Mathematical Model for Pneumonia Dynamics among Children

by

Jacob Otieno Ong'ala

Strathmore University, Nairobi (Kenya)

at

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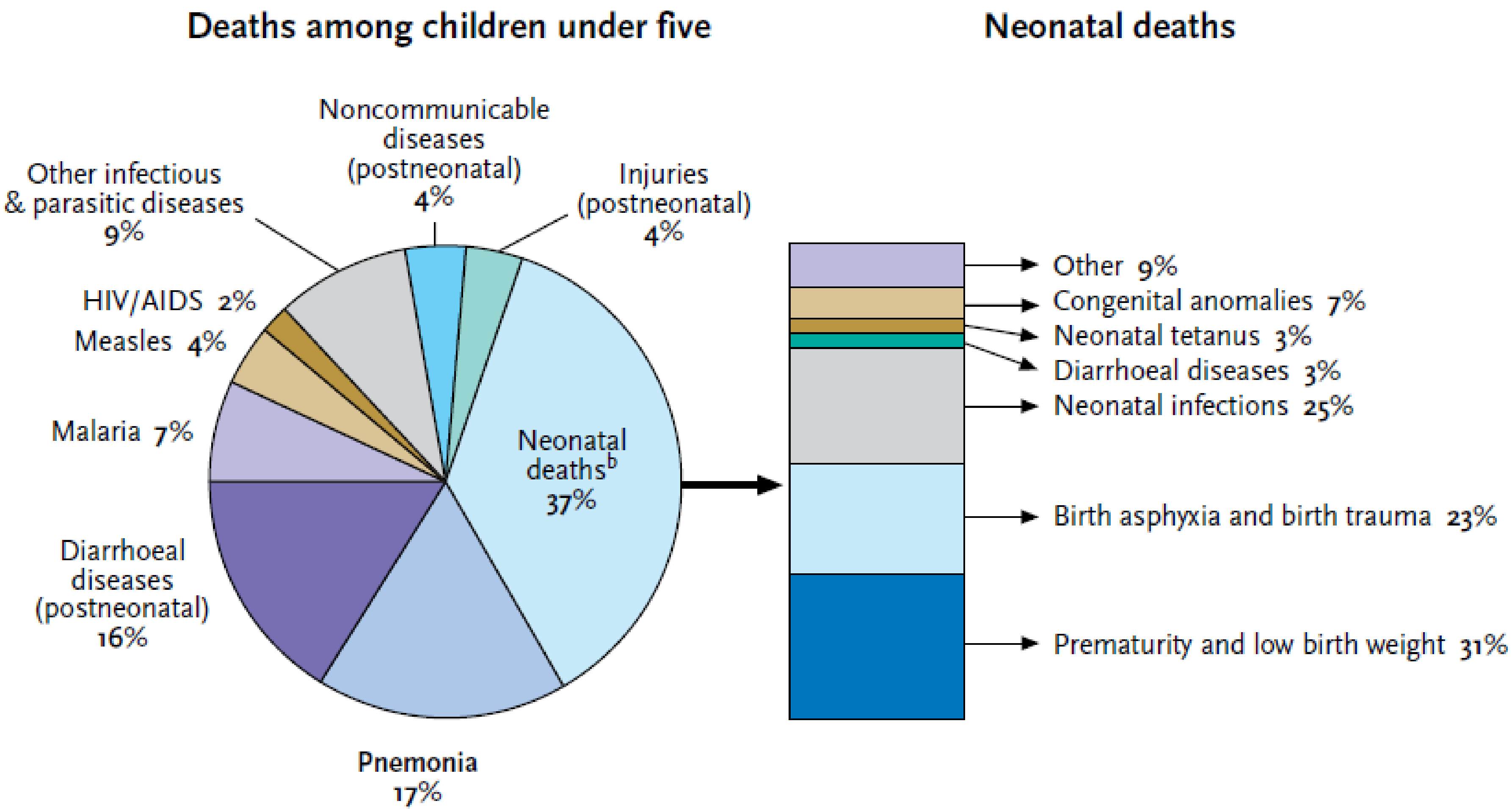
Outline

1. Background information of pneumonia
2. Derivation of the model
3. Analysis and simulation of the model
4. Results and Discussion

Motivation


Deaths among children under five
Neonatal deaths
Noncommunicable diseases (postneonatal) 4%
Other infectious & parasitic diseases 9%
Injuries (postneonatal) 4%
HIV/AIDS 2%
Measles 4%
Malaria 7%
Neonatal deaths[b] 37%
Diarrhoeal diseases (postneonatal) 16%
Pnemonia 17%
Other 9%
Congenital anomalies 7%
Neonatal tetanus 3%
Diarrhoeal diseases 3%
Neonatal infections 25%
Birth asphyxia and birth trauma 23%
Prematurity and low birth weight 31%

Background Information of Pneumonia

- Pneumonia is a high inflammatory disease characterized by inflammatory condition of the lungs.
- Cases of pneumonia can be as a result of inhaling droplets of coughs and sneezes of an infected person.
- Some of the risk factors associated to the spread of pneumonia are: malnutrition, lack of exclusive breastfeeding, indoor pollution, antecedent viral infection amongst others
- It is majorly caused by bacteria (*Streptococcus Pneumoniae*) amongst other micro-organisms such us: virus, parasites and fungi.
- Once the bacteria gets into the lungs, they settle in the alveoli and passages of the lungs where they invades by multiplying in number and cause infection.

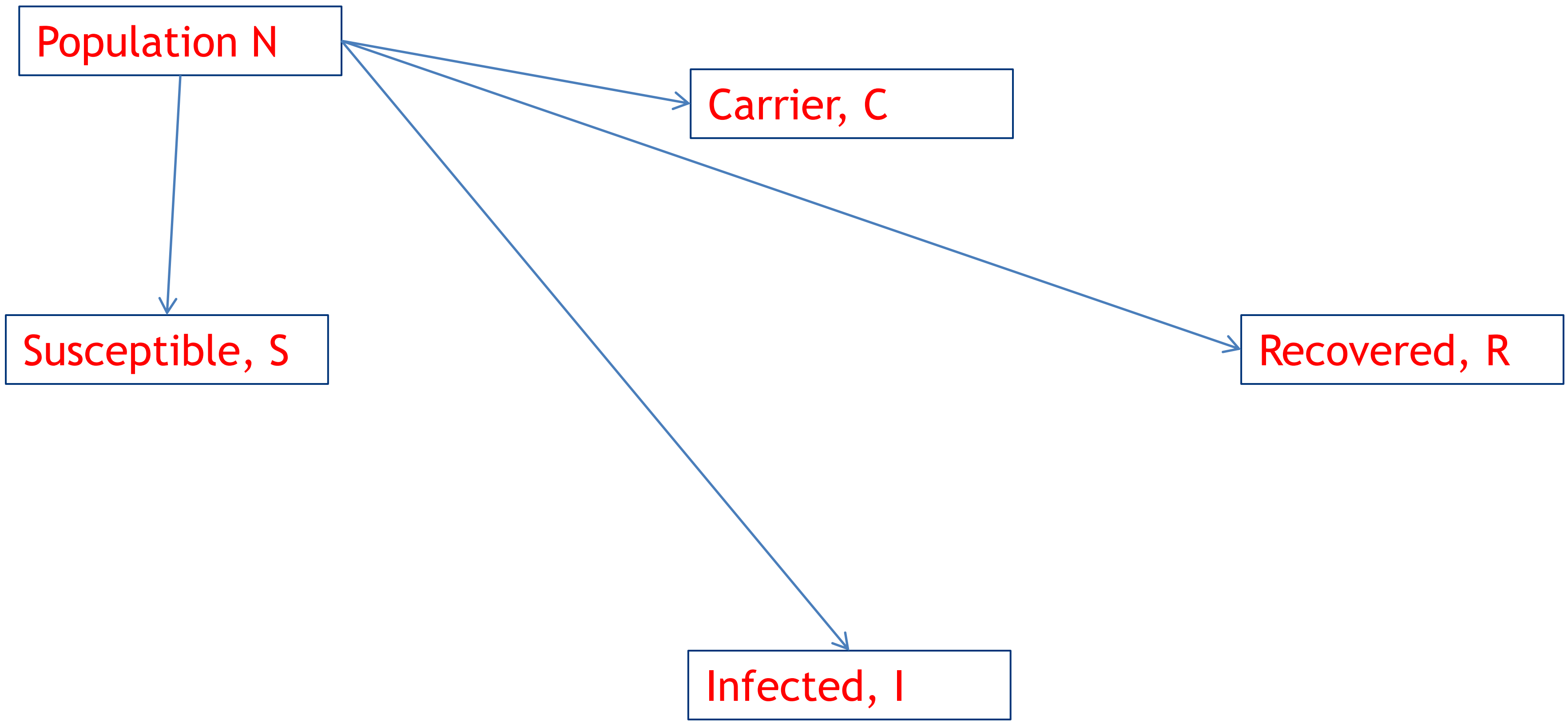
Background Information of Pneumonia (Cont…)

- As the body attempts fight the bacteria, the area infected get filled with fluids and pus which makes breathing difficult.
- Medical literature also indicates that a healthy person can carry the bacteria in his nasopharynx (These are referred to as **Carriers**)
- The carriers can developed infection if the bacteria find its way to the lungs. This is possible when immunity of an individual is lowered.
- There is a possibility of the carrier to clear the bacteria due to self immunity or with antibiotic treatment.
- Individuals showing symptoms (infected) can also recover due to self immunity or through antibiotic treatment.
- The recovered individuals can only acquire temporary immunity.

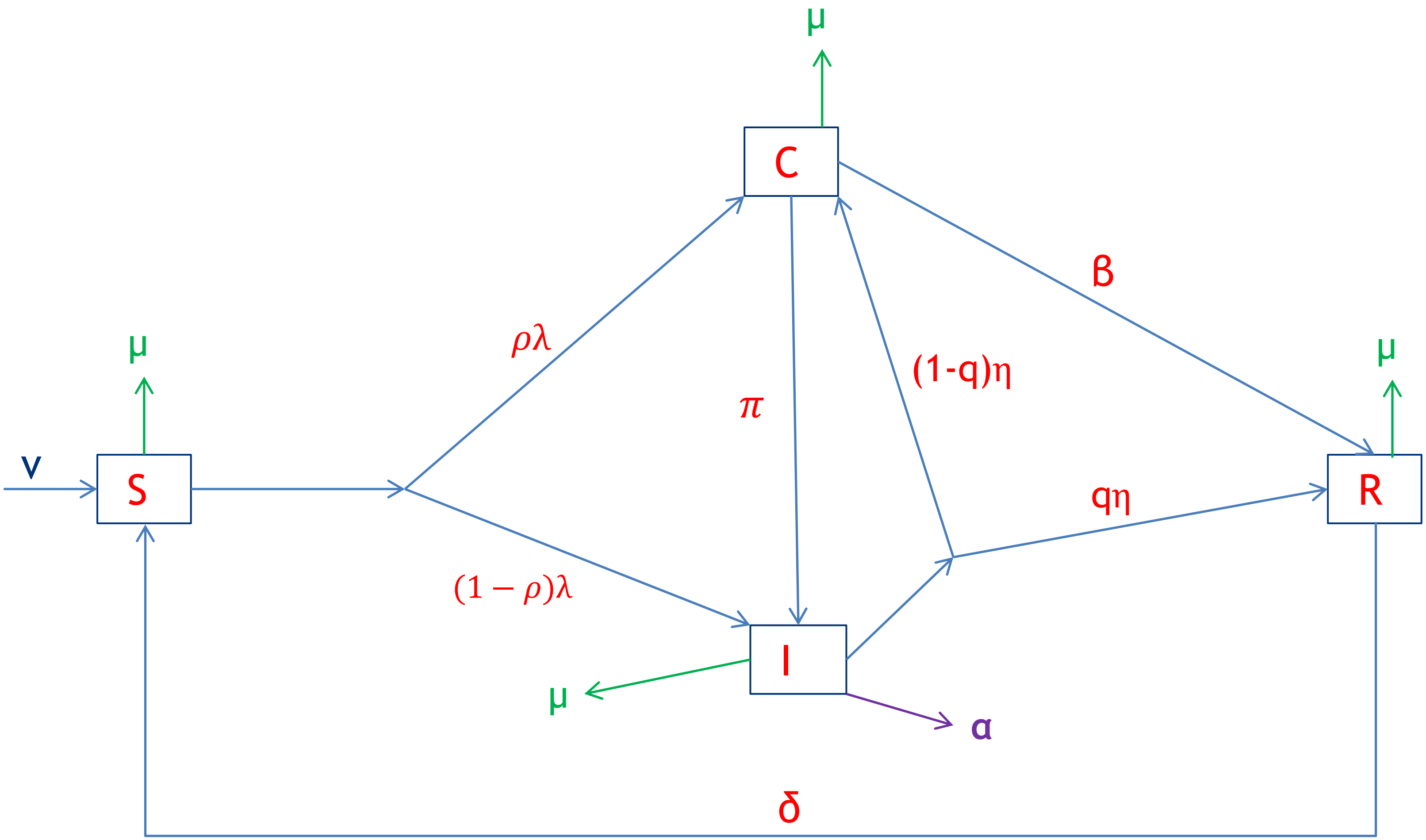
Background Information of Pneumonia (Cont…)

- The control strategies available for pneumonia are Vaccination, treatments and ‘Isolation’.
- Despite an increasing resistance of sp to antibiotics, it is still the major treatment procedure used for pneumonia.
- Today the vaccines used are a 7-valent, 13-valent and 23-valent polysaccharide.
- These can cover upto 23 strains of sp (30% of all strains available)
- This means that if vaccination is used, we are not be 100% sure that the disease would be contained.
- An understanding of the transmission dynamic is therefore required to determine the optimal control measures

Derivation of the model



Derivation of the model



System of Equations

$$\lambda = \psi\left(\frac{I(t) + \varepsilon C(t)}{N(t)}\right) \; : \; \psi = \kappa P$$

$$\frac{dS(t)}{dt} = \nu + \delta R(t) - (\lambda + \mu)S(t)$$

$$\frac{dI(t)}{dt} = (1 - \rho)\lambda S(t) + \pi C(t) - (\mu + \alpha + \eta)I(t)$$

$$\frac{dC(t)}{dt} = \rho(\lambda)S(t) + (1 - q)\eta I(t) - (\mu + \pi + \beta)C(t)$$

$$\frac{dR(t)}{dt} = q\eta I(t) + \beta C(t) - (\mu + \delta)R(t)$$

Analysis of the model

- We checked the positivity and boundedness of the solutions
- Determination of the basic reproduction number R_0 using the next generation operator method.
- Existence and Stability of the disease free equilibrium (DFE)
- Existence and Stability of the Endemic equilibrium (EE)
- Bifurcation Analysis.
- Numerical Analysis.

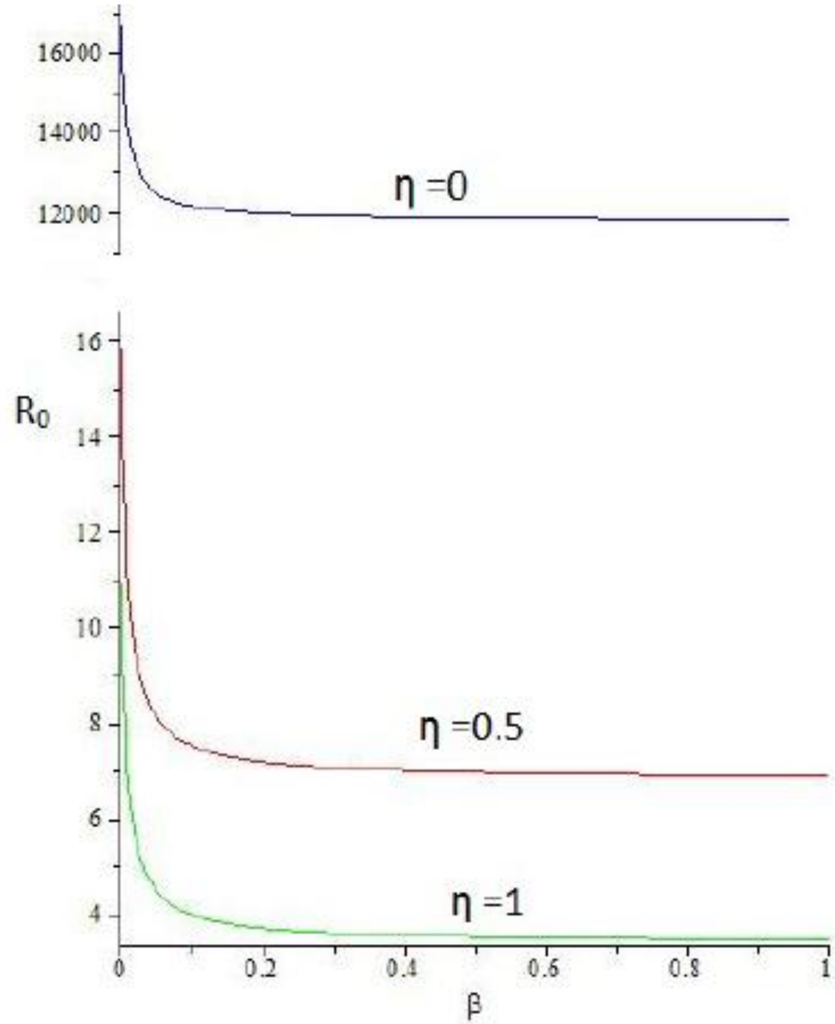
Results

$$R_0 = \kappa \mathcal{P} \left(\frac{\rho[\varepsilon(\mu+\alpha+\eta)+\pi]+(1-\rho)[\mu+\beta+\pi+(1-q)\varepsilon\eta]}{(\mu+\alpha+\eta)(\mu+\beta+\pi)-(1-q)\pi\eta} \right)$$

- R_0 is directly proportional to the contact rate κ and to the mean time spent in the diseased classes. $\frac{1}{(\mu+\alpha+\eta)(\mu+\beta+\pi)-(1-q)\pi\eta}$

- Moreover, a partial derivative of R_0 w.r.t β and κ is a decreasing function

Results



Results

- From the jacobian matrix of the model and DFE;

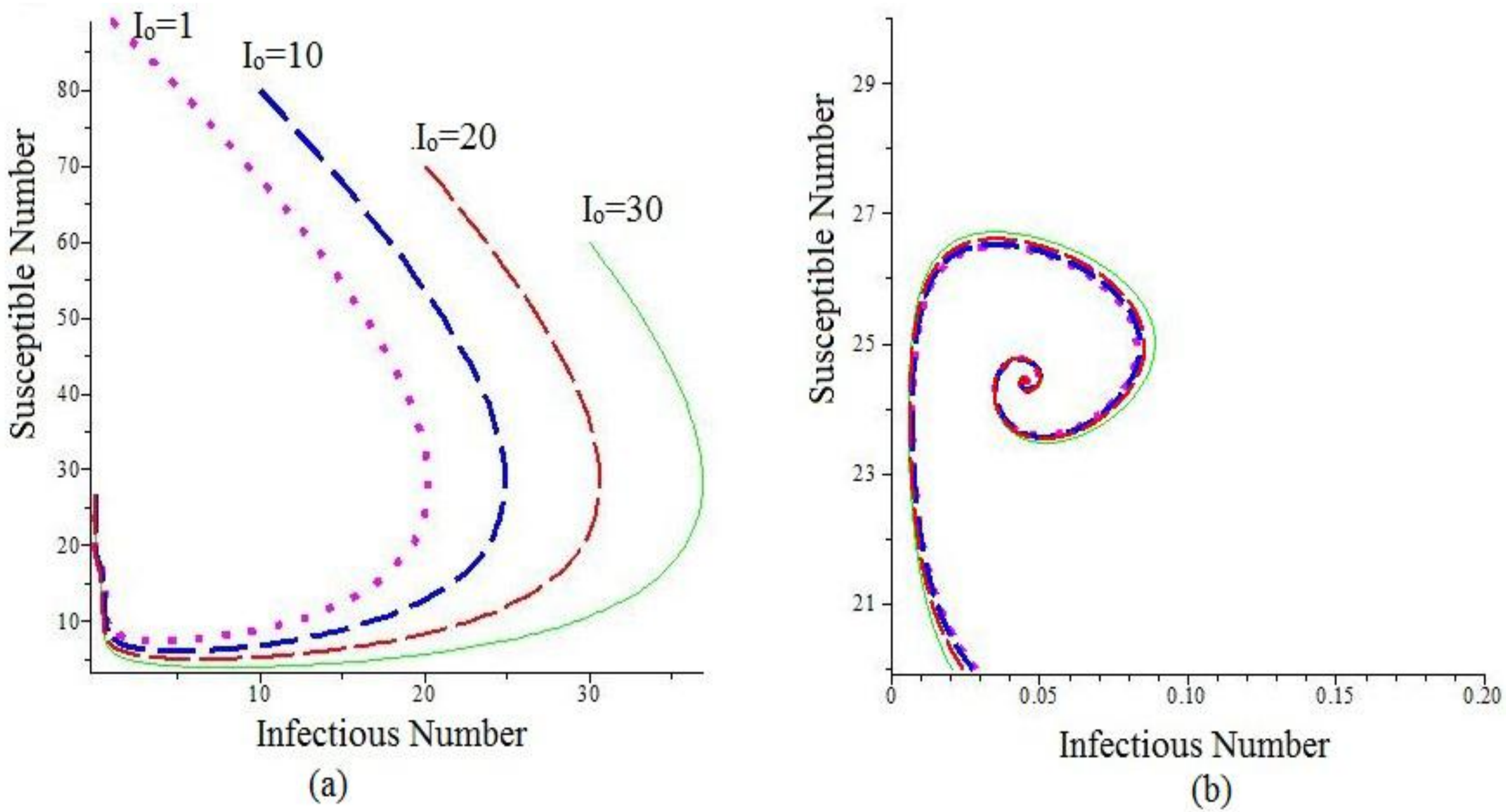
$$\mathcal{J}(E^f) = \begin{pmatrix} -\mu & -\psi & -\psi\varepsilon & \delta \\ 0 & -h_1 & \pi & 0 \\ 0 & (1-q)\eta & -h_2 & 0 \\ 0 & q\eta & \beta & -(\mu+\delta) \end{pmatrix}$$

$$Trace\left[\mathcal{J}(E^f)\right] = -(2\mu + \delta + h_1 + h_2) < 0$$
$$Det\left[\mathcal{J}(E^f)\right] = \mu(\delta + \mu)[h_1 h_2 - (1-q)\pi\eta] > 0$$

- Implying that DFE is stable:

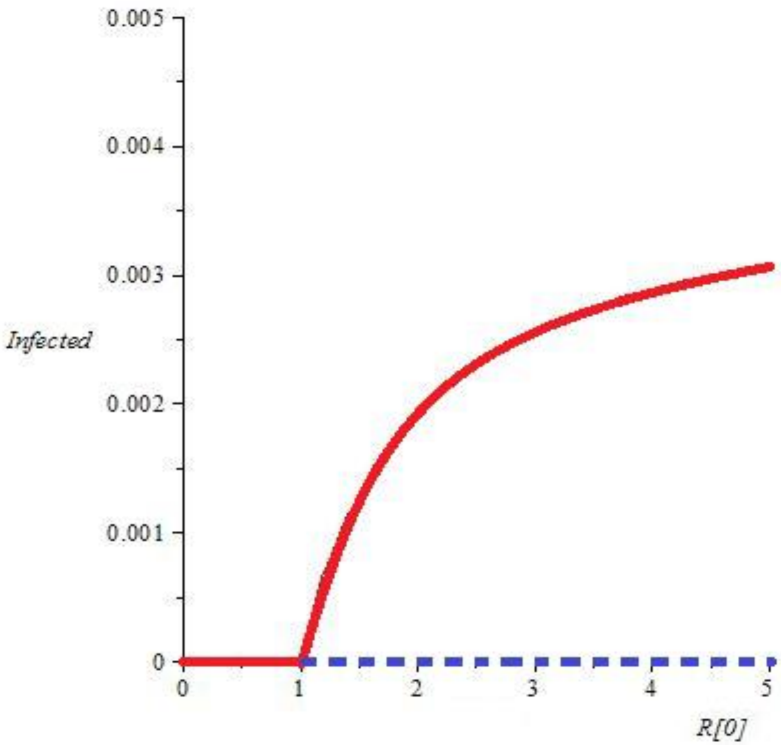
Results

- The stability of EE is assessed using the graph below.

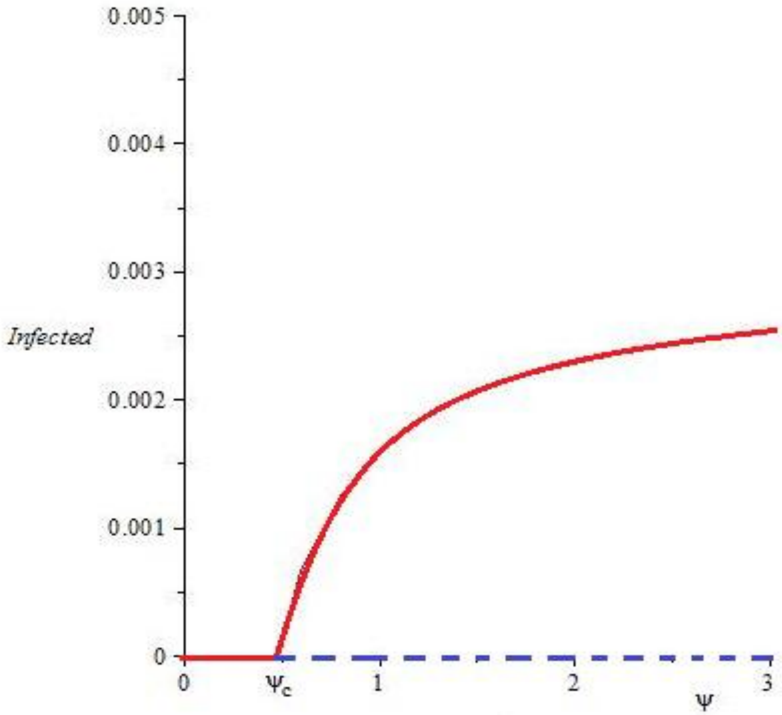


Results

Bifurcation analysis.



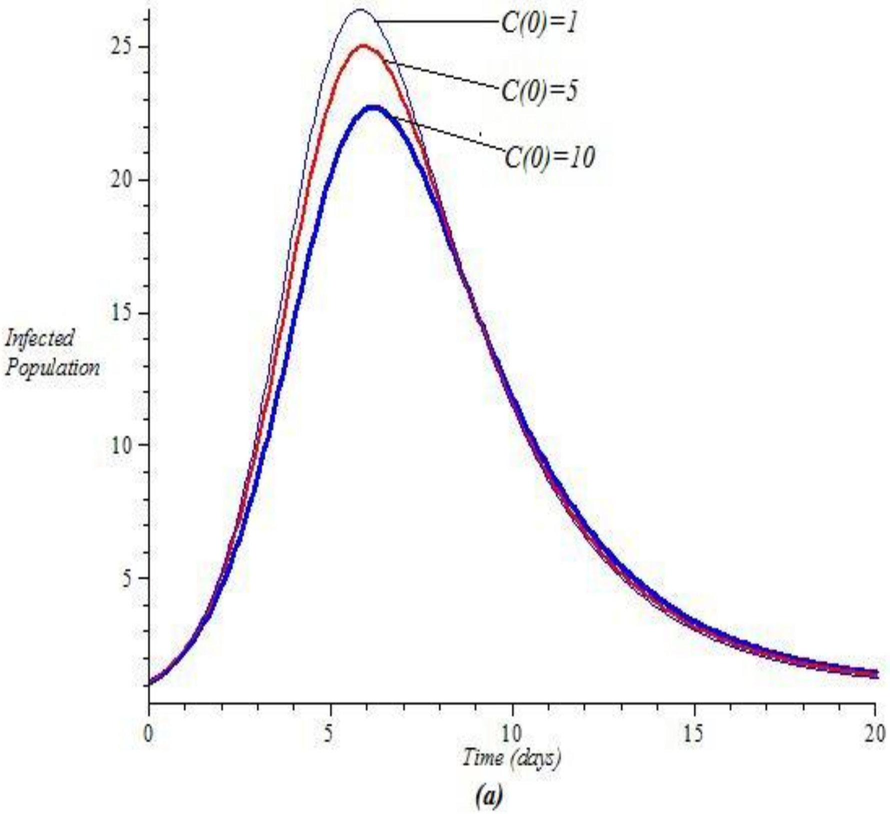
(a)

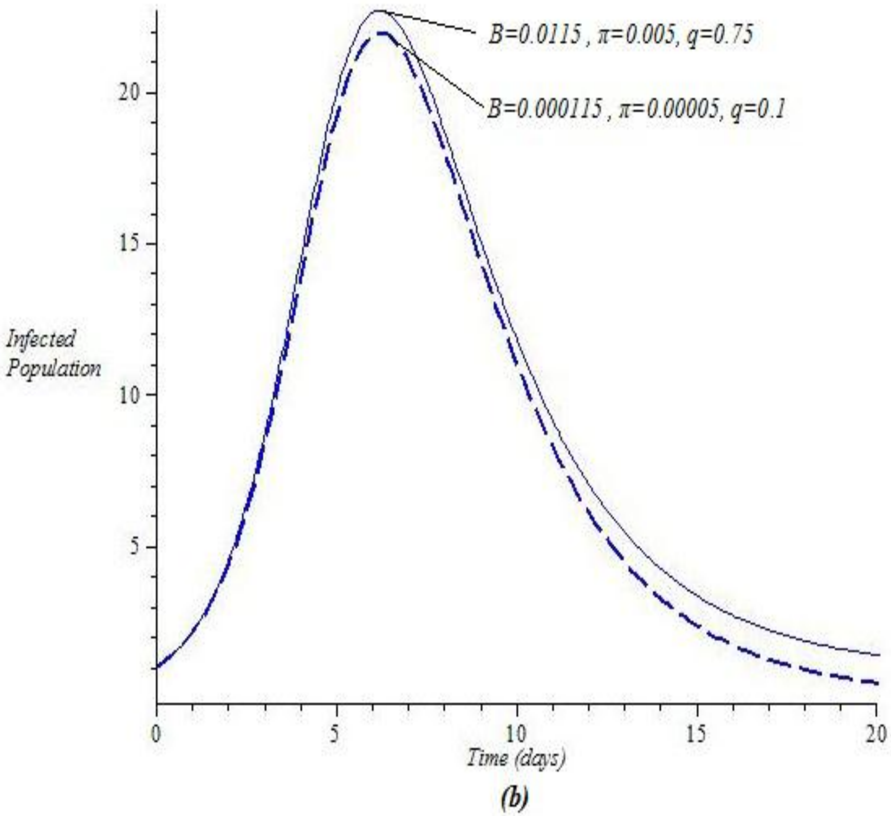


(b)

Results

Effect of Carrier proportion





Discussion

- The existence of forward bifurcation indicates that when $R0 < 1$ then the disease free equilibrium is stable and become unstable when $R0 > 1$.
- The Local stability of the endemic equilibrium point EE changes its nature to unstable when it crosses the critical value via a forward bifurcation.
- This is a clear indication that the effective control measure for pneumonia is achieved when Ro is reduced.
- The first sure way of reducing the progression of the disease is to isolate
- It is also possible to reduce the incidence by increasing recovery rates of the diseased class
- Increasing the carriers proportion in a closed population reduced the Infected proportions.

Further working

- Optimal control strategies (Using probabilistic approach)
- Effect on misdiagnosis and mistreatment of pneumonia in transmission dynamics

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Strathmore

UNIVERSITY

Ole Sangale Road, Madaraka Estate. PO Box 59857-00200, Nairobi, Kenya
Tel +254 (0)20 606155, 606268, 606380 Fax +254 (0)20 607498
Mobile +254 (0)722 25 428, (0)733 618 135 Email info@strathmore.edu
www.strathmore.edu