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Portfolio optimization in the Kenyan stock market: a comparison between mean-variance optimization and threshold accepting

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Portfolio Optimization in the Kenyan Stock Market: A Comparison between Mean-Variance Optimization and Threshold Accepting

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Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other university. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Masese Josephine Mokeira

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Approval

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Abstract

The Mean-Variance Optimization (MVO) model has been used in asset allocation problems since the inception of Modern Portfolio Theory in 1952. Several improvements and alternatives to MVO have been suggested and used since then. These include adding constraints to the traditional MVO model, using alternative risk measures and use of non risk-reward models.

This study seeks to compare this risk-reward model against the Threshold Accepting model, which is a general optimization model, in portfolio selection in the Kenyan stock market to establish optimal stock portfolios to be held by investors in The Nairobi Securities Exchange (NSE). A comparison is done between the two models by measuring their performance using the following performance ratios: Sharpe Ratio, Sortino Ratio and Information Ratio using 29 stocks in the NSE from 1998 - 2016.

Using portfolio performance ratios, it is concluded that the Threshold Accepting (TA) model outperforms the Mean-Variance Optimization model but the latter is observed as a more consistent model. The TA model has portfolios with generally more superior returns relative to the risk taken for the full period; however, this is not consistent over varying time estimates. This observation implies that attention should be given to the TA model rather than the classical MVO approach with the aim of improving optimal portfolio selection.

Key words: Portfolio optimization, Mean-Variance Optimization, risk measure, Threshold Accepting (TA).

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List of Abbreviations

CVaR	Conditional Value-at-Risk
IR	Information Ratio
MAD	Mean Absolute Deviation
MVO	Mean-Variance Optimization
NASI	Nairobi All Share Index
NSE	Nairobi Securities Exchange
TA	Threshold Accepting
VaR	Value-at-Risk

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"A bird does not sing because it has an answer. It sings because it has a song."

Chinese Proverb

Chapter 1

Introduction

1.1. Background to the study

Traditionally, mean-variance analysis as proposed by Markowitz (1952) has been used in portfolio optimization where risk and return are traded off. Mean-variance analysis gives a theory of investor behavior regardless of whether the market as a whole is in equilibrium[1]. Every reasonable investor would want a portfolio where the return is maximized and the risk minimized; that is, for a given level of risk, an investor would choose the portfolio with the maximum return.

Markowitz (2010) states that in some instances, mean-variance analysis does not apply; for example, when return distributions are too spread out. Alternate ways to portfolio selection are suggested, three of which stand out. Firstly, the use of other measures of risk or return in a risk-return analysis, secondly, determining the investor's utility function explicitly and maximizing its expected value and thirdly, use of constraints and guidelines to select preferred portfolio instead of risk-return optimization.

Alternative risk measures to the variance and use of techniques that do not require the mean or variance of returns by utilizing objective functions have since been introduced to offer better optimization results. The semi-variance, mean absolute deviation (MAD), Minimax, Maximum Loss, Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR) and partial moments are some of the main alternative risk measures that have been used. These are further explained in section 2.1.2 of chapter 2. Further to these, models that are non risk-reward but are based on simulations incorporating set requirements have been researched on; most of these do not rely on the returns distribution.

Linear simulations and other algorithms have been run on data for portfolio selection with the most common ones including, Threshold Accepting (TA)(Gilli and Schumann, 2009) ;and, the Genetic Algorithm(GA) used together with a risk measure (Chang, Yang, and Chang, 2009). These are non risk-reward heuristic models[2]; risk and reward

[1]When a market is in equilibrium, the demand is equal to the supply and commodity prices match on the buyer and seller side, that is, the prices are stable (Whitelaw, 2000).

[2]A heuristic optimization model is a general purpose optimization method which searches for a solution in a systematic programmed way. More about this can be read in Gilli (2004) and Gilli and Schumann (2012).

as measures are not dismissed but are incorporated as part of specifications in these models.

In the Kenyan market, portfolio managers mainly use the mean-variance analysis and factor models in portfolio selection. These traditional models are viewed as useful and acceptable since they have been applied frequently for a long time. Caution is taken in using newer techniques due to the uncertainty involved.

In addition to the MVO model, this study will use Threshold Accepting model in constructing optimal portfolios and compare performance across the portfolios selected by the MVO model to determine the best model for optimization. The results of this study determine whether the TA model works better than the MVO model hence providing an alternative optimal portfolio selection method to investors.

The Threshold Accepting (TA) model was introduced by Dueck and Scheuer (1990) as a general purpose optimization algorithm. TA has since then been severally applied to optimization problems and compared to other optimization models. It looks for solutions by optimizing the set objective function through reiterations that improve the solution at each subsequent iteration hence meeting any set constraints Gilli, 2004. TA has been recommended as an optimization model that leads to better optimization results over classical optimization approaches (Fastrich and Winker, 2012; Gilli and Schumann, 2009).

1.2. Investment in the Kenyan stock market

The Nairobi Securities Exchange (NSE) was established in 1954 as Nairobi Stock Exchange and has grown in bounds to date. In 2008, the NSE All-Share index (NASI) was introduced to enable investors to have a better measure of the stock market performance (NSE, 2016a). Both local and foreign investors are attracted to investments in the share market and hold a number of shares in which they trade in.

Over the years, investors in the Kenyan stock market have speculated on stocks expected to gain value and in the process, some have made losses on their strategies. Many pension fund managers in Kenya invest a big portion of the portfolio that they manage in the Kenyan stock market which gives a fairly good return – between 18% to 29% (Njeru, Njeru, and Kasomi, 2015). A survey conducted by Alexander Forbes Consulting Actuaries Schemes (AFCASS) also revealed that among the asset classes in which funds were invested, stock returns were the highest. The survey covered 381 out of a total of 390 fund schemes in Kenya, for the half-year period ending June, 30th, 2014 (Okoth, 2014).

Currently, the NSE has six market indices that can be used investors to measure the performance of the major industry segments of the securities market. These include the NSE 20 Share Index, NSE 5 Share Index, NSE All Share Index (NASI), Financial Times Stock Exchange (FTSE) NSE Kenya 15 Index, FTSE NSE Kenya 25 Index and the FTSE NSE KE (Kenya) Government Bond Index.

1.3. Research problem

Mean-variance analysis is the commonly used model for portfolio selection in the Kenyan stock market (Njeru et al., 2015; Nyokangi, 2016). However, the model has its own weaknesses, the main ones being the way it disregards the overall shape of the distribution of the returns; and, it is only concerned about the final wealth and not the path the wealth takes between time '0' (the start of the investment period) and time 'T' (the end of the investment period) (Gilli and Schumann, 2009). These weaknesses have led to further research on alternative ways of portfolio optimization.

Other models used in Kenya and globally include the multifactor model based on the Price to Earnings ratio (P/E). Factor models were introduced by Fama and French (1992) who used a 3-factor model where portfolio ranking is based on book-to-market equity (BE/ME) and Price to Earnings ratio when comparing the cross-section of expected returns. Nyokangi (2016) used the Single index model against the Mean-Variance (MV) model for selecting portfolios of stocks in the NSE for the years 2002 - 2015. The Single Index model outperformed MV model in two out of three sub-periods under the study.

This study seeks to determine whether there is a model that works best in portfolio optimization in the Kenyan Stock Market and hence provide an optimal portfolio selection method to investors.

1.4. Research objective

1.4.1. Main objective

The main objective of this study is to compare an Threshold Accepting (TA) model over the Mean Variance Optimization (MVO) model in optimal portfolio selection.

1.4.2. Specific objectives

The specific objectives of this study are:

1. To determine which model between MVO and TA gives a better performing optimal portfolio.
2. To determine performance of the models over different time periods.

1.5. Research hypotheses

The research hypotheses for this study are:

1. The TA model builds a better optimized portfolio as compared to the MVO model.
2. The TA model outperforms the MVO model over different time periods.

1.6. Scope of the Study

The study uses weekly and monthly data spanning 19 years between 1998 and 2016 from the Nairobi Securities Exchange to construct optimal portfolios using The Mean Variance Optimization model and Threshold Accepting (TA) model.

The Mean-Variance model was chosen because it is the most commonly used model by fund managers in Kenya as they select portfolios in which to invest. Additionally, it is the standard model for solving portfolio problems in finance (Steinbach, 2001). The TA Model has been selected because of its objectivity of taking into account unique characteristics contained in the different stock returns being considered for building a portfolio.

The study considers weekly and monthly horizons since daily data on stocks is too noisy for optimization purposes. The weekly and monthly analysis are for comparison purposes.

1.7. Significance of the study

The study gives information about the best model to use in portfolio optimization by comparing the performance of the optimization models to determine the one which best applies for the Kenyan stock market. The Kenyan stock market is of interest since it is one of the best performing stock markets in Africa (Gachiri, 2014).

The outcome shows an alternative way in which investors in the Kenyan stock market can build their portfolios and hence maximize their return.

Chapter 2

Literature Review

2.1. Background of the Study

2.1.1. Mean-Variance Optimization

Markowitz (1952) proposed mean-variance analysis as a hypothesis about investor behavior and as a recommendation for portfolio selection. Variance (or its root: standard deviation) is used as a measure of risk in this model and MVO was fronted to have portfolio selection based on reasonable beliefs about future rather than past performance only.

In MVO, at every given level of risk an investor would choose the portfolio with the maximum return. Making a choice based on past performance only assumes that the average returns of the past are good estimates of the likely return in the future; variability of return in the past is a good measure of the uncertainty of return in the future.

This being a solution to a resource allocation problem, the investor needs to choose a portfolio of *n* risky assets with weights

$$w_1, w_2, ..., w_n$$

that would minimize the risk at a given level of return.

2.1.1.1 Improvements to Mean-Variance Optimization

The inclusion of constraints in the MVO problem can lead to better out-of-sample performance when compared to portfolios constructed without constraints. In practice, most portfolio optimization problems have a number of constraints including regulatory (reflect restrictions imposed by market regulators), guideline (limits or conditions specified by client), exposure constraints made at the discretion of the portfolio manager, trading constraints (discretionary limits on positions or trades), risk management constraints and transfer coefficients (Kolm, Tütüncü, and Fabozzi, 2014). However, constraints and guidelines have to be imposed carefully since they may distort robustness and stability of the portfolio allocation. Judgment and constraints should be incorporated cautiously to have the estimates of return and risk as more forward looking rather than historical (Markowitz, 2010).

A different approach is given by the Black-Litterman Model (Idzorek, 2004). This uses a Bayesian approach by combining investors' views regarding performance of stocks in a way that allows intuitive diversified portfolios. Investors' views, being subjective, are modelled together with the market equilibrium of expected returns. In addition, confidence intervals on the views are specified for incorporation in adjustments for final portfolio weights. When an investor does not have any view about a stock, he is taken to hold the market equilibrium. Views on a few assets imply changes to the expected returns on all assets due to correlation of securities. This ensures that the portfolio composition is diverse and not in just a few assets. Having obtained these inputs, the MVO technique is run.

2.1.1.2 Alternatives to Mean-Variance Optimization

One of the alternatives to MVO is determining an investor's utility function and maximizing it. However, there is great difficulty in finding an appropriate utility function to use due to changing perception of the investor (Markowitz, 2010).

The use of alternative measures of risk or return in a risk-return analysis is another way of constructing better optimized portfolios and in overcoming the shortcomings of mean-variance analysis (Biglova, Ortobelli, Rachev, and Stoyanov, 2004; Jaaman, Lam, and Isa, 2013; Bonyo, 2015; Rollinger and Hoffman, 2013). The VaR, CVaR, semi-variance, MAD, Minimax, Maximum Loss and lower partial moments are some of the alternative risk measures used together with the mean in portfolio optimization. These are discussed in section 2.1.2.

It is worthwhile to note that some risk measures give the same result in portfolio selection when the portfolio distributions depend only on the first two moments; but, if the returns distributions is skewed, the risk measures perform differently (Ortobelli, Rachev, Stoyanov, Fabozzi, and Biglova, 2005). Furthermore, risk-averse investors prefer mean-variance in optimal asset allocation while less risk-averse ones prefer where the mean is combined with an alternative risk measure.

In looking at other ways of diversifying the risks and returns in a portfolio, Kolm et al. (2014) suggested a number of alternatives. Firstly, allocation of weights equally across selected securities which will not be subject to estimation errors. Secondly, using the risk-parity approach where the total portfolio risk is shared equally across the investment securities (risk parity approach), and; thirdly, using a risk model without a return model, for example measuring risk using the variance of the portfolio return as in the global minimum variance portfolio. The technique of risk allocation across different risk factors in an equity portfolio as a risk management constraint was fronted.

Fabozzi, Gupta, and Markowitz (2002) found that construction of efficient portfolios is better done by using risk factors rather than using mean-variance analysis. This technique takes the fundamental factor model approach where risk factors that explain company attributes were determined and used as indices.

Lately, heuristic models which act as general optimization models are increasingly being applied in portfolio optimization problems (Gilli and Schumann, 2011) because of their ability to take into account different characteristics of return distributions. This study uses one such model, the Threshold Accepting model, for optimization and compares its performance against the MVO model for optimal portfolio selection.

2.1.2. Alternative risk measures

The variance and MAD are risk measures based on dispersion of the asset return distribution while the VaR, CVaR, semi-variance, Minimax, Maximum Loss and lower partial moments are downside risk measures. Downside risk is the risk of an actual return being below the expected return.

These risk measures, among many others, were introduced to improve portfolio selection after experiencing undesired characteristics of having variance as a risk measure. The variance does not distinguish between upside and downside risk hence misrepresenting risk exposure and it is not a good measure for low probability events (Krokhmal et al., 2011).

The variance has been used widely and is associated with normal distribution hence generally understood by many. However it ignores the extremes in data which compromises study results (Gorard, 2015).

The Mean Absolute Deviation is the expected absolute difference between a random variable and its mean and because of its computational ease in portfolio optimization problems, it’s use is increasing. It is more efficient than variance when the data is not in an ideal normal distribution (Biglova et al., 2004; Jaaman et al., 2013).

Many researchers have found that alternative risk measures offer an improvement to the mean-variance portfolio selection (Gilli and Schumann, 2009; Tian, Cox, Lin, and Zuluaga, 2010). Bonyo (2015) studied portfolio optimization in the NSE using the variance and CVaR risk measures. From the research, the CVaR model outperformed the variance for the years 2007 to 2014.

The CVaR is a percentile risk measure and is defined as the conditional expected loss under the condition that it exceeds the VaR. It is also known as mean expected loss, average value at risk, or expected shortfall. It calculates the average of the losses that occur beyond the VaR cutoff point in the distribution. CVaR was introduced as an improvement to VaR which is widely used. CVaR accounts for risks beyond VaR and is applicable to non-symmetrical distributions. The VaR is an upper percentile of the loss distribution (Rockafellar and Uryasev, 2000).

A study on portfolio optimization in the Malaysian share market using four risk measures was conducted by Jaaman et al. (2013). The risk measures used were mean-variance, semi-variance, mean absolute deviation and conditional value-at risk. They observed portfolio performance under the four measures in three economic periods spanning 14 years and found that the CVaR(0.99) model was the most appropriate portfolio optimization model in the market.

The downside risk measure, CVaR, effectiveness against the variance was tested by Bonyo (2015) who inferred that CVaR outperformed variance; and, was a better measure in asset allocation in that it simultaneously minimizes downside risk and achieves similar or better returns.

The alternative risk measures discussed here are just a few of the many others which have been introduced and used for portfolio optimization problems. This is important in this study because it shows that other researchers have been looking for alternative ways for optimal portfolio selection other than the commonly used mean-variance model in an attempt to have improved portfolio allocations.

2.1.3. Threshold Accepting (TA)

This optimization method does not use mean or variance in considering composition of a portfolio but uses a set of rules incorporating objective functions and includes risk and reward measures as constraints in portfolio selection. Several asset combination scenarios for a portfolio are considered and the most optimal one is chosen with checks put in place to minimize uncertainty of the model hence making it robust.

This model involves creating different scenarios and comparing the portfolios formed to select the optimal portfolio. TA works by always accepting a solution that improves the objective function and only accepts a deterioration only if it is not worse than the set threshold (Gilli and Schumann, 2009). The pseudo-code used to run this model is shown in appendix A.

In Threshold Accepting, creating scenarios acts as a form of modelling data hence improving the portfolio selection procedure. The first solution of TA is a random solution that meets the thresholds set and from this solution improvements are ’created’ until the best solution is found; this solution is then chosen as the optimum (Patalia and Kulkarni, 2012).

2.1.4. Previous research on portfolio optimization in Kenya

Selection of portfolios by the single index model and the MVO model in the NSE-20 Share Index from 2002 to 2015 was compared by Nyokangi (2016). The Sharpe ratio was used to measure performance of the portfolios under the two models, and it was established that the MVO model is superior in investments over longer time periods while the single index model is superior over short time horizons and generally had higher portfolio returns than the MVO model but at a higher risk. Hence risk averse investors would prefer the MVO model. It was however suggested that other models could be considered and a different performance measure could be used because of the weaknesses of the Sharpe ratio in accuracy when stocks are skewed.

A comparison of the performance of the NSE-20 share index and an optimal portfolio formed from eight of the equities constituting the index was done by Abdalla (2013). Using the Sharpe measure, it was observed that the portfolio formed outperformed the index. The conclusion was that a new market portfolio index which surpasses the NSE 20 Index could be formed by use of portfolio optimization techniques under Modern Portfolio Theory. Ogutu (2014) recommended use of portfolio optimization techniques in investment decisions as these lead to better absolute and risk adjusted returns. The MVO, the Single Index model and the naive (1/N) portfolio models were used in the study.

Other research done in Kenya on portfolio optimization is on determining the optimal portfolio size for purpose of risk reduction in portfolio selection (Mbithi, Kisaka, and Kitur, 2015; Mbogo and Aduda, 2016). Mbithi et al. conducted their research on the NSE from 2009 to 2013 where they concluded that the optimal portfolio size is between 18 and 22 securities while Mbogo and Aduda considered stock holdings by investment firms in the years 2007 to 2011 and concluded that 16 to 20 securities offer the risk minimizing portfolio.

This study will enhance the findings of previous research by testing the use of the TA model in stock portfolio optimization since there is no previous research on this model in the Kenyan stock market.

2.2. Literature Review summary

Different risk measures have been used together with the mean for a risk-reward model of portfolio optimization. Markowitz (1952) introduced the commonly used MVO which due to its undesired properties led to suggestions and use of alternative risk measures. In addition to the improvements suggested to MVO, other models have been fronted. These include the Black Litterman model where investors' views are incorporated in the asset allocation problem and heuristic models such as the Genetic Algorithm and Threshold Accepting. Gilli and Schumann (2009) recommended that portfolio optimization should have more focus on the potential of modelling data since they had better results with resampled scenarios. What if these heuristic models can give a better optimized portfolio than the mean-risk models?

The focus of this research is on using the MVO and the TA models for portfolio selection on the equities in the NASI and comparing the optimal portfolio selected by the different models. The performance of the optimal portfolios formed is compared using the Sharpe ratio, information ratio and Sortino ratio as portfolio performance measures.

Chapter 3

Methodology

3.1. Research design

This study adopted a quantitative and exploratory research design. It is quantitative since it is focussed on collection and analysis of stock data statistics in order to construct optimal portfolios. The study is also exploratory since it also does a comparison between two models to consider which one performs best in portfolios selection.

3.2. Population and sampling

The population of the study are the securities listed on the NSE all share index (NASI) which are 66 (sixty six) in number. The sample used in this research is 29 stocks. The securities selected as the research sample are those which have full information on the prices, exhibiting superior excess return to risk when compared to the risk-free asset and were trading as at December 2016. Portfolios are constructed from this sample and then performance is compared using three different performance ratios.

The data used is from the daily price lists of the all share index (ASI) of the Kenyan Stock market represented as NASI. The weekly and monthly ASI prices data from the exchange were translated to corresponding weekly and monthly stock returns over the research period from which the returns, standard deviations and covariances can be obtained. Monthly and weekly returns are used rather than daily returns since the latter are more volatile.

The interest rate on the 91-day Treasury Bills issued by the Central Bank of Kenya is used as a proxy for the risk-free rate.

3.3. Data collection methods

Secondary data on the daily price lists of the stocks trading on the NSE was obtained from the Nairobi Securities Exchange (NSE) and the Treasury Bill interest rate was obtained from the Central Bank of Kenya website.

3.4. Data analysis

3.4.1. Mean-Variance Optimization (MVO) model

In portfolio selection using the mean-variance optimization model as explained in section 2.1.1, the mean (r), variance (σ^2) and covariance (σ_{ij}) of each stock return in the sample is calculated and used in the optimization procedure to obtain the optimal portfolio.

The optimization procedure works by allocating weights

$$w_1, w_2, ..., w_n$$

to a portfolio of *n* risky assets with the aim of minimizing the variance, which is the objective function, for a given level of return subject to other constraints as shown in equation 3.1 below.

The objective of MVO in portfolio selection is:

$$Min \quad w^{'} \sum w$$

$$\begin{aligned} \text{s.t.} \quad w^{'}R \geq \bar{R}, \\ w^{'}\mathbf{1} = 1, \\ w_i \geq 0 \end{aligned} \tag{3.1}$$

where $\bar{R}$ is the minimal rate of return required by an investor and $\sum$ is the covariance matrix. By having $w_i \geq 0$, it means no short-selling is allowed. This is because the focus of this study is on maximizing returns and having a bounded variance which short-selling does not allow.

The risk-free rate is used as the minimal rate of return required by an investor.

3.4.2. Threshold Accepting (TA) model

Threshold Accepting is a non risk-reward optimization technique. The objective of the model is:

$$
\begin{aligned}
&Min_w \quad \phi(r) \\
&\qquad\qquad \text{s. t.} \quad w_j^{\text{inf}} \leq w_j \geq w_j^{\text{sup}} \qquad j \in P, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad n_{\text{inf}} \leq n_{\text{sup}}
\end{aligned}
\tag{3.2}
$$

where w_j^{inf} and w_j^{sup} are minimum and maximum weights for the stock in the portfolio; P is the set of individual stocks in the portfolio; and, n_{inf} and n_{sup} are constraints setting the minimum and maximum number of stocks in the portfolio, P (Gilli and Schumann, 2009; Patalia and Kulkarni, 2012).

The implementation of the TA model/algorithm requires the definition of the objective function *f*, the neighbourhood *N*(x^c) - where x^c is the current best solution - and the threshold sequence τ which gradually reduces to zero in a given number of rounds (Gilli, Këllezi, and Hysi, 2006). The neighbourhood is defined as the subsequent portfolios that offer a better return at a given minimum risk or portfolios that have a lower risk at a given return as compared to the portfolios already formed.

In this study, the thresholds that are applied are capping the weights for stock in the portfolio at 30%, restricting the number of stock in the portfolio to between 3 and 10 stocks and setting the minimum return acceptable as the NASI return.

These thresholds represent bounds/expectations of an investor in the Kenyan stock market. An investor typically would not want to invest in many stocks at a go but would also wish to diversify their portfolio to select a small number of stocks.

The NASI return is used as the minimum acceptable return since one would wish to invest in stocks which at least perform in the same way as the market index. This is the benchmark value against which the returns portfolios formed will be compared to.

3.4.3. Measuring portfolio performance

When comparing the performance of portfolios, risk and return are the main measures combined to form ratios or indices that are used to determine the most optimal portfolio (Tarasi, Bolton, Hutt, and Walker, 2011). It is recommended to use more than one performance measure in comparing portfolios (Gerken, 2015).

The performance of the portfolios formed under the two models is tested by use of performance ratios. These ratios are calculated by utilizing the portfolio's mean ($\bar{R}_P$) and variance (σ_P^2). These are given by:

$$\bar{R}_P = E(P) = \sum_{i=1}^{n} w_i \mu_i \tag{3.3}$$

$$\sigma_P^2 = V(P) = E[\sum_{i=1}^{n} w_i \mu_i - E(P)]^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{3.4}$$

The portfolios are then ranked under the different performance ratios to determine the optimal asset allocation model. The performance of the weekly and monthly portfolios selected is also compared to determine whether the investment horizon matters in optimal allocation.

Varying time periods are also considered to check the consistency of the performance of the MVO and TA models in optimal portfolio selection.

When comparing the performance of two optimization models, the better performing model has portfolios with higher performance ratios (Pekár, Čičková, and Brezina, 2016). Georgiev (2014) maximized the Sharpe and information ratios in selecting optimal portfolios.

In stock optimization in the Kenyan market, Nyokangi (2016) maximized the Sharpe ratio to determine the better optimization model between the Single index and the MVO models. Abdalla (2013) also used the Sharpe measure in comparing performance of the NSE 20 index against a new market portfolio index formed. In addition to the Sharpe ratio, Ogutu (2014) used the Jensen and Treynor ratios to determine the best portfolio optimization model among the MVO, Single index and (1/N) naive portfolio models.

In this study, the following three ratios are used:

3.4.3.1 Sharpe Ratio

This ratio was introduced by Sharpe (1966) as a means of evaluating and measuring performance of mutual funds. It is defined as the average return earned in excess of the risk-free rate per unit of volatility and is calculated as:

$$\frac{\bar{R}_P - \bar{r}_f}{\sigma_P} \tag{3.5}$$

where $\bar{r}_f$ is the average risk-free rate.

The portfolio with a higher Sharpe Ratio is preferred since it has better returns relative to the risk taken.

3.4.3.2 Sortino Ratio

The Sortino ratio is a modification of the Sharpe ratio, using downside deviation instead of standard deviation as a measure of risk. It thus considers the riskiness of only those returns falling below the required rate of return (Rollinger and Hoffman, 2013).

It was introduced by Frank Sortino in 1981 and is obtained as follows:

$$\frac{\bar{R}_P - \bar{r}_f}{\sigma_L(R_p)} \tag{3.6}$$

where $\sigma_L(R_p) = \sqrt{\frac{1}{n_L}\sum^n (Min(R_P - R_f), 0)}$

The higher the Sortino Ratio, the better the portfolio.

3.4.3.3 Information Ratio (IR)

The IR is a measure of the average excess return per unit of volatility in excess return:

$$\frac{\bar{R}_P - \bar{r}_b}{TE} \tag{3.7}$$

where $\bar{r}_b$ is the average return for the benchmark and *TE* is the Tracking Error (standard deviation of the excess return).

It shows the extent to which a portfolio performs better than the benchmark index to which it is compared against (Kidd, 2011). The higher the IR, the better the portfolio performance.

Chapter 4

Data Analysis, Findings and Discussion

This chapter presents the results of the study. The analysis employs the use of two models in the selection of optimal stock portfolios over the period 1998 to 2016. This will inform the decision to accept or reject the null hypothesis.

4.1. Variable selection and transformation

Stock data was obtained as daily price lists from the NASI in the Nairobi Securities Exchange (NSE). The prices used are the closing prices of Friday in every week. The monthly prices are then obtained as every fourth week's price from the weekly prices used. Log returns are then obtained from these prices and used in the portfolio selection models. Log returns are used in this study because they allow for easier aggregation over time when compared to simple returns.

The 91-day Treasury Bill rate was obtained from the Central Bank of Kenya website. The rate is then converted into effective interest rates to be used as the risk free rate.

A total of twenty nine stocks are used in this study; these are the ones which have trading information for a period of 957 weeks out of the total 992 weeks considered in the study and exhibit superior average excess returns over the risk free rate per unit of volatility. The monthly data comprises 242 data points. A summary of the descriptive statistics of these stocks is shown in Appendix C.

4.2. Optimal portfolio selection

The mean-variance optimization and Threshold Accepting models are used in construction of the optimal portfolio from the selected stocks in the NASI.

4.2.1. Mean Variance Optimization model

The study follows the process described under equation (3.1) to find the optimal portfolio under this model. The Solver function in Microsoft Excel is used and has the following stocks and weights allocated for the optimal portfolio:

i	w_i	$\bar{R}_P$	σ_P
STOCK	WEIGHTS	Portfolio Return	Portfolio Std Dev
BAMB	10.2217%	2.7165%	0.8116%
BBK	3.6704%		
BERG	1.6732%		
BOC	25.5548%		
C&G	5.9519%		
CFC	2.7401%		
DTK	3.1351%		
EABL	10.3260%		
JUB	2.5483%		
KENO	2.7209%		
KQ	0.6804%		
KUKZ	4.9227%		
NMG	1.3584%		
OCH	1.2744%		
SASN	0.6090%		
SCBK	5.7813%		
SGL	0.4096%		
SNLM	4.9117%		
TOTL	5.5638%		
UNGA	0.1421%		
XPRS	5.8044%		

TABLE 4.1: Stock composition and return & risk of optimal portfolio selected by the MVO model on weekly stock returns data 1998 - 2016

The portfolio majorly consists of four stocks (**BOC**, **EABL**, **BAMB**, and **C&G**) which belong to three different industry sectors - Manufacturing, Construction and Automobiles sector.

where i represents a stock, w_i is the weight of the stock in the portfolio, $\bar{R}_P$ is the portfolio return and σ_P is the portfolio standard deviation.

Over the weekly analysis, we can see in Table 4.1 above, that **BOC**, **EABL**, **BAMB**, and **C&G** take the largest allocations respectively. **BOC** and **EABL** are in the Manufacturing sector, **BAMB** is in the Construction sector while **C&G** is in the Automobiles sector. Looking at the statistics in Table C.1, we can see **BOC** is selected for its low risk while the other stocks selected have high returns with relatively low risks.

The monthly optimized portfolio comprises the same stocks but also includes **SCBK** (Banking Sector) among its largest allocations (Table 4.2). This shows that this model is consistent regardless of the time period considered.

Despite the monthly portfolio having more stock hence being more diversified, it has lower expected returns with higher variance as compared to the weekly portfolio.

i STOCK	w_i WEIGHTS	$\bar{R}_P$ Portfolio Return	σ_P Portfolio Std Dev
BAMB	7.0698%	2.2270%	1.848%
BOC	30.5265%		
C&G	9.9547%		
CFC	1.3551%		
EABL	14.875%		
KENO	1.7713%		
KQ	4.4192%		
KUKZ	3.6636%		
NMG	0.7691%		
OCH	1.0371%		
SASN	2.7608%		
SCBK	10.4343%		
SGL	0.2527%		
SNLM	2.4087%		

TABLE 4.2: Stock composition and return & risk of optimal portfolio selected by the MVO model on monthly stock returns data 1998 - 2016

The greatest weights are given to **BOC** (Manufacturing), **EABL** (Manufacturing), **SCBK** (Banking), **C&G** (Automobiles) and **BAMB** (Construction); these selected stocks are in different sectors except for the first two.

4.2.2. Threshold Accepting Optimization model

This model employs the thresholds mentioned in section 3.4.2. The optimization results are as shown in tables 4.3 and 4.4 below.

i	w_i	$\bar{R}_P$	σ_P
STOCK	WEIGHTS	Portfolio Return	Portfolio Std Dev
EABL	30.0000%	7.9377%	1.3824%
ICDC	10.0000%		
JUB	30.0000%		
KENO	30.0000%		

TABLE 4.3: Stock composition and return & risk of optimal portfolio selected by the TA model on weekly stock returns data 1998 - 2016

Portfolio selects **EABL** (Manufacturing), **ICDC** (Investment), **JUB** (Insurance) and **KENO** (Energy). All stocks selected are in different sectors with the maximum weight in the TA threshold, (30%), allocated to three stocks. The selected stocks are a subset of stocks selected by the monthly TA portfolio in Table 4.4.

Only four stocks are selected in the weekly analysis for the TA portfolio; these can be seen in Table 4.3 - **EABL** (Manufacturing sector), **JUB** (Insurance sector), **KENO**

i	w_i	$\bar{R}_P$	σ_P
STOCK	WEIGHTS	Portfolio Return	Portfolio Std Dev
ARM	11.717%	5.4409%	2.5819%
BOC	4.5711%		
CG	3.2149%		
DTK	10.0202%		
EABL	28.3888%		
HFCK	2.1922%		
ICDC	1.786%		
JUB	12.9509%		
KENO	17.1471%		
OCH	6.3906%		

TABLE 4.4: Stock composition and return & risk of optimal portfolio selected by the TA model on monthly stock returns data 1998 - 2016

The greatest weights are given to **EABL** (Manufacturing), **KENO** (Energy), **JUB** (Insurance), **ARM** (Manufacturing) and **DTK** (Banking). Portfolio return is less than the weekly TA portfolio in table 4.3 and also has higher risk.

(Energy sector) and **ICDC** (Investment sector). All the stocks selected are in different industry sectors.

In the monthly TA portfolio, we have the largest allocation to **EABL**, **KENO**, **JUB**, **ARM** (Manufacturing sector) and **DTK** (Banking sector) stocks (Table 4.4). Most of these stocks are also in different sectors for the respective models' portfolios. The weekly TA portfolio is a subset of the monthly TA portfolio.

4.3. Portfolio performance

The portfolios formed by the two optimization models considered, have been tested and ranked using the three performance ratios explained in section 3.4.3 as shown in the tables 4.5 and 4.6 below.

Most of the ratios obtained are negative since the average stock returns are less than the risk free rate for the period considered (Tables C.1, C.2).

The Sharpe ratios are negative numbers indicating that holding the risk free security is superior to holding stock portfolios. The TA portfolio offers a better performance when compared to the MVO portfolio since it has higher Sharpe ratios in comparison.

In order to obtain the IR and Sortino ratios, portfolios of the stocks are formed over each data point using the weights shown in tables 4.1, 4.2, 4.3 and 4.4. The IR gives

	Performance Ratios				
	$\bar{R}_P$	σ_P	Sharpe ratio	IR	Sortino Ratio
MVO Portfolio - weekly	2.7165%	0.8116%	-8.8401%	-0.0182%	-0.2749%
TA Portfolio - weekly	7.9397%	1.3824%	-1.4118%	0.2983%	-0.0696%
MVO Portfolio - monthly	2.227%	1.8482%	-4.1149%	-0.6865%	-0.7738%
TA Portfolio - monthly	5.4409%	2.5819%	-1.7007%	0.7684%	-0.4507%

TABLE 4.5: Performance Ratios of portfolios (weekly and monthly) selected by the MVO and TA models

A summary of the returns, standard deviation & performance ratios values of the portfolios selected by the MVO and TA models for the full period 1998 - 2016. Performance ratios of the TA portfolios are higher than those for the MVO portfolios, with the TA portfolios exhibiting higher risk and returns.

positive values for the TA portfolios unlike the MVO portfolios, showing that the TA model portfolio outperformed the benchmark index. The Sortino ratios are negative ratios for both models but the TA model is still potrayed as superior since it has higher values.

It is however noted that the TA portfolios carry a higher risk as compared to the MVO portfolios. This is due to the risk return trade-off.

The ranking obtained is:

RANKING			
Weekly Analysis			
	Sharpe ratio	IR	Sortino Ratio
MVO Portfolio	2	2	2
TA Optimization Portfolio	1	1	1
Monthly Analysis			
	Sharpe ratio	IR	Sortino Ratio
MVO Portfolio	2	2	2
TA Portfolio	1	1	1

TABLE 4.6: Portfolio optimization models ranking

Ranking of the MVO and TA models by the Sharpe ratio, information ratio and Sortino ratio for the full period 1998 - 2016.

This ranking is based on the values of these three performance ratios as explained in section 3.4.3, where a better ranking is given to a portfolio with a higher performance ratio. The portfolio with a higher ranking is considered to have better performance.

Figures 4.1 and 4.2 below give a summary of returns of portfolios formed by the two optimization models used in this study.

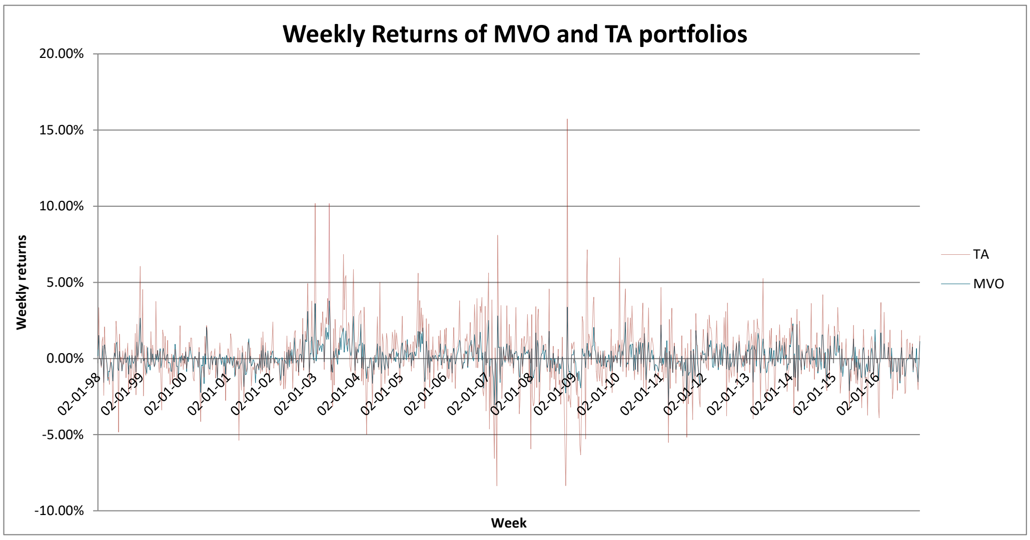

FIGURE 4.1: Portfolio Returns of portfolios selected by MVO & TA models in the weekly stock returns data 1998 - 2016

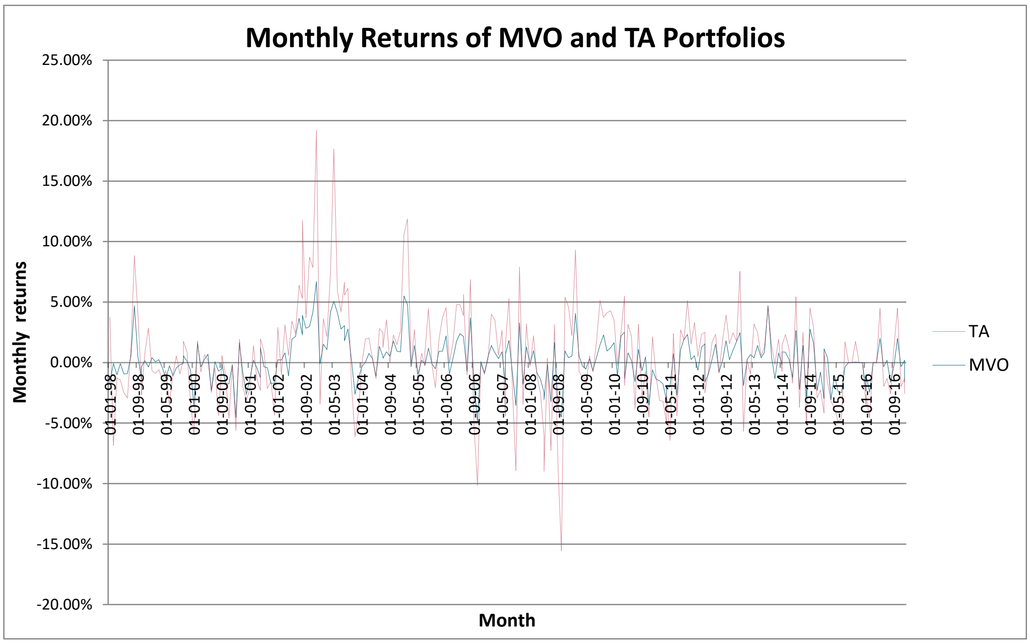

FIGURE 4.2: Portfolio Returns of portfolios selected by MVO & TA models in the monthly stock returns data 1998 - 2016

4.3.1. Performance of the models in different time periods

The period 1998 to 2016 is divided into four periods to test consistency in performance of the MVO and TA optimization models. The four sub-periods are selected based on Kenya's election cycle. This is because research has shown that the stock market performance is affected by the uncertainty during election periods where there are abnormal returns (negative and positive) in the periods immediately preceeding and superceeding the elections (Kabiru, Ochieng, and Kinyua, 2015; Menge et al., 2014) and stock prices become less informative (Durnev, 2010).

Weekly analysis as explained in sections 4.2.1 and 4.2.2 is conducted over the sub-periods as follows:

4.3.1.1 Period One: 1998 - 2002

The optimization results are as shown in table 4.7 below:

Portfolio composition for 1998 - 2002

MVO model				TA Model			
i	w_i	$\bar{R}_P$	σ_P	i	w_i	$\bar{R}_P$	σ_P
BAMB	11.2778%	0.00000007%	0.8531%	BOC	29.97%	0.034993%	1.0833%
BBK	3.4963%			EABL	29.63%		
BERG	2.7179%			SCBK	9.93%		
BOC	23.9029%			TOTL	29.63%		
C&G	5.8803%						
CFC	1.8022%						
EABL	18.3178%						
JUB	1.5118%						
KENO	6.0926%						
NMG	2.879%						
SCBK	10.3863%						
SNLM	3.7827%						
TOTL	5.0586%						

TABLE 4.7: Stock composition of portfolios selected by the MVO and TA Models in the period 1998 - 2002

The greatest weights are given to similar stocks by both models' portfolios. The MVO portfolio has **BOC** (Manufacturing), **EABL** (Manufacturing), **BAMB** (Construction) and **SCBK** (Banking) while the TA portfolio has **BOC**, **EABL**, **SCBK** and **TOTL** (Energy). This period had poor stock performance with the MVO portfolio return being very low.

The MVO model consists of various stocks with the largest allocations in **BOC** (Manufacturing sector), **EABL** (Manufacturing sector), **BAMB** (Construction sector) and **SCBK** (Banking sector) consisting of three industry sectors as is shown in appendix B; while the TA model portfolio consists of four stocks, **BOC** (Manufacturing sector),

EABL (Manufacturing sector), **SCBK** (Banking sector) and **TOTL** (Energy sector) which are companies in three different sectors also. The two portfolios have a similar composition with only one different stock among the ones selected in each.

The TA model outperforms the MVO model in this period since all its performance ratios are higher than the MVO model as is shown in table 4.11. The TA model portfolio has higher returns but at a higher risk as compared to the MVO model.

The performance of the stock market during this period was poor. This can be seen in table 4.11 where the portfolios performance ratios for this period are high negative values.

4.3.1.2 Period Two: 2003 - 2007

The optimization results are as shown in table 4.8 below:

Portfolio composition for 2003 - 2007

MVO model				TA Model			
i	w_i	$\bar{R}_P$	σ_P	i	w_i	$\bar{R}_P$	σ_P
ARM	11.9324%	6.5%	1.5635%	ARM	27.6421%	6%	1.47%
CABL	36.6295%			BAMB	0.951%		
CFC	8.1925%			CABL	11.9844%		
DTK	8.1195%			CFC	17.1365%		
JUB	16.5975%			DTK	2.0104%		
KCB	6.447%			EABL	2.246%		
KENO	1.5244%			JUB	25.1877%		
SNLM	10.5456%			KCB	4.9982%		
				KENO	0.5088%		
				SNLM	6.3375%		

TABLE 4.8: Stock composition of portfolios selected by the MVO and TA Models in the period 2003 to 2007

The greatest weights are also given to similar stocks by both models' portfolios as is the case in Period One (Table 4.7. The MVO portfolio has **CABL** (Construction), **JUB** (Insurance), **ARM** (Construction) and **SNLM** (Insurance) while the TA portfolio has **ARM**, **JUB**, **CABL** and **CFC** (Banking). This period had superior stock performance with the MVO portfolio having higher performance ratios than the TA portfolio.

The highest allocations for the MVO model portfolio are in **CABL** (Construction sector), **JUB** (Insurance sector), **ARM** (Construction sector) and **SNLM** (Insurance sector); while the TA model portfolio has **ARM** (Construction sector), **JUB** (Insurance sector), **CFC** (Banking sector) and **CABL** (Construction sector) with the highest allocations.

According to the performance ratios, the MVO model outperforms the TA model in this period (Table 4.11). The performance ratios are also positive indicating that the stock market was performing well.

4.3.1.3 Period Three: 2008 - 2012

The optimization results are as shown in table 4.9 below:

Portfolio composition for 2008 - 2012

MVO model				TA Model			
i	w_i	$\bar{R}_P$	σ_P	i	w_i	$\bar{R}_P$	σ_P
ARM	24.9392%	1.2%	1.3607%	ARM	28.5528%	1.2%	1.4384%
DTK	12.2033%			DTK	10.0118%		
EABL	32.5624%			EABL	29.9954%		
KUKZ	24.7311%			KQ	1.0864%		
NMG	5.5639%			KUKZ	29.9944%		

TABLE 4.9: Stock composition of portfolios selected by the MVO and TA Models in the period 2008 to 2012

Highest portfolio weights are allocated to similar stocks by both the MVO and TA models. The MVO portfolio selects **EABL** (Manufacturing), **ARM** (Construction), **KUKZ** (Agricultural), **DTK** (Banking) and **NMG** (Commercial) while the TA portfolio is a sub-set of the MVO portfolio comprising **EABL**, **ARM**, **KUKZ** and **DTK**.

The MVO model portfolio consists of the following stocks: **EABL** (Manufacturing sector), **ARM** (Construction sector), **KUKZ** (Agricultural sector), **DTK** (Banking sector) and **NMG** (Commercial sector). The highest allocations in the TA portfolio model are a sub-set of the MVO model stocks with **NMG** excluded.

The MVO model outperforms the TA model in this period as can be seen by the performance ratios in table 4.11.

4.3.1.4 Period Four: 2013 - 2016

The optimization results are as shown in table 4.10 below.

The MVO model portfolio has **KUKZ** (Agricultural sector), **JUB** (Insurance sector) and **ICDC** (Investment sector) stocks with the highest allocations while the TA model portfolio comprises the same stocks but also has **CFC** (Banking sector) and **UNGA** (Manufacturing sector) with sizeable allocations.

The MVO model outperforms the TA model in this period as can be seen from the performance ratios in table 4.11.

Portfolio composition for 2013 - 2016

MVO model				TA Model			
i	w_i	$\bar{R}_P$	σ_P	i	w_i	$\bar{R}_P$	σ_P
CFC	2.072%	3.8%	1.6271%	CFC	6.9636%	3.5%	1.52%
ICDC	19.0275%			ICDC	27.1942%		
JUB	26.1958%			JUB	29.9951%		
KUKZ	47.7456%			KUKZ	29.9994%		
UNGA	4.955%			UNGA	5.7775%		

TABLE 4.10: Stock composition of portfolios selected by the MVO and TA Models in the period 2013 to 2016

The greatest weights are given to the same stocks by both models' portfolios: **KUKZ** (Agricultural), **JUB** (Insurance) and **ICDC** (Investment) but at different proportions. The MVO model has higher performance ratios as shown in Table 4.11.

4.4. Summary

4.4.1. Full period analysis

Over the study period, 6 of the 29 stocks have negative weekly average returns and 5 have negative monthly average returns (Tables C.1 and C.2). Furthermore, the average risk free rate is higher than the average weekly and monthly returns for all stocks considered in the full period 1998 - 2016; but, many stock returns are above the average NASI return.

This leads to negative performance ratios for the portfolios formed with all the three performance ratios indicating that the TA portfolio is better than the MVO portfolio. It should however be noted that the stock returns vary within the study period with maximum returns as high as 108.06% and 54.41%, to minimum returns as low as -76.07% and -73.98%, for the weekly and monthly analysis respectively (Appendices C.1, C.2); hence, the stock markets are still a good avenue for an investor to get a good return on their investments.

The two optimization models mainly selected stocks from different sectors implying that optimal stock portfolios are well diversified by considering stocks of companies in different industries. This is as shown in tables 4.1 - 4.4. This finding agrees with the conclusion of Dou, Gallagher, Schneider, and Walter (2014), that diversification across sectors leads to better portfolio performance. Dou et al. also recommended analysis over weekly basis which could signal more timely information as compared to the monthly analysis that was only considered then.

In this study, optimization over the weekly periods yields better portfolios, in terms of higher returns and lower risk, as compared to the monthly period.

The MVO model has a similar set of stock in the weekly and monthly return portfolios indicating that it is a consistent model.

In the TA model, the monthly portfolio considers more stocks than the weekly portfolio which seems to have naively selected high return stocks.

4.4.2. Sub-periods analysis

	Performance measure values				
	$\bar{R}_P$	σ_P	Sharpe ratio	IR	Sortino Ratio
MVO Portfolio - 1998-2002	0.00000007%	0.8531%	-18.1128%	-20.4355%	-10.954%
TA Portfolio - 1998-2002	0.035%	1.0833%	-14.2313%	-13.5473%	-9.6618%
MVO Portfolio - 2003-2007	6.5%	1.5635%	0.3378%	0.2496%	0.0906%
TA Portfolio - 2003-2007	6%	1.4744%	0.0191%	0.0144%	0.0048%
MVO Portfolio - 2008-2012	1.2%	1.3607%	-5.337%	-1.9485%	-0.8575%
TA Portfolio - 2008-2012	1.2%	1.4384%	-5.0487%	-1.7117%	-0.8387%
MVO Portfolio - 2013-2016	3.8%	1.6271%	-3.7446%	0.0295%	-0.6282%
TA Portfolio - 2013-2016	3.5%	1.5177%	-4.2121%	-0.1564%	-0.6283%

TABLE 4.11: Performance ratios values of MVO & TA model portfolios for the four sub-periods: 1998 - 2002, 2003 - 2007, 2008 - 2012 and 2013 - 2016

These performance ratios are used to rank the MVO and TA models by their portfolios shown in Tables 4.8, 4.9, 4.7 & 4.10 for the sub-period analysis. The MVO model performs better in the periods 2003 - 2007 and 2013 - 2016 where its portfolios have higher performance ratios, while the TA model performs better in the other two sub-periods.

The MVO model outperforms the TA model in two out of the four subperiods as can be seen by the ratio values in table 4.11 above. These periods (2003 - 2007 & 2013 - 2016) also exhibit better stock market performance than the other two subperiods.

The stocks comprising the portfolios formed are also majorly from different sectors, hence the portfolios are diversified. Over the different sub-periods we mainly have stocks from the Construction, Manufacturing, Insurance and Banking sectors selected.

Chapter 5

Conclusion and Recommendations

This study was based on using a risk-reward model and a heuristic measure to construct optimal portfolios of stocks listed in the NASI for the period 1998 - 2016. The models' performance was compared using different performance ratios, two of which (IR and Sortino Ratio) focussed on constructing portfolios over each data point and summarizing the performance as a single measure. A comparison of the ratios then led to the conclusion as to which model performs better.

This study concludes that the TA model outperforms the MVO model for portfolio selection. This is in line with what Fastrich and Winker (2012) found, that heuristic models lead to overall superior results over the MVO approach and portfolio compostions from heuristic approaches are more stable. Gilli and Schumann (2009) also concluded that employing techniques that lead to running optimization over created scenarios and different risk measures other than variance, offers an improvement to the traditional mean-variance optimization models.

It is also shown that the TA model portfolios have higher returns than MVO model but at a higher risk cost. This would be appealing for investors who are risk takers. Nyokangi (2016) had the same conclusion on the single index model in comparison with the MVO model.

As mentioned in section 4.4, stock returns are volatile and can have both very high returns and large negative returns across time. Thus, with proper watch on movement of stock prices, an investor in the Kenyan stock market can yield high returns.

In summary, this study found that TA optimization model is better for use in optimal portfolio construction as compared to the MVO model. Therefore, the first research hypothesis, as stated in section 1.5, is not rejected.

It is also observed that in comparing performance over time that each of the models performs best in two out of the four sub-periods. Hence the second research hypothesis is rejected. The MVO model particularly performs best in the periods where stock returns exhibit higher returns.

Investors in the Kenyan stock market, especially fund managers who act on behalf of their clients can explore the benefits of using the TA model in making decisions on stock investment holdings.

5.1. Limitations of the Study

This study has some limitations. To begin with, only 29 stocks out of 66 are sampled for use in portfolio analysis and also, the only securities considered are stocks. This could have left out other stocks and investment in other viable securities like Treasury Bills and Bonds which may have led to a better performing portfolio.

Another limitation is inadequate information over the thresholds that are applicable in the TA optimization model for the Kenyan stock market because investors' views were not incorporated in the study. This can be an avenue for further study by incorporating the Black-Litterman model as explained by Idzorek (2004).

5.2. Suggestions for further Studies

This study found that the TA model outperforms the MVO model in selection of optimal stock portfolios in the Kenyan stock market for the full period 1998 - 2016. However, in the four subperiods considered to test the consistency of the two models, both the MVO and TA models perfom well with each performing better than the other in two out of the four sub-periods. The MVO model exhibits more stability than the TA model because the stocks selected over the weekly and monthly portfolios is similar.

There is need for further research on portfolio optimization especially when considering other securities like Bonds and Bills which were not used in this study. The effect of transaction costs - brokerage fees and taxes, on portfolio optimization can also be considered for further study.

The approach of the study was historical, additional studies can be conducted on a forward-looking design.

Further studies can also be done by considering other risk measures like CVaR, partial drawdown, Mean Absolute Deviation and semi-variance in a risk-return analysis as an improvement over MVO as suggested by Jaaman et al. (2013) and A. H. Chen, Fabozzi, and Huang (2012). Z. Chen, Li, and Wang (2015) recommended use of a new risk measure which takes investors' views into account; this direction of research can also be considered for the Kenyan stock market.

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Appendix A

Formulas

A.1. Return measures

The return measures are obtained as follows:

$r_i = log(\frac{P_{i,t+1}}{P_{i,t}})$ where $P_{i,t}$ is the stock price of the i^{th} asset at time t.

$\sigma_i^2 = E[(r_i - \mu_i)^2]$ where $\mu_i = E(r_i)$, the expected value of the return.

$\rho_{ij} = cov(r_i r_j)/(\sigma_i \sigma_j)$ where $cov(r_i r_j) = \sigma_{ij} = E[(r_i - \mu_i)(r_j - \mu_j)]$

A.2. Threshold Accepting pseudo-code

1: Initialize n_{Rounds} and n_{Steps}

2: Compute threshold sequence, τ_r

3: Randomly generate current solution $x^c \epsilon X$

4: **for** r = 1: n_{Rounds} **do**

5: **for** i = 1: n_{Steps} **do**

6: Generate $x^c \epsilon N(x^c)$ and compute $\Delta = f(x^n) - f(x^c)$

7: **if** $< \tau_r$ then $x^c = x^n$

8: **end for**

9: **end for**

10: $x^{sol} = x^c$

where *f* is the objective function, *X* is the set of feasible solutions, x^c is the current solution, x^n is an alternative solution close to x^c

Source: (Schumann, 2011)

Appendix B

Codes of Stocks used

Code	Company	Company Sector
ARM	ARM Cement Ltd	Construction and Allied
BAMB	Bamburi Cement Ltd	Construction and Allied
BBK	Barclays Bank of Kenya Ltd	Banking
BERG	Crown Paints Kenya Ltd	Construction and Allied
BOC	B.O.C Kenya Ltd	Manufacturing and Allied
C&G	Car and General (K) Ltd	Automobiles and Accessories
CABL	E.A.Cables Ltd	Construction and Allied
CFC	CFC Stanbic of Kenya Holdings Ltd	Banking
DTK	Diamond Trust Bank Kenya Ltd	Banking
EABL	East African Breweries Ltd	Manufacturing and Allied
FIRE	Sameer Africa Ltd	Automobiles and Accessories
HFCK	Housing Finance Group Ltd	Banking
ICDC	Centum Investment Co Ltd	Investment
JUB	Jubilee Holdings Ltd	Insurance
KCB	KCB Group Ltd	Banking
KENO	KenolKobil Ltd	Energy and Petroleum
KQ	Kenya Airways Ltd	Commercial and Services
KUKZ	Kakuzi Ltd	Agricultural
NBK	National Bank of Kenya Ltd	Banking
NIC	NIC Bank Ltd	Banking
NMG	Nation Media Group Ltd	Commercial and Services
OCH	Olympia Capital Holdings Ltd	Investment
SASN	Sasini Ltd	Agricultural
SCBK	Standard Chartered Bank Kenya Ltd	Banking
SGL	Standard Group Ltd	Commercial and Services
SNLM	Pan Africa Insurance Holdings Ltd	Insurance
TOTL	Total Kenya Ltd	Energy and Petroleum
UNGA	Unga Group Ltd	Manufacturing and Allied
XPRS	Express Kenya Ltd	Commercial and Services

TABLE B.1: Stocks considered in the optimization models

Source of Codes: (NSE, 2016b)

Appendix C

Descriptive Statistics of Stocks

Stock	Mean	Std Dev	Kurtosis	Skewness	Minimum	Maximum
ARM	5.9972%	2.8035%	5.6961	0.6212	-14.738%	15.448%
BAMB	3.9627%	1.822%	10.5123	0.0279	-15.4902%	10.8728%
BBK	3.1903%	2.0326%	9.9103	0.9219	-10.8507%	14.6651%
BERG	3.4387%	3.1661%	20.2291	0.1681	-27.2635%	30.2211%
BOC	1.1563%	1.4738%	5.9545	0.2442	-8.3337%	8.0268%
C&G	2.7505%	2.6265%	29.956	0.2998	-26.6268%	27.703%
CABL	3.2855%	2.9104%	13.399	1.2577	-17.4498%	22.1849%
CFC	3.6931%	2.8426%	56.7374	3.4467	-22.0111%	40.056%
DTK	6.0209%	2.1917%	5.9984	0.6396	-8.9237%	15.613%
EABL	8.6763%	1.6833%	3.9284	0.163	-7.5476%	9.2617%
FIRE	-2.6814%	2.799%	7.0817	0.8198	-12.0681%	18.0203%
HFCK	0.6072%	2.9659%	4.5044	0.3276	-17.7306%	17.8432%
ICDC	6.8275%	2.5753%	10.3058	0.8052	-13.8564%	21.7688%
JUB	8.0286%	2.3335%	6.1038	0.4807	-12.4939%	14.3614%
KCB	3.3852%	2.7068%	7.3225	0.3189	-15.1404%	15.1502%
KENO	7.4849%	2.4363%	13.9769	0.5612	-17.6091%	18.8733%
KQ	-0.9333%	2.6253%	6.7353	0.5703	-13.1012%	18.187%
KUKZ	2.77%	2.5496%	7.1451	0.5595	-15.7084%	16.986%
NBK	0.6951%	3.4335%	5.5235	0.7085	-17.5054%	19.9069%
NIC	2.8686%	2.5147%	6.8059	0.4262	-13.6344%	17.0614%
NMG	6.0489%	3.2548%	142.7422	4.6528	-40.5173%	58.3288%
OCH	-8.3061%	4.1304%	68.8808	-2.7586	-61.3489%	38.4448%
SASN	1.5056%	2.8267%	11.0447	1.7532	-11.5039%	19.6473%
SCBK	4.5722%	1.8517%	6.351	-0.2206	-11.0642%	9.3725%
SGL	-1.7908%	3.6611%	20.8912	-1.1685	-36.386%	20.9802%
SNLM	3.3501%	2.7869%	14.7089	0.4521	-23.0069%	21.3416%
TOTL	-2.2692%	2.4179%	10.1838	0.6937	-15.7608%	17.2265%
UNGA	1.2951%	5.4424%	202.793	5.1573	-76.0697%	108.0571%
XPRS	-5.6698%	2.4434%	8.971	-0.3246	-20.6419%	14.2995%

TABLE C.1: Summary of the descriptive statistics for weekly returns: 1998-2016

	Mean	Std Dev	Kurtosis	Skewness	Minimum	Maximum
ARM	6.8000 %	5.3043%	3.5785	68.27%	-15.1469%	26.7172%
BAMB	3.1876%	3.5103%	5.496	116.32%	-9.241%	18.406%
BBK	3.4398%	3.9016%	4.1704	52.12%	-12.6398%	21.2861%
BERG	3.9757%	6.1724%	14.096	-160.91%	-45.5932%	20.536%
BOC	0.3264%	3.072%	4.3032	-19.88%	-15.4902%	13.0078%
C&G	2.7581%	5.1004%	9.0048	59.7%	-26.6268%	26.8552%
CABL	2.6422%	5.8213%	4.4543	117.99%	-16.4273%	26.2872%
CFC	3.0683%	5.08%	8.4035	58.34%	-27.986%	24.6436%
DTK	5.663%	4.4207%	4.3438	49.27%	-16.2974%	24.6106%
EABL	8.2715%	3.4231%	1.7133	-0.26%	-12.3818%	13.8809%
FIRE	-3.3313%	5.1715%	2.08	67.14%	-16.4886%	22.7244%
HFCK	0.5435%	5.6535%	5.2349	120.54%	-14.8939%	29.9476%
ICDC	6.2288%	5.5378%	8.4885	105.92%	-18.4394%	36.4265%
JUB	7.6603%	4.5041%	14.3296	219.52%	-14.9785%	30.5618%
KCB	3.8577%	5.2256%	4.408	55.83%	-19.025%	27.6056%
KENO	5.7214%	4.3007%	1.9512	18.77%	-14.1329%	15.3536%
KQ	0.1837%	5.3241%	3.9553	90.25%	-20.4863%	22.8236%
KUKZ	2.9532%	5.3348%	4.3835	73.43%	-20.4719%	22.3088%
NBK	1.4437%	5.7578%	3.4733	89.05%	-20.2427%	26.2848%
NIC	3.1475%	4.7774%	1.9293	46.71%	-12.6997%	20.2695%
NMG	4.8775%	5.1253%	14.1255	74.27%	-31.2719%	30.4369%
OCH	-6.7155%	8.0556%	30.2514	-241.46%	-73.9836%	31.3867%
SASN	1.4345%	5.4065%	5.4443	99.14%	-17.7442%	31.1896%
SCBK	4.1453%	3.2797%	2.8704	2.6%	-13.9947%	14.0619%
SGL	-0.3273%	7.4021%	11.3891	35.25%	-37.6956%	46.812%
SNLM	2.6349%	6.0687%	7.0772	40.47%	-26.7606%	31.8759%
TOTL	-1.6613%	4.241%	3.6125	54.78%	-17.1935%	19.4733%
UNGA	1.0015%	7.4022%	15.5572	228.34%	-23.3901%	54.4068%
XPRS	-5.7808%	4.8701%	6.2061	14.25%	-22.5258%	24.4401%

TABLE C.2: Summary of the descriptive statistics for monthly returns: 1998 - 2016