Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

Bachelor of Business Science Finance & Financial Economics
END OF SEMESTER EXAMINATION
BSF 4130:- FOUNDATIONS OF ASSET PRICING

Date: Tuesday 16th July 2019 Time: 2 Hours

Instructions

1) This examination consists of FIVE questions.

2) Answer Question ONE (COMPULSORY) and any other TWO questions.

3) Question One carries 30 marks.

4) All other question carry 20 marks each.

Question 1 - Compulsory

1. (a) The two statements below have been made in relation to our discussion on Asset Pricing:

- An asset pricing theory is simply a set of hypotheses that implies some particular form for the Stochastic Discount Factor (SDF).
- Asset pricing theory is concerned with explaining the risk premia of different assets

With the help of relevant workings, prove that these two statements are equivalent. *(4 marks)*

(b) Discuss the following statement: *A market admits No Arbitrage, if and only if, the market has a Risk-Neutral (Martingale) Measure.* *(3 marks)*

(c) Suppose there is an existing economy with 3 states of nature and 3 assets. The payoff matrix X of the assets (an $S \times N$ matrix) is given by:

$$X = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

The associated price vector is $p = \begin{bmatrix} 1 & 2 & 4 \end{bmatrix}'$.

i. Does the equilibrium price measure q exist for this economy? If so derive this measure. *(4 marks)*

ii. Is the equilibrium price measure derived in (i) above unique? Explain. *(2 marks)*

iii. In this economy, what is the no-arbitrage price of a derivative security with payoffs equal to $X = (5\ 10\ 15)$ *(2 marks)*

(d) In a paper that attracted intense scrutiny when it was published, Fama and French (1992) focus on the impact of size (ME) and book to market equity (BE/ME) on monthly cross-sections of individual company returns for over 2000 US non-financial firms. An illustration of results from their Fama-Macbeth regressions is:

$$\begin{aligned} \text{average return} \ =& 2.07 - 0.17\widehat{\beta}_j - 0.12\ln(\text{ME}) + 0.33\ln(\text{BE/ME}) \\ &+ \text{ residual error} \end{aligned}$$

The t -statistics for the three coefficients are -0.62, -2.52 and 4.80

Required: What do the above results mean for the Empirical Appraisal of the Capital Asset Pricing Model? Discuss, using a significance level of 95%. *(4 marks)*

(e) Consider a CARA investor who chooses among multiple assets. Suppose there exists a risk-free asset with return R_f. The investor has to choose between the risk free asset and the n risky assets with jointly normally distributed returns $\tilde{R}_i$.

The following details are also provided for your use:

- Let $\tilde{R}^{vec}$ denote the n-dimensional column vector of returns with $\tilde{R}_i$ as its $i-th$ element
- μ is the n-dimensional column vector of expected returns with $E[\tilde{R}_i])$ as its $i-th$ element.
- Let ϕ_f denote the investment (in \$) in the risk-free asset
- Let ϕ_i denote the investment in risky asset i, AND

- Let ϕ denote the n -dimensional column vector with ϕ_i as its i -th element.

The investor needs to know the optimal amount ϕ to invest in the risky assets. Show that the investor's optimal allocation to the risky assets is a direct/positive function of the risk premium *(11 marks)*

Total for Question 1: 30 marks

Question 2

2. (a) Roll's Critique of Asset Pricing Theory Tests in 1976 argues that testing the capital asset pricing theory is difficult (and *"currently infeasible*). Roll's main concern seems to be in the choice of market portfolio r_m. Discuss further, highlighting two key statements made by Roll (1976) regarding the market portfolio *(4 marks)*

(b) Georgina is carrying out a study to estimate the preference parameters for a representative Kenyan investor within the framework of a Consumption-based Asset Pricing Model. Georgina plans to use the Generalized Method of Moments (GMM) approach of Hansen and Singleton (1982) to estimate two preference parameters: the subjective discount factor (beta) and the coefficient of RRA (gamma). She has quarterly Gross Household Consumption Growth Rate and quarterly Stock Market Returns (NSE 20 Returns) from 1991-2017. The purpose of her study is to estimate the parameters (beta and gamma) that best fit the data.

Assuming that the investor's utility function is iso-elastic of the form:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$

How will she go about the Empirical Estimation of the Parameters in a GMM framework? Make sure to highlight at least three moment conditions she needs to specify to estimate the parameters *(8 marks)*

(c) There is a clear sense that the level of volatility of the overall stock market cannot be well explained with any variant of the efficient markets model in which stock prices are formed by looking at the present discounted value of future returns. The empirical debate in this area has largely sought to find some definition of discount rates that produces a present value series that fits the actual price behaviour better. Briefly explain the role played by the behavioural finance heuristics below, in an attempt to better explain the volatility in equity markets.

i. Style Investing *(4 marks)*

ii. Prospect Theory *(4 marks)*

Total for Question 2: 20 marks

Question 3

3. (a) The most tested random walk hypothesis in the recent empirical literature-may be obtained by relaxing the independence assumption of Random Walk 2 to include processes with dependent but uncorrelated increments, otherwise known as Random Walk 3. Discuss how Variance Ratios can be used to test for the plausibility of Random Walk 3 *(4 marks)*

(b) Fama and MacBeth (1973) proposed a method of testing some asset pricing implications that arise from the CAPM assumption of normally distributed portfolio returns and risk-averse investors.

i. The authors proposed the below specification to test three implications of the CAPM:

$$\tilde{R}_p = \hat{\gamma}_{0,t} + \hat{\gamma}_{1,t}\hat{\beta}_p + \hat{\gamma}_{2,t}\hat{\beta}_p^2 + \hat{\gamma}_{3,t}\overline{s}_p(\hat{e}_i) + \hat{\eta}_{p,t} \quad \forall p$$

where $\hat{\beta}_p$ are the market betas, $\hat{\beta}_p^2$ is the squared market betas and $\overline{s}_p(\hat{e}_i)$ are residual variances from time series regressions of the stock returns on the market return.

Discuss three testable implications of the Capital Asset Pricing Model that arise from this model specification *(6 marks)*

ii. In testing the Capital Asset Pricing Model, empirical evidence shows that:

- Stock level time-series regression are way too noisy.
- The single cross sectional regression approach used by Black, Jensen, Scholes (1972) is susceptible to correlated errors.

With the help of relevant model specifications, discuss the two-step methodology adopted by Fama Macbeth (1973) with special mentions of the innovations used to solve the two concerns above. *(6 marks)*

(c) Consider a stock whose dividends grow at a constant rate:

$$\frac{D_{t+1}}{D_t} = 1 + G = Exp(g)$$

Although dividends are known in advance, the price of the stock varies because discount rates vary over time. The log dividend-price ratio, $x_t = log(\frac{D_{t+1}}{P_t})$, follows a random walk:

$$x_t = x_{t-1} + \varepsilon_t$$

where ε_t is normally distributed with mean zero and variance σ^2.

i. Write a formula for the simple gross return on the stock, held from time t to time $t+1$, in terms of x_t, x_{t-1}, and g *(1 mark)*

ii. Take conditional expectations and show that the expected gross return on the stock at time t can be written as the dividend-price ratio $\frac{D_{t+1}}{P_t}$ plus a constant term reflecting the growth rate of dividends and the volatility of the stock price. *(3 marks)*

Total for Question 3: 20 marks

Question 4

4. (a) A few simple rearrangements of the basic pricing equation $p = E(mx)$ can give a lot of intuition and introduce some classic issues in finance, including the determinants of the interest rate and risk adjustments.

Show that the basic pricing equation with the stochastic discount factor, m, is equivalent to a single-factor expected return - beta model *(3 marks)*

(b) Can the "Sharpe ratio" between two risky assets exceed the slope of the mean-variance frontier? i.e. if R^{mv} is on the frontier, is it possible that *(3 marks)*

$$\frac{E\left(R^i\right) - E\left(R^j\right)}{\sigma\left(R^i - R^j\right)} > \frac{E\left(R^{mv}\right) - R^f}{\sigma\left(R^{mv}\right)}?$$

(c) Show that an investor who has an isoelastic utility function (also known as a power utility function) has both a Decreasing Absolute Risk Aversion and Constant Relative Risk Aversion *(4 marks)*

(d) Consider an investor with an optimal investment problem. Let w_0 denote the amount he invested at the beginning of the period, and let θ_i denote the number of shares the investor chooses to hold of asset i. Assume this investor has some possibly random endowment $\tilde{y}$ at the end of the period (for example, labor income), which he consumes in addition to the end-of-period portfolio value. Letting n denote the number of assets, the investor's choice problem is:

$$maxE[u(\tilde{w})]$$

subject to $\sum_{i=1}^{n} \theta_i p_i = w_0$ and $\tilde{w} = \tilde{y} + \sum_{i=1}^{n} \theta_i \tilde{x}_i$

i. You are given the investor's choice problem above. Construct a Lagrangean function for the investor's Choice problem *(2 marks)*

ii. Derive the first order condition for this investor's optimal portfolio *(4 marks)*

iii. What is the intution behind this First Order Condition. Discuss *(4 marks)*

Total for Question 4: 20 marks

Question 5

5. (a) The central task of asset pricing is to figure out what are the real risks that drive asset prices and expected returns. Take something like the Consumption-Based model – investors' first order conditions for savings and portfolio choice – has to be the starting point. However, rather than dream up models, test them and reject them, financial economists have worked backwards to characterise the SDF, for example, since the work of Mehra and Prescott (1986) and Hansen and Jagannathan (1991).

Required: Provide a detailed discussion of the *research problem, the methodology and the results* from Mehra & Prescott (1985) Equity Premium Puzzle study, clearly highlighting the implications of the study on the Stochastic Discount Factor. *(12 marks)*

(b) Conditional asset pricing presumes the existence of some return predictability. Some studies in the area support predictability in step-ahead or outof-sample exercises (e.g. Pesaran and Timmerman, 1995).

Discuss the findings and conclusions that Pesaran and Timmerman (1995) made in their paper regarding predictability of asset returns, in line with the 3 pointers provided below, highligting the implications on risk premia of these assets

i. Importance of the business cycle *(3 marks)*

ii. Time Varying Predictability of returns *(3 marks)*

iii. Choice of Trading rule *(2 marks)*

Total for Question 5: 20 marks