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APPLICATION OF MARKOV CHAIN THEORY AND NETWORK THEORY TO A MANPOWER SYSTEM

A PROJECT SUBMITTED TO THE SCHOOL OF MATHEMATICS IN PARTIAL FULFILLMENT

FOR A DEGREE OF MASTER OF SCIENCE IN SOCIAL STATISTICS.

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Declaration

I the undersigned declare that this project is my original work and to the best of my knowledge has not been presented for the award of a degree in any other University.

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ABSTRACT

Labour is one of the key resources in any institution that if managed well can lead to increased productivity. Regular evaluation of existing structures and policies is necessary to avoid wastage and stagnation.

Alot of work has been done in analysis of manpower systems using Markov models and other models such as computer simulation models,optimization models,supply chain models and holonic models. In this study Markov chain theory has been integrated with network theory in evaluating effective administration of the current schemes of service for teachers by the Teachers Service Commission. The growth and development of the teacher progression system has also been assessed.

Acknowledgments.

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Dedicated to:

To my husband,Walter Ochieng Aoko who has offered me a lot of support and encouragement throughout the study.

Contents

Chapter 1

INTRODUCTION

As recently as two centuries ago per capita incomes were not very different across countries. Today's vast differences in living standards between the richest and poorest nations is a reflection of sustained differences in economic growth. Economic growth is the increase in value of goods and services produced by a country. The ability of a country to produce goods and services depends primarily on the quantity and quality of its resources. The extent of economic growth depends on how well a country's resources are managed. Labour is one of the key resources in any country or organization. Proper management of labour or human resource has a direct bearing on productivity in any institution.

In Kenya growth in the Gross Domestic Product has risen from less than 2 percent in 2002 to 6.2 percent in 2007(Kenya Fact Sheet, May 2007). Nevertheless Kenya as one of the developing nations still faces a major challenge of improving the living standards of its people. Education is one of the key areas that contribute to development by equipping people with the necessary knowledge and skills for existing industries and entrepreneurship. Proper management of resources available in this area is important if education is to remain relevant to the ever changing economic environment.One of the important resources in education is teachers. In Kenya the government is the main employer of teachers through the Teachers Service Commission. The Teachers Service Commission was established in 1967 by an Act of Parliament(Cap 212 of the laws of Kenya)to provide services to teachers and was mandated to perform the following teacher management functions:-registration,recruitment,deployment,remuneration,promotion,discipline and maintenance of teaching standards. The mission of the Teachers Service Commission is " Establish and maintain,in partnership with all stakeholder a sufficient professional teaching service for educational institutions-responsive to environmental changes."

Analysis of manpower systems have become a very important component of planned economic development of any organization or nation. However manpower planning depends on highly unpredictable behaviour and uncertain environment. As a result the use of mathematical models of manpower systems is very important.

1.1 Aim and Objectives of the Study

The overall aim of this study is to integrate Markov chain theory and network theory in evaluation of a manpower system.

1.2 Specific Objectives

a) Obtain transition matrix of movement of teachers between job groups.

b) Obtain flow matrix using transition matrix.

c) Compute network indices from flow matrix and relate them to a manpower system.

d) Give suggestions on areas of further application of network theory to manpower systems.

1.3 Significance of the Current Study

A Chinese proverb says " If you are planning for a year plant rice, if you are planning for ten years plant trees, if you are planning for a lifetime educate people". In Kenya the national goals of education include: national unity, national development, individual development, social equality and national consciousness. How far we achieve these goals is to a great extent a measure of the productivity of the education system. The contribution of teachers towards the achievement of these goals cannot be overemphasized. If productivity is to be maximized then efficiency in management of teachers is key.

In Kenya the main employer of teachers is the Teachers Service Commission. The Commission has a scheme of service for

a) Non-Graduate Teachers

b) Technical Teachers and Lecturers

c) Graduate Teachers.

The objectives of the schemes of service are

a) To provide for a well defined career structure which will attract and facilitate retention of suitably qualified teachers with adequate professional training and competence in the Teaching Service.

b) To establish standards for recruitment,training and advancement within the career structure on the basis of qualifications, merit and ability as reflected in work performance and results.

c) To provide for clearly defined job descriptions and specifications at all levels within the career and grading structure which will ensure proper deployment and utilization of teachers.

Development may be defined as increase in organization. Webster(New Collegiate Dictionary,G.C Merriam Co.,1981) defines the verb "to organize" as "to arrange or form into a coherent unity or functioning whole." This is synonymous with one meaning of the verb "to articulate," as to form or fit a systematic whole." Articulation also means clear and precise communication. In the schemes of work TSC has articulated responsibility, career structure and advancement for teachers at all levels. Development of the manpower system for teachers depends largely on the effective administration of the schemes of service. The growth and development of an organization is reflected in its power to attract and retain able people. Attraction and retention of qualified teachers is one of the objectives of the schemes of service. This study evaluates the effective administration of the schemes of service for teachers and also assesses the growth and development of the Teachers Service Commission.

1.4 Modeling Teachers promotion

In this study the Markov chain model is applied to the Teachers Service Commission manpower system from job group K through to job group R. The study has been restricted to these job groups (leaving out job groups F to J) because all the data required to compute transition probabilities for F to J was not available within the working time frame for this project.

Assumptions

a) Promotion from K to L is automatic after three years.

b) Promotion to M,N,P,Q and R is through interviews.

c) A teacher is eligible for promotion to the next job group after having been in the current job group for at least three years.

d) Chances of transition to a lower job group are negligible.

e) Promotion is only to the next higher level job group.

f) Entry into the system is only at $\boldsymbol{K}$.

g) Teachers leave at any of the job groups by resignation, retirement or death.

These assumptions are only valid for job groups $\boldsymbol{K}$ to $\boldsymbol{R}$. Those who leave the system are considered to have been absorbed into the absorbing state as chances of re-employment by the Commission are negligible. According to current recruitment guidelines preference is given to those not previously employed by the commission which highly limits the chances for those seeking re-employment.

The data used to compute transitional probabilities is for 2004 to 2007 for number of teachers per job group and teachers promoted per job group obtained from the Teachers Service Commission. Transition matrices were obtained for 2004-2005,2005-2006 and 2006-2007. Averages of the transitions were obtained to give a transition matrix whose transitional probabilities are assumed to be time homogeneous in the study for computational convenience. The transition probabilities represent the proportions of the number of teachers remaining in the same job group, moving to the next job group or dropping out of the system. These proportions and the number of teachers per job group were used to obtain the flow matrix. The unusable exports (flow out of the teaching service) in the flow matrix are the teachers who drop out of the system through death. The data on the number of teacher deaths per job group obtained from the commission was incomplete and therefore the figures used are estimates. The proportions of teachers who die per job group were obtained from the data available on deaths per job group. These proportions and the total number of deaths were used to get the estimates used for unusable exports. From the flow matrix the indices used in ecosystems were computed using a script implemented in ***R*2.5.0**(See appendix B)

1.5 Literature Review

Analysis of manpower systems is an important component of planned economic development of any organization or nation.Efficiency in management of a manpower system in any institution or organization has a bearing on productivity.Various mathematical models have been used in analysis of manpower systems such as Markov chain models,computer simulation models,optimization models and supply chain models . Wang (2005)reviewed operations research applications in workforce planning and potential Modeling of military training.The models were decomposed into four major categories: Markov chain models, computer simulation models,optimization models and supply chain management through system dynamics.Each category was reviewed for underlying mathematical formalism and concepts,advantages and potential limitations.

Markov chain models have been applied to very many areas. In Education stochastic models have been applied in various studies.Gani(1963) used the Markovian model to forecast the total enrollment in Australian Universities and the number of degrees to be awarded in future.Thonstad (1967)applied the Markovian model to the Norwegian education system.Marshall and Oliver(1970)applied the model to the student population of the University of California.Uche(1980)derived the retention properties of the Nigerian education system using the Markov chain model and later using the cohort analysis approach.In Kenya the Markov chain model has been applied to the education system in various studies.Owino(1982)computed the various retention properties of the system.Odhiambo and Owino(1985)applied the model in estimating academic survival.Odhiambo and Khogali(1986)developed a transitional model for estimating academic survival by cohort analysis.Odondo (1985)compared education characteristics within provinces in the country.Owino and Philips(1988)applied the Markov chain model in comparing retention properties of the Kenyan primary education system before and after 1972. Kones(2000)compared the retention properties of pupils in rural and urban schools.Mbugua (2005)studied the implications of free primary education in terms of expected length of schooling,cost,staffing and capital requirements. Markov chain models have been applied in examining the structure of manpower systems in terms of the proportion of staff in each grade or age profile under a variety of conditions and evaluating policies for controlling manpower systems,Young and Almond (1961),Young (1971),Forbes (1971 a,b),Bartholomew(1973)and Gani(1973).

Vassiliou (1976) came up with a model for wastage in manpower systems. He modeled the phenomenon of voluntary leavers with the concept of cumulative acceleration of the overall growth of an organization. He assumed that the number of "normal" resigning workers is increased by a number of "frustrated" leavers, proportional to the cumulative acceleration of expansion of the system. Raghavendra (1991)employed a Markov chain model in obtaining the transition probabilities for promotion in a bivariate framework consisting of seniority and performance rating. Georgiou and Vassiliou (1997)have introduced phases in a Markov chain model and investigated the input policies subject to cost objective functions.Yadavalli and Natarajan (2001)studied a semi Markov model in which a single grade system allows for wastage and recruitment.Yadavalli et al(2002) subsequently studied the time dependant behaviour of stochastic models of manpower systems with the impact of pressure on promotion.Setlhare (2007) has studied stochastic models of manpower systems with reference to recruitment, promotion, training and wastage. Setlhare derives expressions for relevant measures of system performance and develops appropriate cost models.

Anke Richter et al(2005)developed a Markov model of personnel flow of junior officers to increase Navy Nurse Corps manpower management efficiency.

Markov chains have also been used in studies on ecosystems. Leguerrier et al (2006)used Markov chains to assess residence time,first passage time,rate of transfers between compartments,recycling index with a general mathematical formalism. Network analysis is a powerful, general analytical tool that makes it possible to study objects as part of a connected system and to identify and quantify the direct and indirect effects in that system. It is an area that is growing and gaining acceptance as a way to answer important questions about the connectivity of system components. Network analysis has the advantage over other methods of analysis for its ability to quantify direct,indirect and integral relationships within a system (Fath and Patten 1998). Some of the studies advancing this approach include, Platt et al(1981),Ulanowicz and Platt(1985),Wulff et al (1989),Higashi and Burns (1981)and Patten and Jørgensen (1996).Network analysis is an environmental application of input-output analysis which was developed by Leontief(1936,1951,1966).Shannon (1979) proposed that network analysis could be used as a goal function in that ecosystems operate to maximize their total direct and indirect storages. Herendeen(1981)applied network formulation to determine energy intensity in ecological and economic systems.He also used network analysis to compare goal functions (Herendeen 1989;Brown and Herendeen 1996)such as exergy (Jørgensen and Mejer 1983;Jørgensen 1986),ascendency(Ulanowicz 1980, 1986),and emergy (Odum 1983,1988).

Patten(1978,1982)and subsequent coworkers developed a line of ecological network analysis called environ analysis. Patten (1978) set the foundations of network environ analysis by applying the previously developed general systems theory to ecological systems and established three key ideas toward an environmental system theory. First, every object within a system has two environments that within the system boundaries, could be specified and quantified as environs. Second, an environmental(external)reference state is needed to account for the internal causation of a system.Third,the propagation of flow along each pathway is uniquely targeted for and derived from a particular component.

The concept of ecological indicators is developing rapidly literally on a daily basis.A lot of work has been done on development and selection of ecological indicators. Some have been borrowed from other areas such as Communication theory which was founded by Shannon(1948) and further developed by Gallager(1968). Rutledge et al (1976) applied an index of communication theory, the Average Mutual Information (AMI) to ecological networks. The contribution of Rutledge et al (1976)to the development of communication theory in Biology led Ulanowicz (1980)to develop an index Ascendency which would encompass the natural growth and development of ecosystems. Finn (1976)developed standard methods to calculate total system through flow,average path length and cycling index that has been widely used in ecology. Jørgensen (2006)proposes three holistic thermodynamic indicators eco-exergy, specific exergy and buffer capacity to cover essential ecosystem health properties. Ecological indicators have been used in evaluating the effects of environmental policies. Turnhout et al(2006)approaches the concept of ecological indicators from a social science perspective. The focus is on ecological indicators that attempt to measure the ecological quality of ecosystems and are specifically developed to be used as instruments to evaluate the effects of policies.

Chapter 2

THE MARKOV CHAIN PROCESS

2.1 Introduction

A stochastic process is one defined by $(\boldsymbol{X_t, t} \in \boldsymbol{N})$and takes values in the a space $\boldsymbol{E}$ called the state space where $\boldsymbol{T}$ is the time space.Its future outcomes cannot be stated with certainty. The best that can be done is to assign probabilities to the various outcomes.Consider a discrete time stochastic process $(\boldsymbol{X_t, t} \in \boldsymbol{N})$ taking values in a discrete state space E = {1,2,3 ...,N} which is finite.

The process is called a Markov chain if

$$\boldsymbol{Pr[X_{t+1} = j/X_0 = i_0, X_1 = i_1, X_2 = i_2, ..., X_n = i_n] = Pr[X_{t+1} = j/X_t = i]} \tag{2.1}$$

i.e the future outcome at time $\boldsymbol{t+1}$ depends only on the outcome at time $\boldsymbol{t}$ and not on any previous outcomes.

$$\boldsymbol{p_{ij}(t) = Pr[X_{t+1} = j/X_t = i]} \tag{2.2}$$

is the transition probability from state $\boldsymbol{i}$ to $\boldsymbol{j}$ at time $\boldsymbol{t}$ for $\boldsymbol{i}$,$\boldsymbol{j}$=1,2,...,N The transition probabilities can be arranged in matrix form.The one step transition matrix for the Markov chain is given by

$$P(t) = (p_{ij}(t)) = \begin{pmatrix} p_{11}(t) & p_{12}(t) & . & . & . & p_{1N}(t) \\ p_{21}(t) & p_{22}(t) & . & . & . & p_{2N}(t) \\ & & & . & & \\ & & & . & & \\ & & & . & & \\ p_{N1}(t) & p_{N2}(t) & . & . & . & p_{NN}(t) \end{pmatrix} \tag{2.3}$$

2.2 Properties of the transition matrix

The transition matrix is a stochastic matrix,that is

$$p_{ij} \geq 0 \tag{2.4}$$

for all i and j

$$\sum_{j=1}^{N} p_{ij}(t) = 1 \tag{2.5}$$

The probability of moving from state i to state j in n steps is

$$p_{ij}^{(n)}(t) = Pr[X_{t+n} = j / X_t = i] \tag{2.6}$$

This gives rise to the $\boldsymbol{n}$-step transition matrix

$$P^{(n)}(t) = (p_{ij}^{(n)}(t)) = \begin{pmatrix} p_{11}^{(n)}(t) & p_{12}^{(n)}(t) & . & . & . & p_{1N}^{(n)}(t) \\ p_{21}^{(n)}(t) & p_{22}^{(n)}(t) & . & . & . & p_{2N}^{(n)}(t) \\ & & & . & & \\ & & & . & & \\ & & & . & & \\ p_{N1}^{(n)}(t) & p_{N2}^{(n)}(t) & . & . & . & p_{NN}^{(n)}(t) \end{pmatrix}$$

The probability vector at time $t+n$ is defined by

$$p'(t+n) = (p_1(t+n), p_2(t+n), ..., p_j(t+n), ..., p_n(t+n))' \qquad (2.7)$$

where$p_j(t+n) = Pr[X_{t+n} = j]$ i.e probability that at time $t+n$ the process is in state j.

2.2.1 Chapman-Kolmogrov Equations

The Chapman-Kolmogrov equations essentially define what happens after many steps.

$$P^{(n)}(t) = (p_{ij}^{(n)}(t)) = \prod_{n=0}^{n-1} P(t+n) \qquad (2.8)$$

$$p'(t+n) = p'(t)P^{(n)}(t) \qquad (2.9)$$

In general,n-step transitional probabilities can be obtained by computing the n-th power of the one-step transition matrix i.e $P^{(n)}(t) = P(t).P^{(n-1)}$

A Markov chain is ***homogeneous*** if its transition probabilities do not depend on time.

$p_{ij}(t) = P_{ij}$ for all t

This implies that $P(t) = (p_{ij}(t)) = (p_{ij}) = P$

$$\boldsymbol{P} = (p_{ij}) = \begin{pmatrix} p_{11} & p_{12} & \cdot & \cdot & \cdot & p_{1m} \\ p_{21} & p_{22} & \cdot & \cdot & \cdot & p_{2m} \\ & & & \cdot & & \\ & & & \cdot & & \\ & & & \cdot & & \\ p_{m1} & p_{m2} & \cdot & \cdot & \cdot & p_{mm} \end{pmatrix} \tag{2.10}$$

The Chapman-Kolmogrov results for the homogeneous Markov chain are

(i)$\boldsymbol{P}^{(n)} = ((p_{ij}^{n})) = \boldsymbol{P}^{n}$

(ii)$p'(t+n) = p'(t)\boldsymbol{P}^{n}$

2.3 Classification of states and chains

A state j is said to be ***absorbing*** if upon reaching this state, the process will never leave this state again .State i is an absorbing state if and only if $P_{jj} = 1$. A Markov chain is said to be an absorbing Markov chain if it has at least one absorbing state.

A Markov chain is ***ergodic*** if it is possible to go from every state to any other state i.e

$p_{ij}^{n} > 0$for all i and j and for some positive integer n.

A Markov chain is ***irreducible*** if every state can be reached from every other state.

A state is ***transient*** if upon entering this state the process may never return to it again i.e there exists a state j $(j \neq i)$ that is accessible from state i but not vice-versa.

A state is ***periodic*** if upon entering this state, the process will definitely return to this state in a fixed number of steps.

A state is said to be ***reccurent*** if upon entering this state, the process will definitely return to this state again.

2.4 Transition Matrix in Canonical Form

Consider a Markov chain with $\boldsymbol{r}$ absorbing states and $\boldsymbol{s}$ non-absorbing states. The transition probability matrix can be represented in canonical form:

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{I}| & \boldsymbol{O} \\ \boldsymbol{D}| & \boldsymbol{Q} \end{pmatrix} \tag{2.11}$$

where,

$\boldsymbol{I}$ is an $\boldsymbol{r} \times \boldsymbol{r}$ identity matrix representing transition probabilities between absorbing states.

$\boldsymbol{O}$ is an $\boldsymbol{r} \times \boldsymbol{s}$ matrix of zeros giving transition probabilities from absorbing to non-absorbing states.

$\boldsymbol{D} = ((\boldsymbol{d}_(\boldsymbol{ik})))$ is an $\boldsymbol{s} \times \boldsymbol{s}$ matrix which gives transition probabilities from non-absorbing to absorbing states.

$\boldsymbol{Q} = ((\boldsymbol{q}_(\boldsymbol{ij})))$ is an $\boldsymbol{s} \times \boldsymbol{r}$ matrix of transition probabilities between non-absorbing states.

Using the Chapman-Kolmogrov equation, $\boldsymbol{P^n = P.P^{n-1}}$,the n-step transition matrix in canonical form for the process is given by:

$$\boldsymbol{P^n} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{O} \\ \boldsymbol{D + QD + \dots + Q^{n-1}D} & \boldsymbol{Q^n} \end{pmatrix}$$

$$(2.12)$$

The elements of Q^n give the probability of transiting from state i to state j in n steps,where i and j are non-absorbing states.The elements of $(D + QD + ...Q^{n-1}D)$ denoted by d^n_{jk} give the probability of dropping out from grade j in n years.

$D + QD + Q^2D + ... + Q^{n-1}D = (1 + Q + Q^2 + ... + Q^{n-1})D$

For an absorbing Markov chain, $Lim_{n\to\infty}Q^n = 0$

thus $I + Q + Q^2 + ...$ is an infinite series in Q whose sum is given by

$$L = (I - Q)^{-1} \tag{2.13}$$

L is known as the fundamental matrix of the absorbing Markov chain.

2.5 Some properties of the fundamental matrix

Consider a random variable X^n_{ij} where

$$X^n_{ij} = \begin{cases} 1, & \text{if one moves from } i \text{to } j \text{ in } n \text{ time units} \\ 0, & \text{otherwise} \end{cases}$$

$E[X^n_{ij}] = 1 \times Pr[X^n_{ij} = 1] + 0 \times Pr[X^n_{ij} = 0] = Pr[X^n_{ij} = 1] = q^n_{ij}$ which is the ijth entry of Q^n.

Consider the sum

$$T^n_{ij} = \sum_{n=0}^{n} X^n_{ij} \tag{2.14}$$

which is the total time one in state i will spend in state j within n time units.

$$E[T^n_{ij}] = \sum_{n=0}^{n} EX^n_{ij} = \sum_{n=0}^{n} q^n_{ij} = q^0_{ij} + q^1_{ij} + ... + q^n_{ij} \tag{2.15}$$

which is the ijth element of $I + Q + Q^2 + ... + Q^n$

$lim_{n\to\infty}E(T^n_{ij}) = q^0_{ij} + q^1_{ij} + q^2_{ij} + ... + q^n_{ij} = L = (I - Q)^{-1}$

The elements of $\boldsymbol{L}$, $\boldsymbol{l_{ij}}$ give the expected length of stay in state $\boldsymbol{j}$ from $\boldsymbol{i}$ before leaving the system.

$$\boldsymbol{L} = \begin{pmatrix} \boldsymbol{l_{11}} & \boldsymbol{l_{12}} & . & . & . & \boldsymbol{l_{1m}} \\ \boldsymbol{l_{21}} & \boldsymbol{l_{22}} & . & . & . & \boldsymbol{l_{2m}} \\ & & & . & & \\ & & & . & & \\ & & & . & & \\ \boldsymbol{l_{m1}} & \boldsymbol{l_{m2}} & . & . & . & \boldsymbol{l_{mm}} \end{pmatrix} \tag{2.16}$$

Summing the rows of $\boldsymbol{L}$ e.g row one; $\sum_{\boldsymbol{j=1}}^{\boldsymbol{m}} \boldsymbol{l_{ij}}$ gives the expected length of stay for an entrant in grade one.

These properties can be exploited in the study of a manpower system as is the case in the foregoing chapters.

Chapter 3

NETWORK THEORY IN ECOSYSTEM STUDIES

3.1 Introduction

Network theory concerns properties that arise in systems of many objects linked together. It is central to understanding and managing complexity because many features of systems arise from their underlying network structure,rather than the specifics of the objects and interactions. The universal nature of networks means that there is potential applications to many natural and artificial systems.Examples include communication networks,control systems, food webs,gene regulatory networks,disease spread and epidemics,neural networks and social systems. In this study network theory is applied to a manpower system.

An ecosystem is a community of organisms, interacting with one another, plus the environment in which they live and with which they also interact e.g lake,forest e.t.c. Such a system includes abiotic components such as mineral ions, organic compounds and the climatic regime. The biotic components generally include representatives from several trophic levels(M Thain and M Hickman 2001). Ecosystems can be thought of as networks. In a spatially and temporally defined ecosystem,groups of individuals of the same species are called populations. Interactions are model led as transfers of material or energy between populations and /or groups of populations of similar function. In symbolic representation of the system, each population or group of populations appear as one node. The nodes are connected if there is a direct interspecific transfer of material or energy. Modeled this way ,ecological systems lend themselves to mathematical analysis for patterns.

3.2 Ecosystem Indices

Several indices have been developed and used to evaluate ecosystem health, sustainability, growth and development. As already mentioned in the literature review the concept of ecological indicators is developing rapidly literally on a daily basis. In this study we will only consider ecosystem indices that are applicable to a heirachical system which can be seen as a network with flow in one direction only.

3.2.1 Notation and nomenclature for equations

i) $\boldsymbol{n}$: number of internal compartments.

ii) $\boldsymbol{n+1}$: destination of usable exports.

iii) $\boldsymbol{n+2}$: destination of unusable exports(respiration/dissipation)

iv) $\mathbf{0}$: source of imports into the internal network.

v) $\boldsymbol{T_{ij}}$: flow from compartment $\boldsymbol{j}$ to $\boldsymbol{i}$ where $\boldsymbol{j}$ represents the columns of the flow matrix and $\boldsymbol{i}$ the rows.

vi) $\boldsymbol{T_{i.}}$: total inflows to compartment $\boldsymbol{i}$

vii) $\boldsymbol{T_{.j}}$: total outflows from compartment $\boldsymbol{i}$.

viii) $\boldsymbol{z_{i0}}$: flow into compartment $\boldsymbol{i}$ from outside the network.

ix) $\boldsymbol{y_{n+1,j}}, \boldsymbol{y_{n+2,j}}$:flow out of the network for compartment $\boldsymbol{j}$ to compartment $\boldsymbol{n+1}$ and $\boldsymbol{n+2}$ respectively.

3.2.2 Flow Matrix

The following is a general flow matrix. The columns are the source of flow while the rows are the destination of flow.Compartments $\boldsymbol{n+1}$ and $\boldsymbol{n+2}$ are the export compartments for usable and unusable exports respectively. **0** is the source of imports into the system.

$$\boldsymbol{F} = \begin{array}{c} \\ 1 \\ 2 \\ \\ \\ \\ n \\ n+1 \\ n+2 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & . & . & . & n \end{array} \\ \begin{pmatrix} T_{10} & T_{11} & T_{12} & . & . & . & T_{1n} \\ T_{20} & T_{21} & T_{22} & . & . & . & T_{2n} \\ & & & . & & & \\ & & & . & & & \\ & & & . & & & \\ T_{n0} & T_{n1} & T_{n2} & . & . & . & T_{nn} \\ T_{n+1,0} & T_{n+1,1} & T_{n+1,2} & . & . & . & T_{n+1,n} \\ T_{n+2,0} & T_{n+2,1} & T_{n+2,2} & . & . & . & T_{n+2,n} \end{pmatrix} \end{array} \tag{3.1}$$

We will consider the indices under three categories

a) Network Uncertainty.

b) System growth and development.

c) Environ analysis.

3.2.3 Network Uncertainty

The indices considered under this category are Average Mutual Information,Statistical Uncertainty,Conditional Uncertainty and Realized Uncertainty.

3.2.4 Average Mutual InformationAMI

The $\boldsymbol{AMI}$ was borrowed from communication theory and applied to ecological works by Rutledge et al (1976).It is a measure of the average amount of constraint placed upon an arbitrary unit of flow anywhere in the network.Rutledge et al hypothesized that rigid diets and highly constrained flow in an ecosystem would form a brittle system, unable to persist in the face of changing environmental conditions.They suggested that as a system matures to form a web-like pattern of energy and material flow, the AMI should fall.

$$\boldsymbol{AMI = k \sum_{i=1}^{n+2} \sum_{j=0}^{n} \frac{T_{ij}}{T_{..}} log_2 \frac{T_{ij}T_{..}}{T_{i.}T_{.j}}} \tag{3.2}$$

3.2.5 Statistical Uncertainty H_R

Uncertainty is the inability to say a priori what the exact outcome of an event will be. Uncertainty about a particular outcome is related inversely to the probability of that event taking place and is measured directly by the logarithm of the probability of that outcome.

$$\boldsymbol{H_i = Klog(1/p_i)} \tag{3.3}$$

or

$$\boldsymbol{H_i = -Klogp_i} \tag{3.4}$$

where $\boldsymbol{p_i}$ is the probability of outcome $\boldsymbol{i}$,$\boldsymbol{K}$ is a constant of proportionality, and $\boldsymbol{H_i}$ is the degree of uncertainty one assigns to outcome $\boldsymbol{i}$. Logarithms are invoked because there are certain utilitarian properties that a valid measure of uncertainty should possess.

- the measure be non-negative, $\boldsymbol{H(p_i) \geq 0}$

- the measure should be decisive in instances involving certainty,$\boldsymbol{H(1) = 0}$

- the uncertainty about co-occurences of two unrelated outcomes should equal the sum of the uncertainties about the individual outcomes.$\boldsymbol{H(p_i q_j) = H(p_i) + H(q_j)}$,where $\boldsymbol{p_i}$ and $\boldsymbol{q_j}$ are the probabilities of the independent outcomes $\boldsymbol{i}$ and $\boldsymbol{j}$ respectively.

Statistical uncertainty is a measure of diversity. It refers to the effective number of choices for energy flow in the system (Rutledge et al 1976). A system of populations with flexible diets and high choice of energy flow would respond to disturbance by shifting in a direction to counteract the disturbance and return to equilibrium (Odum, 1953 and McArthur, 1955). For an ecosystem a higher value of Statistical Uncertainty means more flexibility and therefore better coping mechanisms in the face of disturbances. It is the upper bound on $\boldsymbol{AMI}$.

$$\boldsymbol{H_R = -\sum_{j=0}^{n} \frac{T_{.j}}{T_{..}} log_2 \frac{T_{.j}}{T_{..}}} \tag{3.5}$$

3.2.6 Conditional Uncertainty D_R

This is the difference between Statistical Uncertainty and Average Mutual Information. It is a measure of the stability of the system. According to Rutledge et al (1976)as the $\boldsymbol{AMI}$ falls in a system with increasing 'choice'(i.e web-like structure)from the system's statistical Uncertainty the stability increase.

$$D_R = H_R - AMI \tag{3.6}$$

3.2.7 Realized Uncertainty

$$AMI/H_R \tag{3.7}$$

This index expresses the total fraction of total uncertainty accounted by the network structure as measured by the $\boldsymbol{AMI}$. It can be used to compare the degree of constraint across systems.

3.2.8 System Growth and Development

Growth implies increase or expansion. Increase can be either in spatial extent or as accretion of the medium of flow. Expansion in spatial context increase the number of nodes. Compartmental throughput(the greater between the total inflow and total outflow) may characterize the size of the node. The size of the entire system is the sum of the individual throughput or total system throughput $\boldsymbol{T..}$.

Development is the increase in organization. In a highly organized system flow from one compartment should engender an effect at another site over a highly articulated (jointed and unambitious)pathway. If one knows that a flow has left compartment $\boldsymbol{i}$ at time $\boldsymbol{t}$ then in a highly organized system this provides a great deal of information about which compartment $\boldsymbol{j}$ will receive the flow at time $\boldsymbol{t+\theta}$.

The indices that measure the systems growth and development considered here are ascendency,development capacity, overhead and extent of development.

3.2.9 Ascendency $\boldsymbol{A}$

Ascendency quantifies both system size and organization. The size component is measured by the Total System Throughput ($\boldsymbol{T_{..}}$)and the organization is measured by the $\boldsymbol{AMI}$. Ascendency increase during development of an ecosystem from an early undeveloped stage to a climax stage (Ulanowicz 1997).

$$\boldsymbol{A = AMI \times T_{..} = \sum_{i=1}^{n+2} \sum_{j=0}^{n} T_{ij} log_2 \frac{T_{ij}T_{..}}{T_{i.}T_{.j}}} \tag{3.8}$$

3.2.10 Development Capacity $\boldsymbol{C}$

This is the upper bound on ascendency.

$$\boldsymbol{C = -\sum_{i=1}^{n+2} \sum_{j=0}^{n} T_{ij} log_2 \frac{T_{ij}}{T_{..}}} \tag{3.9}$$

3.2.11 Overhead $\boldsymbol{\Phi}$

This is the difference between Development Capacity and Ascendency. It reflects the multiplicity of pathways in the network.

$$\boldsymbol{\Phi = C - A} \tag{3.10}$$

3.2.12 Extent of Development $\boldsymbol{A/C}$

This is a ratio of ascendency to capacity.

$$\boldsymbol{A/C = \frac{A}{C}} \tag{3.11}$$

It gives a measure of the extent of development of the system. It is useful for comparing ascendency across networks.

3.2.13 Environ Analysis

In this category we consider one index, synergy

3.2.14 Synergy b/c

Synergy occurs when the net sum of all relations in a model is positive.A transaction is the direct,observable transfer of conservative resources between two organisms. A relation is the direct and indirect consequence of these transfers. Whereas a transaction is physical, a relation is conceptual and can occur between two organisms which are not connected via direct transactions. Network Synergism (Patten 1991,1992)is based on utility analysis.$\boldsymbol{D_P}$ is the non-dimensional direct flow-based matrix whose elements are described by

$$\boldsymbol{d_{ij} = \frac{T_{ij} - T_{ji}}{T_i}} \tag{3.12}$$

where

$$\boldsymbol{T_i = \sum_{j=0}^{n} f_{ij} = \sum_{j=0}^{n} f_{ji}} \tag{3.13}$$

where $\boldsymbol{f_{ij}}$ is flow from $\boldsymbol{j}$ to $\boldsymbol{i}$ and $\boldsymbol{f_{ji}}$ is flow from $\boldsymbol{i}$ to $\boldsymbol{j}$. $\boldsymbol{U_P}$ is the utility matrix

$$\boldsymbol{U_P = (I - D_P)^{-1}} \tag{3.14}$$

The positive elements of the $\boldsymbol{U_P}$ matrix denote positive interactions (benefits and mutualism), while the negative elements denote negative interactions (costs and competition). The $\boldsymbol{b/c}$ index is the ratio of positive to negative interactions.

$$\boldsymbol{\frac{b}{c} = \frac{\sum + utility in U_P matrix}{\mid \sum - utility in U_P matrix \mid}} \tag{3.15}$$

A value of $\boldsymbol{b/c}$ greater than one describes a system with greater positive than negative interactions.Such a system has synergy.

Chapter 4

ADAPTATION OF MARKOV THEORY AND NETWORK THEORY TO A MANPOWER SYSTEM

4.1 The Markov chain model of a manpower system

A manpower system can be viewed as a hierarchical system with a finite number of states which changes only at a discrete point of time. The states are the grades or job groups in the system. In this study the manpower system for the Teachers Service Commission(TSC)is evaluated from job group K to R. The states therefore are K,L,M,N,P,Q and R. Each year a number of teachers are promoted in accordance with the schemes of service which clearly outline criteria for promotion. Based on the assumptions on teachers entry and promotion earlier stated and assuming homogeneity in the system , the probability,$\boldsymbol{p_{ij}}$ of a teacher in state $\boldsymbol{i}$ at time $\boldsymbol{t}$moving to state $\boldsymbol{j}$ at time $\boldsymbol{t+1}$ is fixed. This deduces the transition matrix

$\boldsymbol{P = ((p_{ij})), i, j = K, L, M, N, P, Q, R}$

Remark **:** for simplicity we shall use the integers $\boldsymbol{1, 2, 3, ..., N}$ to denote the job groups in the remaining part of this section. Assuming that there are $\boldsymbol{r}$ absorbing states and $\boldsymbol{s}$

non-absorbing states in the system, the transition matrix will have the canonical form

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{I}| & \boldsymbol{O} \\ \boldsymbol{D}| & \boldsymbol{Q} \end{pmatrix} \tag{4.1}$$

The $\boldsymbol{q_{ii}}$ diagonal elements of $\boldsymbol{Q}$ are the probabilities of a teacher remaining in the same grade. The sum of the rows of $\boldsymbol{Q}$ and $\boldsymbol{R}$ is one because a teacher is either in one of the job groups or leaves the system and enters one of the absorbing states.

We now define $\boldsymbol{n_{ij}(t)}$ to be the number of teachers in state $\boldsymbol{i}$ at time $\boldsymbol{t}$, who go to state $\boldsymbol{j}$ at time $\boldsymbol{t+1}$ and $\boldsymbol{n_i(t)}$ to be the number of teachers in state $\boldsymbol{i}$ at time $\boldsymbol{t}$, then

$$\sum_{j=1}^{N} \boldsymbol{n_{ij}(t) = n_i(t)} \tag{4.2}$$

Let $\boldsymbol{p_{ij}}$ be the probability of moving from state $\boldsymbol{i}$ at time $\boldsymbol{t}$ to state $\boldsymbol{j}$ at time $\boldsymbol{t+1}$. Then

$$\sum_{j=1}^{N} \boldsymbol{p_{ij}} = \boldsymbol{1} \tag{4.3}$$

for a large population, the sequence $\boldsymbol{n_{i1}(t), n_{i2}(t), ..., n_{iN}(t)}$ has a multinomial distribution given by

$$\boldsymbol{f(n_{i1}(t), n_{i2}(t), ..., n_{iN}(t))} = \frac{n_i(t)! p_{i1}^{n_{i1}(t)} p_{i2}^{n_{i2}(t)} ... p_{iN}^{n_{iN}(t)}}{n_{i1}(t)! n_{i2}(t)! ... n_{iN}(t)!}$$

The maximum likelihood estimator of $\boldsymbol{p_{ij}}$ is thus

$$\hat{p}_{ij} = \frac{\boldsymbol{n_{ij}(t)}}{\boldsymbol{n_i(t)}} \tag{4.4}$$

If observations are made over T time periods,denoted as $\boldsymbol{t = 1, 2, ..., T}$, then the maximum likelihood estimator of the transition probabilities is

$$\hat{p}_{ij} = \frac{\sum_{t=1}^{T} \boldsymbol{n_{ij}(t)}}{\sum_{t=1}^{T} \boldsymbol{n_i(t)}} \tag{4.5}$$

The value in (4.5)can be interpreted as the average transition ratio from state $\boldsymbol{i}$ to $\boldsymbol{j}$ at any given time interval.

4.2 Teacher Promotion Transitional Matrix

Using data on number of teachers per job group for the years 2004,2005,2006,and 2007 obtained from the Teachers Service Commission as shown in (appendix A), we obtain the maximum likelihood estimator as defined by equation(4.5). Using this estimator we obtain a one step transitional matrix $\boldsymbol{P}$ shown in table 4.1

Table 4.1: One step transition matrix.

Job group	Abs	K	L	M	N	P	Q	R
Abs	1.0000	0	0	0	0	0	0	0
K	0.0234	0.4976	0.4790	0	0	0	0	0
L	0.0165	0	0.8820	0.1015	0	0	0	0
M	0.1169	0	0	0.7525	0.1306	0	0	0
N	0.1070	0	0	0	0.8022	0.0908	0	0
P	0.1207	0	0	0	0	0.6929	0.1864	0
Q	0.2295	0	0	0	0	0	0.5744	0.1961
R	0.3800	0	0	0	0	0	0	0.6200

Table 4.2: Three- step transition matrix.

Job group	Abs	K	L	M	N	P	Q	R
Abs	1.0000	0	0	0	0	0	0	0
K	0.0653	0.1232	0.7015	0.1037	0.0063	0	0	0
L	0.0766	0	0.6861	0.2038	0.0323	0.0012	0	0
M	0.3082	0	0	0.4261	0.2368	0.0267	0.0022	0
N	0.2929	0	0	0	0.5162	0.1525	0.0350	0.0033
P	0.3732	0	0	0	0	0.3327	0.2252	0.0690
Q	0.6006	0	0	0	0	0	0.1895	0.2099
R	0.7617	0	0	0	0	0	0	0.2383

We obtain $\boldsymbol{P^3}$ the 3-step transition matrix in table 4.2 using the Chapman-Kolmogrov equation.

4.2.1 Discussion

From the one-step transition matrix 47.9% of the teachers in K are promoted to L. This is the highest transition rate compared to the rest which are very low e.g N to P is the lowest with a transition rate of 9.08%. The high transition rate from K to L is because promotion from K to L is automatic after three years unlike the rest of the grades where it is through interview and therefore highly competitive. The very low transition rate for N to P could be because of non-graduate teachers specifically P1 who according to the scheme of service can only go up to Job group N. Rate of dropout is highest in R at 38% as expected since by the time most teachers make it to R they are about to retire.

Teachers are eligible for promotion after having been in the present grade for three years. The $\boldsymbol{q_{ij}^3}$ elements of the $\boldsymbol{P^3}$ give the probability that a teacher now in any Job group

i will be in another Job group ***j***, three years later. 70.15% of the teachers currently in job group K will be in Job group L three years later. Job group L has 68.61% of the teachers remaining in job group L three years later. This is the highest percentage of teachers remaining in the same job group after three years compared to the other job groups. This could be explained by the fact that it is from L that promotion is competitive and there is a glut at L. It is the job group with the highest number of teachers (see appendix A).

Chances of promotion to a job group beyond the next one in three years is small as reflected by the probabilities. Only 10.37% of the teachers currently in Job group K will be in Job group M three years later. This is the highest compared to the rest e.g 3.23% of the teachers currently in Job group L will be in job group N three years later. These are exceptional cases.

4.3 Fundamental Matrix

Using the equation $\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$ we obtain the fundamental matrix $\boldsymbol{L}$.

Table 4.3: Fundamental Matrix

Job group	K	L	M	N	P	Q	R
K	1.9904	8.0799	3.3136	2.1878	0.6469	0.2833	0.1462
L	0	8.4746	3.4754	2.2947	0.6785	0.2972	0.1533
M	0	0	4.0404	2.6677	0.7888	0.3455	0.1783
N	0	0	0	5.0556	1.4948	0.6547	0.3378
P	0	0	0	0	3.2563	1.4261	0.7360
Q	0	0	0	0	0	2.3496	1.2125
R	0	0	0	0	0	0	2.6316

The elements of L, $\boldsymbol{l_{ij}}$ give the expected length of stay in grade $\boldsymbol{j}$ from $\boldsymbol{i}$ before leaving the system.The sum of the elements of the rows of $\boldsymbol{L}$ e.g sum of the elements for the first row will give the expected length of stay for an entrant in job group K.This can be used to estimate the length of stay of teachers in the system which can be used further in estimating the cost of keeping a teacher till they leave the system.

Estimated cost per teacher per year= Ksh 790555. This figure has been estimated from the recently implemented teachers salary award(TSC,Circular No 8/2007) taking the average of the maximum salary in each job group plus house allowance in each job group.

Table 4.4: Estimated length of stay and estimated cost

Job group	Length of stay(yrs)	Estimated of cost(K sh)
K	16.65	13,162,741
L	15.37	12,150,830
M	8.02	6,340,251
N	7.54	5,960,785
P	5.42	4,284,808
Q	3.56	2,814,376
R	2.63	2,079,160

4.4 Application of Network Theory to a Manpower System

A manpower system can be viewed as a network with flow in only one direction.The job groups or grades are the nodes or compartments.For the Teachers Service Commission the compartments are job groups K,L,M,N,P,Q and R.Teacher transitions between job groups can be presented using a flow matrix.Unlike the Markov chain transition matrix whose elements are probabilities of transition the elements of the flow matrix are the numbers of teachers transiting between the different job groups. For the Markov chain both time and state are discrete, while for the flow matrix the states are discrete but time is continuous. The probabilities in the Markov chain transition matrix are proportions of teachers transiting between the different job groups and have been used in this study to obtain the elements of the flow matrix as follows,

$\boldsymbol{p_{ij}} \times \boldsymbol{N_i}$ gives the number of teachers transiting from Job group $\boldsymbol{i}$ to Job group $\boldsymbol{j}$ where $\boldsymbol{p_{ij}}$ is the probability of transition from $\boldsymbol{i}$ to $\boldsymbol{j}$ and $\boldsymbol{N_i}$ is the number of teachers in $\boldsymbol{i}$. The internal elements of the flow matrix could have also been obtained from the data on teachers in appendix A but the dropouts cannot be obtained this way since data on dropouts was not available and could only be computed from the probabilities of the absorbing states. The flow matrix obtained is as given in Table 4.5.

Table 4.5: Flow Matrix

Job groups	Import	K	L	M	N	P	Q	R
K	996	4144	0	0	0	0	0	0
L	0	3989	39292	0	0	0	0	0
M	0	0	4522	6554	0	0	0	0
N	0	0	0	1137	1517	0	0	0
P	0	0	0	0	172	142	0	0
Q	0	0	0	0	0	38	45	0
R	0	0	0	0	0	0	15	24
Retired/Resigned(n+1)	0	120	597	974	189	19	12	15
Deaths(n+2)	0	75	138	44	13	6	6	0

Compartment $\boldsymbol{n+1}$ represent usable exports. These are the teachers who leave the system alive through resignation,sacking retirement etc.Compartment $\boldsymbol{n+2}$ represents unusable exports. These are the teachers who leave the system through death.Recruitment into the system between K and R is only at K therefore the other elements for the import compartment are all zero except at K which is 996(this is the average of recruitments for the years 2005 and 2006). The columns of the flow matrix are the source of flow while the rows the destination of flow e.g 4144 teachers remain in K,3989 teachers flow from K to L, 120 teachers resign, are sacked or retire at Job group K and 75 teachers from Job group K die.

Network indices can be computed manually from tne flow matrix using the formulae given in the previous chapter or using a software. In this study ***R2.5.0*** has been used to compute the relevant indices.

4.4.1 Application of network indices to a manpower system

We now consider the indices described in the previous chapter relating them to a manpower system.

(a)***Average Mutual Information (AMI)*** in an ecosystem is a measure of constraint on an arbitrary unit of flow anywhere in the network. Constraint here implies control of the direction of flow. As ***AMI*** falls the system forms a weblike structure meaning flow in all possible directions which is good for an ecosystem but for a manpower system this would mean chaos as employees would move to any job group. Therefore ***AMI*** for a manpower system is a measure of organization/coherence in the way transitions between job groups are effected. For a manpower system increasing ***AMI*** implies better organization and articulation of policies governing movement from one grade to the next.

(b)***Statistical Uncertainty*** $(\boldsymbol{H_R})$

This is a measure of diversity in an ecosystem. It is the effective number of choices of energy flow in system.For a manpower system this would represent the options in terms of grades available for a person moving from a particular grade.

(c)***Conditional Uncertainity*** (D_R)

This is a measure of stability in an ecosystem. It is the difference between Statistical Uncertainty and $\boldsymbol{AMI}$. The bigger the difference the more stable an ecosystem.For a manpower system the smaller the difference the better the organization of the system because it implies a higher value of $\boldsymbol{AMI}$.

(d)***Realised Uncertainty*** (AMI/H_R)

This is the fraction of total uncertainty accounted by network structure as measured by the $\boldsymbol{AMI}$. For a manpower system, this proportion of disorganisation that can be attributed to the network structure, may reveal need for evaluation and reorganization of structure.

For the data on the TSC manpower system we have the following results for the four indices discussed above.

a) Average Mutual Information = 0.80

b) Statistical Uncertainty = 1.43

c) Realized Uncertainty = 0.56

d) Conditional Uncertainty = 0.63

Most of the network indices above by themselves do not give much information about a system unless looked at in relation to another index. An $\boldsymbol{AMI}$ of 0.80 unless looked at in relation to its upper bound, the $\boldsymbol{H_R}$ may not reveal much.The $\boldsymbol{AMI}$ for the teachers manpower system is 0.80 compared to its upper bound which is 1.43. The difference is the conditional uncertainty, 0.63 which when expressed as a percentage of $\boldsymbol{H_R}$is 44%. This means about 44% of the movements of teachers between job groups are not constrained or controlled in accordance with the relevant schemes of service. The value for Realized Uncertainty shows that 56% of the lack of organization/coherence in movement of teachers is accounted for by the structure of the system.

The next five indices relate to system size and develoment.

(e)***Total System Thruoghput*** $(\boldsymbol{T}_{..})$

This is a measure of the size of the system. It is the sum of the compartmental throughput . The greater between the inflow of persons into a grade and outflow from a grade is the compartmental throughput.

(f)***Ascendency***$(\boldsymbol{A})$

This is the product of $\boldsymbol{T}_{..}$ and $\boldsymbol{AMI}$.It a measure of the organization and development of a system. The bigger the better for a manpower system but by itself may not reveal much unless compared with development capacity.

(g)***Development Capacity***$(\boldsymbol{C})$ is the upper bound on ascendency. It gives an idea of the potential for development for the system which when compared with ascendency can reveal where the system is in relation to where it should be.

(h)***Overhead***$(\boldsymbol{\Phi})$ is the difference between $\boldsymbol{C}$ and $\boldsymbol{A}$. A big value of$\boldsymbol{\Phi}$ means there alot of room for improvement in terms of organization and also in the numbers of persons moving from one Job group to the next.

(i)***Extent of development*** $(\boldsymbol{A/C})$

This index is a proportion which quantifies how far a system has developed. Useful for policy makers within an organization in evaluating effectiveness of existing policies for growth and development in human resource management. Results for the indices discussed above are as follows,

e) Total System throughput = 64795

f) Ascendency = 51832.42

g) Development Capacity = 140686.46

h) Overhead = 88854.03

i) Extent of development = 0.37

The Total System Throughput value gives us the size in terms of the flow of teachers but again by itself does not reveal much. The Ascendency is low compared to its upper bound the Development Capacity as revealed in the ratio of the two,the Extent of Development which is 0.37. This implies that the system is performing way below its potential at only 37%. The overhead quantifies room for improvement which is 63%.

The last index discussed here is synergy

Synergy(b/c)

b/c is the ratio of positive to negative interactions. A value greater than one implies that there are more positive interactions than negative ones. A manpower system with b/c greater than one implies it has synergy i.e movement between job groups are purely on merit and these movements are working for the mutual benefit of the organization. A value less than one would reveal unfair practices in carrying out promotions, unhealthy competition , demotions and conflict that do not benefit the organization. For the TSC manpower system b/c is 9.19. This value is greater than one which means the system has synergy i.e there are more positive interactions than negative. The net result of the flows of teachers is positive and benefits the system.

Chapter 5

CONCLUSION AND RECOMMENDATION

In this study the Markov chain theory and Network theory complement one another. Markov chain theory facilitates the computation of the elements of the export compartments of the flow matrix in this study since the data on dropouts per Job group was not available. The Markov chain theory is descriptive and reflects the application of the schemes of service. Network theory goes beneath the surface to reveal features about the manpower system that arise from its underlying structure.

The probabilities of transition in the Markov transition matrix can be explained to a great extent by the schemes of service which means the schemes of service are being applied in effecting promotions of teachers. How effectively they are applied is answered by the results of the network indices. 44% of the teacher transitions are not constrained or controlled in accordance with the relevant schemes of service which implies ambiguity or presence of loopholes in the system which can be exploited by unscrupulous individuals. 56% of this ambiguity is accounted for by the structure of the system. There is a lot of room for growth and development as revealed by the Extent of Development index which is only at 37%. The system has synergy, which is good though since this index does not have a maximum we are not able to judge whether it is low or could be higher.

There is need to review the structure of the system to eliminate any ambiguity in teacher transitions and where possible put in place policies that will minimize the influence of factors outside the system which contribute to this ambiguity. As revealed by the Extent of Development there is a lot that can be done in terms of organization and development. With regard to development the number of teachers moving from one job group to the next needs to be reviewed upwards especially at the transition from L to M. There seems to be a glut at L which if not addressed could result to an exodus if other jobs are available or despair which will lead to low productivity. From the late nineties when opportunities for university education opened up alot of teachers have gone back to school to upgrade themselves professionally. Many P1 teachers have completed their Bachelor of Education degrees and are therefore getting to Job group K faster than they would, if they had waited to get there under the non-graduate scheme of service. This could be one of the factors contributing to the high number of teachers at L. A healthy system should be able to respond to change by adjusting in a way that nullifies the effect of that change.
Further work can be done in integrating the two models used in this study by comparing the TSC manpower system with other similar systems for countries that Kenya would like to emulate in the area of educational management.

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Appendix A

The Appendix

A.1 Data on Teachers

Table A.1: Teacher distribution per job group

Job group	2004	2005	2006	2007
K	9085	8345	8156	7729
L	46951	44265	43906	43075
M	8132	11339	11982	13535
N	812	903	844	5006
P	190	180	174	276
Q	77	69	76	88
R	56	30	33	36

Table A.2: Total Teacher Attrition from job group D to R

Year	Deaths	Retirement
2004	951	5240
2005	1145	7401
2006	1030	7137

Table A.3: Teacher promotions and new appointments

Year	New appointments at K	K	L	M	N	P	Q	R
2005	1193	954	2477	4382	250	52	25	10
2006	798	1429	2239	1980	0	49	51	14
2007	Non as at 30-6-07	6913	7309	7304	4325	130	28	4

A.2 *R* Commands Script