extended abstract.

A study by McCoy (1994) investigated the problem-solving actions of second and third grade school students as they engaged in solving three unfamiliar problems. She used observation and interviewing technique. The study showed that the students used systematic solution processes to attempt the problems. However, no significant gender differences were evident. Heinze (2005) investigated differences in problem-solving strategies and processes of mathematically gifted and non-gifted elementary students and found that mathematically gifted elementary students were more capable of working systematically and quickly, verbalising and explaining their solutions than non-gifted students. Kim, S.W. and Kim, S. Y. (2003) explored and compared the problem-solving processes of two groups: good novice problem solvers and poor novice problem solvers using the thinking aloud oral method followed by post-interviews. Their findings indicate that good novice problem solvers tended to use knowledge-development strategy, while the poor novice problem solvers have a propensity to use the means-end and random strategies. The two groups also differed in the amount of time spent on the problem and the ratio of the time spent in solving the problem to the total time spent.

In a previous research study I argued that "to be most fruitful, practice in problem-solving should not consist of repeated experiences in solving the same problems with the same techniques, but should consist of the solution of different problems by the same techniques and the application of different techniques to the same problem" (Opolot-Okurut, 1989, p. 44). Unfortunately, there has been lack of success to embed problem-solving in the curriculum despite efforts to introduce problem-solving in teacher education in Uganda (Opolot-Okurut, 1989). However, piecemeal efforts by the Ministry of Education and Sports to introduce problem-solving in primary and secondary schools exist. Whereas in the United States the NCTM (2000) in the *Principles and Standards for Mathematics teaching* recommended that instructional programmes for K-12 grades should enable student to (1) build new mathematical knowledge through problem-solving; (2) solve problems that arise in mathematics and other contexts; (3) apply and adapt a variety of appropriate strategies to solve problems; and (4) monitor and reflect on the process of mathematical problem-solving this practice is rare in mathematics classrooms of the developing world, but are worth investigating.

A brief overview of the Ugandan secondary school system, to give a context for the study is as follows. The secondary education lasts six years subdivided into Ordinary Level (four years) and Advanced level (two years). For O-level, students have to progress from senior one (S1) in the first year of lower secondary schooling at age of about 13 years to senior four (S4) in fourth of secondary school at age about 16 years. Successful students proceed to A-level. For A-level, students have to progress from senior five (S5) in the first year of higher secondary schooling at age of about 17 years to senior six (S6) in second year of higher secondary school at age about 18 years. After A-level students progress to tertiary education, after about 13 years of schooling.

The purpose of this study was to investigate the response patterns of the students from government-aided and private schools and by their gender on non-routine mathematical problems or tasks. In line with the purpose of the study the following research questions were posed:

1. Are there significant differences in achievement scores in mathematics non-routine problems between students from government-aided and private schools?
2. Are there significant differences in achievement scores in mathematics non-routine problems between male and female students?

Methodology

Research design

This exploratory study was conducted in government and private secondary schools in Central Uganda. A survey research design was followed and considered suitable because, we were interested in students problem-solving strategies and processes. The target population for the study was all students enrolled in the lower secondary education classes. Students in senior three participated.

Sample

The data discussed in this paper is from grade-nine secondary students. The students ages ranged from 15 to 21 years in eight secondary schools in Central Uganda. Thirty students, from one class, were randomly selected in classes whose teachers volunteered to participate in the study. Eight schools (four government-aided and four private) schools participated in the study.

Data from a total of 225 students: 109(48.4%) male and 116(51.6%) female participants were used in the analysis.

Instrument

The tasks used in this study were adopted from those developed by Opolot-Okurut (2004) as given in Appendix A. The instrument has two parts. Part A asked respondents for their demographic information: the gender, age and school type and Part B contained three non-routine tasks, which respondents were to provide their best answer in a test format. The students were advised to write their solutions legibly and to show clearly all working steps and strategies that were used in the space provided. These tasks did not need deep mathematical knowledge to solve but they needed the ability to apply insightful basic mathematical skills. The tasks were challenging questions and the time was unlimited. The students took between 20 and 90 minutes to complete.

Procedure

The writing of the test was conducted in each school under strict examination conditions. The time was unlimited to eliminate pressure of time from students. Most of the students finished within an hour. Students who could not do some of the tasks left earlier. The test administration procedure followed strict examinations regulations. The marking guide for individual task rubric followed the same pattern as outlined in Appendix B. The rubric for the three tasks consisted of a paragraph statement of the requirements of an adequate solution followed by the criteria for scoring students solutions from zero (no attempt or leaves a blank page) to five points. One point was given for a solution that was ‘inadequate’; two or three points were given to a ‘satisfactory’ solution; four or five points were given to a solution that is ‘outstanding’. Any solution was expected to indicate a student understands mathematics and application of mathematical knowledge. For example, five marks were awarded to a student who represented houses in two columns with letters A to E and F to J and joined each letter to the other letters without double counting, and counted the number of links between two letters and obtained the correct answer. A session for the coordination of marking of the test was conducted with all the markers. During the marking each script was marked by three independent markers and an agreed mark between the markers for each task obtained after discussion. Each students solutions were scored using analytic scoring in which the marker considered whether specific points on the marking guide were addressed by the solution. The solution was then scored according to the points that it closely similar to that laid down in the scoring rubric for the particular problem.

Data Analysis

The collected data were analysed using descriptive statistics and the means of the achievement scores were compared using the independent samples t-test for the different school types and gender.

Results

Each script of three questions was examined and marked for evidence that satisfied the criteria outlined in the assessment rubric giving a total of 675 questions. These results are given for each task in Tables 1-3. Of the 675 solutions that the students attempted 320 solutions were categorised as blank. Of the 355 non-blank solutions 143(40.3%) were categorised as exceptional; 24(6.8%) were categorised as proficient; 66(18.6%) were categorised as satisfactory; 59(16.6%) were categorised as limited; and 73(20.6%) were categorised as poor. Table 1 shows that about 30 percent of the students wrote exceptional solutions to Task 2, which was Anthony, a school sports prefect, has to plan a football tournament involving ten schoolhouse teams. Each house-team has to play every other house-team once. What is the total number of games to be played that he has to plan for? Show clearly how you worked the total out, were the overall well attempted task even if the majority of the students left it blank. Due to space limitation the results for only Task 2 are presented here as example of solutions.

TABLE 1. Overall student performance on the tasks

Criteria	**Score**	**Tasks**			**Total**	**Percent**
		T1	**T2**	**T3**		
Exceptional	5	41	68	34	143	40.3
Proficient	4	13	07	04	24	6.8
Satisfactory	3	38	13	15	66	18.6
Limited	2	25	09	25	59	16.6
Poor	1	25	03	35	73	20.6
Blank	0	83	125	112	320	47.4
Total		**225**	**225**	**225**	**675**	**100.0**

Q1. Are there significant differences in achievement scores in mathematics non-routine problems between students from government-aided and private schools?

Descriptive statistics were used to capture the government and private schools students responses for each task. For this task, Table 2 shows that most of the students from government-aided schools gave exceptional solutions to this task when compared to those from the private schools. There were nearly 50% of the students from private schools who did not attempt the task. Comparing the mean scores by school type the t-test of independent samples shows that there were significant school type gender differences for this task.

TABLE 2. Scores and percent (in brackets) of student scores by school type for Task 2

School Type	**Exceptional**	**Proficient**	**Satisfactory**	**Limited**	**Poor**	**No Attempt**
Government	44 (36)	05 (04)	09 (07)	07 (12)	01 (06)	56 (46)
Private	24 (23)	02 (02)	04 (04)	02 (02)	02 (02)	69 (67)
Total	68	07	13	09	03	125

Total $N = 225$, Government $n = 122$, Private $n = 103; t(220.62) = 3.01, p < .05$, sig.

Q.2 Are there significant differences in achievement scores in mathematics non-routine problems between male and female students?

Descriptive statistics were used to capture the male and female students responses for each task. For this task Table 3 shows that most of the students turned in exceptional solutions to this task but over 50% of the students did not attempt the task. Comparing the mean scores by gender the t-test of independent samples shows that there were no significant gender differences for this task.

Table 3. Scores and percent (in brackets) of student scores by gender for Task 2

Gender	Exceptional	Proficient	Satisfactory	Limited	Poor	No Attempt
Male	33 (30)	02 (02)	09 (08)	03 (03)	02 (02)	60 (55)
Female	35 (30)	05 (04)	04 (03)	06 (05)	01 (01)	65 (56)
Total	68	07	13	09	03	125

Total $N = 225$, Male $n = 109$, Female $n = 116$; $t(223) = 0.04$, $p > .05$, ns.

Discussion

This study set out to investigate lower secondary students achievement in non-routine mathematics tasks in government and private schools and by their gender. The paper has captured the solutions strategies and process that included for instance taking smaller number of letters to represent houses and working through to 10 houses; writing digits 1 to 10 to represent the houses and drawing loops from each digit to the other digits and counting them; and using 10 dots to represent houses and joining each dot to others and counting the number of lines, for each task using descriptive statistics. The students solutions were rated on six point scale ranging from blank (0) to exceptional solution (5). The mean achievement scores were compared by school type and by gender using an independent samples t-test. One key finding was, students attending both government and private schools were able to submit some exceptional solutions to given tasks. In general there are statistically significant differences in student achievement by school type. This result increases our knowledge of the problem-solving strategies used by students in lower secondary schools in Uganda. There is evidence that the common heuristics that students applied to solve the tasks were drawing diagrams and tables, attempting trial and error methods, listing and counting on strategies. This finding supports the type of strategies students in the McCoys (1994) study that showed that students closely followed Polyas (1957) stages of problem-solving: understanding the problem, devising a plan, carrying out the plan, and looking back. This result suggests that there is need for teachers to encourage students to participate in problem-solving in school mathematics. The national examining body considers including problem-solving items in the public examinations; and text books authors think of incorporating problem-solving content via appropriate situations in their books and to include problem-solving tasks to promote problem-solving. In sum, students could solve problems using different heuristics and strategies if teachers nurture them to do so. This finding reveals that the achievement of students on non-routine tasks did vary significantly according to the type of school they were enrolled.

Another significant finding from this study was that both male and female students turned in exceptional solutions to the given tasks. This finding is not surprising because most of the schools are coeducational and students discuss among themselves some of their work. There was evidence that the heuristics commonly used were guessing and trial and error, algebra, listing counting on and underlining key words in each of the problems for all problems. However, some of the students gave up giving comments like ‘we are not told how many lessons there were before break’, students showing evidence of lack of understanding of the problem claiming ‘It is impossible for the bells to ring at once’, students introducing their own conditions and assumptions such as ‘assume four periods in each section before break’, some students used the contextual knowledge of the timing of lessons in the school timetable and guessing that ‘the bells will all ring at the same time at 1:00 pm for lunch.’ This finding suggests that the performance of students on non-routine tasks did not vary significantly according to their gender.

Conclusion

The findings and their discussion lead to the following conclusions:

1. The achievement of students on non-routine tasks did vary significantly according to the type of school they were enrolled.
2. The performance of students on non-routine tasks did not vary significantly according to their gender.

This study has investigated problem-solving behaviour of lower secondary students in Uganda. Although the study has few findings, several limitations can be cited that affect how generalisable the results of the study are to which the reader is take the results with caution for the following reasons. First, the format of the non-routine tasks in the test maybe a factor that could influence the achievement results in this study. Second, the sample was from one area of the country close to an urban setting. Thus, these results cannot be generalised to students in other districts and schools of the country. And thirdly, the method of data collection did not include interviews and observation but limited to written solutions survey.

Implications for teaching and learning

In summary, the findings of this students problem-solving processes study have several interesting and important implications for both practice and further research. From a practical point of view, three implications are apparent from the findings. First, teachers should be made aware of the need to nurture students mathematical proficiency. For example, more opportunities must be availed to students to develop the mathematical proficiency strands (NRC, 2001). Second, administrators, especially those in the private schools, in light of the findings of this study must provide more teacher support to enable them concentrate less on only having student pass examinations. Third, systematic efforts must be made to promote problem-solving in school mathematics by focusing on teacher pre-service preparation programmes and in-service education through seminars and workshops. Fourth, mathematics teachers need to develop student mathematical understanding through a balance of solving problems and presenting examples to them in the context of a problem-focussed and question-driven classroom conversation often regarded as 'teaching THROUGH problem-solving'. Fifth, in general, teachers need to pay more attention to the development of students problem-solving skills in their classrooms. Finally, encourage textbook authors to include problem-solving tasks in their books and introduce content in these books via appropriate problem situations. In terms of further research the following areas are suggested as needing more research. (1) Future investigations may extend the research of the current study by exploring students achievement and school-related variables in addition to those used here. (2) This study could also be replicated at different levels of education (primary, upper secondary, teacher education and tertiary levels. (3) The use of qualitative research approaches to the investigations could be considered. (4) Other factors that could be causes or associated with the student achievement should be explored.

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APPENDICES

Appendix A: Mathematics Problem Solving Tasks

STUDENT CODE .... GENDER: M .... F......... SCHOOL CODE.......
AGE......... TIME: 1 hour

INSTRUCTIONS:

(a) Attempt all problems. All reasonable solutions are acceptable.
(b) All problems carry the same number of marks.
  (i) Write legibly, and show all your working steps clearly.
  (ii) Show all strategies you use and working in the answer sheets provided.

Task 1

A school is divided into lower, middle and upper sections. The change-of-lesson bell rings after thirty, forty and forty five minutes in the lower, medium and upper sections respectively.

(a) Morning break falls when the bells ring at the same time. If the lessons in the lower and middle section start at 8.00 am at what time does break start?
(b) The break lasts 30 minutes. After break, lessons start at the same time in the lower, middle ad upper sections. At what time will the three bells ring at the same time?

Task 2

Anthony, a school sports prefect, has to plan a football tournament involving ten school house teams. Each house-team has to play every other house-team once. What is the total number of games to be played that he has to plan for? Show clearly how you worked out the total.

Task 3

Children are seated around a table and pass round a packet of sixteen sweets. Ekanya takes the first sweet. Each child then takes one sweet at a time as the packet is passed around. Ekanya also receives the last sweet. Find three possible numbers of children seated on the table and how many sweets each one gets in each case.

(Source: Opolot-Okurut, 2004)

Thank you for your cooperation

Appendix B: Solutions Categories, Indicators and Scores for Non-Routine Tasks

CATEGORY	INDICATORS	RATING
EXCEPTIONAL	-fully acceptable solution	5
	-accurate execution of strategy	
	-appropriate choice of strategy	
	-correct interpretation of problem	
PROFICIENT	-mostly acceptable solution	4
	-accurate execution of strategy	
	-appropriate choice of strategy	
	-correct interpretation of problem	
SATISFACTORY	-partly unacceptable solution	3
	-minor errors in execution of strategy	
	-appropriate choice of strategy	
	-correct interpretation of problem	
LIMITED	-unacceptable solution	2
	-accurate execution of incorrect strategy	
	-inappropriate choice of strategy	
	-correct interpretation of problem	
POOR	-no solution, working abandoned	1
	-inaccurate execution of wrong strategy	
	-inappropriate choice of strategy	
	-incorrect interpretation of problem	
BLANK	No attempt, empty answer script	0

A Generic Rubric for Scoring Non-Routine Tasks

CRITERIA	SCORE	SOLUTION
• Attempts to extend the problem; contains a full complete solution; correct interpretation of problem; correct strategy identified and followed.	5	As given
• Starts with a correct interpretation of the problem; identifies correct strategies; gives a complete solution with minor errors.	4	As given
• Interprets the problem correctly starts with a correct strategy; follows some wrong steps; part correct solution.	3	As given
• Gives incomplete solution; shows some errors; starts with an appropriate strategy.	2	As given
• Begins with an inappropriate strategy; misunderstands the question; shows major errors; incomplete solution.	1	As given
• No attempt or response.	0	Nil

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