Strathmore UNIVERSITY

SCHOOL OF FINANCE AND APPLIED ECONOMICS

Bachelor of Business Science – Actuarial Science, Finance & Financial Economics

END OF SEMESTER EXAMINATION

BSA 2206 FINANCIAL MATHEMATICS II

DATE: 13th December 2017 Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions

Section A

Question One [30 Marks]

a. Distinguish between each of the following:

i. Selling (writing) a call option and buying a put option (use payoff diagrams) [3]

ii. Interest rate swap and a currency swap [3]

iii. Market Risk and credit risk faced by the counter party of a swap [2]

iv. Debenture and unsecured loan stock [2]

b. An investor is considering investing in the shares of a particular company. The shares pay dividends every 6 months, with the next dividend being due in exactly 4 months' time. The next dividend is expected to be d_1, the purchase price of a single share is P and the annual effective rate of return expected from the investment is $100i\%$. Dividends are expected to grow at a rate of $100g\%$ per annum from the level of d_1 where $g < i$. Dividends are expected to be paid in perpetuity. Show that: [5]

$$P = \frac{d_1(1+i)^{\frac{1}{6}}}{(1+i)^{\frac{1}{2}} - (1+g)^{\frac{1}{2}}}$$

c. The expected annual effective rate of return from an insurance company's investments is 6% and the standard deviation of annual returns is 8%. The annual effective returns are independent and $(1 + i_t)$ is lognormally distributed, where i_t is the return in the t^{th} year. Calculate the probability that the accumulation of the investment will be less than 90% of the expected value. [6]

d. A property development company has just purchased a retail outlet for $4,000,000. A further $900,000 will be spent refurbishing the outlet in six months' time. An agreement has been made with a prospective tenant who will occupy the outlet beginning one year after the purchase date. The tenant will pay rent to the owner for five years and will then immediately purchase the outlet from the property development company for $6,800,000. The initial rent will be $360,000 per annum and this will be increased by the same percentage compound rate at the beginning of each successive year. The rental income is received quarterly in advance. Calculate the compound percentage increase in the annual rent required to earn the company an internal rate of return of 12% per annum effective. [9]

SECTION B

Question Two [20 Marks]

a. Define each of the following terms stating one disadvantage of each: [6]

i. Discounted payback
ii. Time-weighted rate of return
iii. Money- weighted rate of return

b. A university offers its students three financing options for a degree course that lasts exactly three years.

Option A

Fees are paid during the term of the course monthly in advance. The fees are £10,000 per annum in the first year and rise by 5% on the first and second anniversaries of the start of the course.

Option B

- The university makes a loan to the students which is repaid in instalments after the end of the course. The instalments are determined as follows:
- No payments are made until three years after the end of the course.
- Over the following 15 years, students pay the university £1,300 per year, quarterly in advance.
- After 15 years of payments, the quarterly instalments are increased to £1,500 per year, quarterly in advance. After a further 15 years of payments, the quarterly instalments are increased to £1,800 per year, quarterly in advance, for a further 15-year period after which there are no more payments.

Option C

Students pay to the university 3% of all their future earnings from work, with the payments made annually in arrear.

A particular student wishes to attend the university. He expects to leave university at the end of the three-year course and immediately obtain employment. The student expects that his earnings will rise by 3% per annum compound at the end of each year for 10 years and then he will take a five-year career break.

After the career break, he expects to restart work on the salary he was earning when the career break started. He then expects to receive salary increases of 1% per annum compound at the end of each year until retiring 45 years after graduating.

The student wishes to take the financing option with the lowest net present value at a rate of interest of 3% per annum effective.

i. Calculate the present value of the payments due under option A. [3]

ii. Calculate the present value of the payments due under option B. [4]

iii. Calculate the initial level of salary that will lead the payments under option C to have the lowest present value of the three options. [7]

Question Three [20 Marks]

a. Describe the No-Arbitrage Assumption as used in Financial Mathematics [2]

b. State what is meant by a "forward contract". Your answer should include reference to the terms "short forward position" and "long forward position". [3]

c. Derive the formula for obtaining delivery price of a forward contract under each of the following circumstances (Define clearly all symbols used):

- A forward contract on a stock that pays no dividend.
- A forward contract on a stock that pays fixed amount of dividend
- A forward contract on a stock that pays dividend yield [9]

d. A one-year forward contract on a share was agreed on 1 September 2015 when the share price was Ksh. 870 and the risk-free force of interest was 7% per annum. The stock was expected to pay a dividend of Ksh. 110 eight months after the date of issue. The price of the share was Ksh. 990 on 1 February 2016 and the risk-free force of interest was 6.5% per annum. The dividend expectation was unchanged. Calculate, showing all working, the value of the contract to the holder of the long forward position on 1 February 2016. [6]

Question Four [20 Marks]

a. In any year, the interest rate per annum effective on monies invested with a given bank has mean value $\boldsymbol{j}$ and standard deviation $\boldsymbol{s}$ and is independent of the interest rates in all previous years. Let S_n be the accumulated amount after n years of a single investment of 1 at time $t = 0$. Show that:

i. $E[S_n] = (1+j)^n$ [2]

ii. $Var[S_n] = (1 + 2j + s^2 + j^2)^n - (1+j)^{2n}$ [3]

b. An individual wishes to receive an annuity which is payable monthly in arrears for 15 years. The annuity is to commence in exactly 10 years at an initial rate of £12,000 per annum. The payments increase at each anniversary by 3% per annum. The individual would like to buy the annuity with a single premium 10 years from now.

i. Calculate the single premium required in 10 years' time to purchase the annuity assuming an interest rate of 6% per annum effective. [5]

The individual wishes to invest a lump sum immediately in an investment product such that, over the next 10 years, it will have accumulated to the premium calculated in b(i). The annual effective returns from the investment product i_t are independent and $(1 + i_t)$ is lognormally distributed, where is the return in the year. The expected annual effective rate of return is 6% and the standard deviation of annual returns is 15%. Calculate the lump sum which the individual should invest immediately in order to have a probability of 0.98 that the proceeds will be sufficient to purchase the annuity in 10 years' time. [10]

Question Five [20 Marks]

a.

i. Describe the Characteristics of an index linked bonds. [3]

ii. The Government of a country issued at par index linked bonds in April 2008 with maturity after exactly 3 years and coupons at 8.25% p.a. payable yearly in arrears. The coupons and redemption amount are linked to an inflation index with a one-year time lag. The index value at 1st April of each year were as follows:

Year	2007	2008	2009	2010	2011
Index	158	167	175	190	230

If an investor had expectations of the inflation index being the same as in the above table, calculate the price he would have paid in order to achieve a gross redemption yield of 7% p.a.

[7]

b. Describe each of the following investment products

i. Certificate of deposit [2]
ii. A repo [2]
iii. Factoring [2]
iv. Commercial Paper [2]
v. A gilt strip [2]