STRATHMORE UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
MASTER OF SCIENCE IN STATISTICAL SCIENCE
END OF SEMESTER EXAM 2020/2021
STA 8201-BAYESIAN MODELLING AND DATA ANALYSIS

Date: April, 2021 Duration: 3 Hours

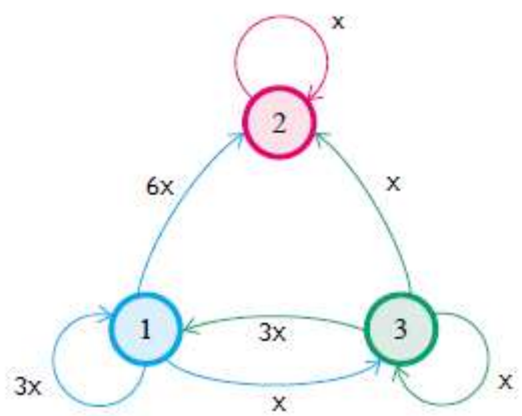
Answer Question ONE and any other Two questions:

Question 1 (30 marks)

a. Briefly describe Gibbs sampler for parameters θ_1, θ_2, and θ_3, with joint posterior distribution $p(\theta_1,\theta_2,\theta_3|y)$ (5 marks)

b. Let y be the number of heads in *n* spins of a coin, whose probability of heads is θ. If you prior for θ is Beta(α,β) distribution show that your posterior mean for θ is between the prior mean $\alpha/(\alpha+\beta)$ and the observed proportion of the heads y/n. (9 marks)

c. Find the transition matrix from the transition diagram below (3 marks)



d. Let $y_1, \ldots, y_{10}$ be i.i.d N (θ, 1). Let the sample mean be $\bar{y} = 1.873$. Assume that $\theta \sim N(0, 5)$ a priori. Compute the posterior distribution of θ (6 marks)

e.

Suppose that $x_1, \ldots, x_n$ is a random sample from a geometric distribution with p.d.f.

$$p(x|\theta) = \begin{cases} \theta(1-\theta)^x & 0 < \theta \leq 1 \quad x = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}.$$

Show that $p(x|\theta)$ is an exponential family and hence find a conjugate prior for θ. Find the posterior distribution for θ using this prior and its mean and variance.

(7 marks)

Question 2 (15 marks)

Consider a bivariate normal posterior distribution of the parameters θ_1 and θ_2:

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right)$$

a. Determine the full conditionals of $\theta_1|\theta_2$ and $\theta_2|\theta_1$.

b. Write a **R code** for Gibbs sampling from the full conditionals in (a.) above.

Question 3 (15 marks)

Suppose we have number of failures (Y_i) for 10 pumps in a nuclear plant. We also have the times (t_i) at which each pump was observed.

(a.) Determine the Poisson likelihood, and hence the posterior distribution of the number of failures where the expected number of failure λ_i differs for each pump. Use Gamma(α,β) prior on λ_i where α=1.8. Let us put Gamma (γ, δ) hyperprior on beta with γ =.01 and δ =1. (11.5 marks)

(b.) Determine the posterior mean and variance. (3.5 marks)

Question 4 (15 marks)

The WinBUGS model below concerns the proportion of seeds that germinated on each of 21 plates arranged according to a 2 by 2 factorial layout by seed and type of root extract. Let r_i and n_i respectively be the number of germinated and the total number of seeds on the *i* th plate, *i* =1,...,N.

```
model
     {
              for( i in 1 : N ) {
                       b[i] ~ dnorm(0.0,tau)
                       logit(p[i]) <- alpha0 + alpha1 * x1[i] + alpha2 * x2[i] +
                                alpha12 * x1[i] * x2[i] + b[i]
              }
              alpha0 ~ dnorm(0.0,1.0E-6)
              alpha1 ~ dnorm(0.0,1.0E-6)
              alpha2 ~ dnorm(0.0,1.0E-6)
              alpha12 ~ dnorm(0.0,1.0E-6)
              # Choice of priors for random effects variance
              # Prior 1: uniform on SD
              sigma~ dunif(0,100)
              tau<-1/(sigma*sigma)

              #Prior 2:
              #tau ~ dgamma(1.0E-3, 1.0E-3);
              #sigma <- 1/sqrt(tau); # s.d. of random effects
     }
```

Describe what component of the model: what is the likelihood and what is the priors.

(5 marks)

Write this model in simple mathematical equation in terms of the underlying predictor function and the statistical model (5 marks)

Sketch the densities of parameter "tau" under Uniform and Gamma (hint: the commented Prior 2) priors? (5 marks)